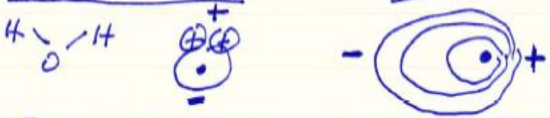
Electrostatics and Dielectrics

(Chapter 6) — a quick summary because you studied this in PHY 481.

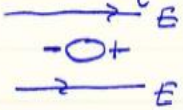
<u>Polarization $\vec{P}(\vec{x})$</u>

All matter is atomic (or molecular) and atoms and molecules contain charged particles. The net charge is 0, but there may be <u>polarization</u>

<u>Polar molecule</u>   <u>Atom</u>

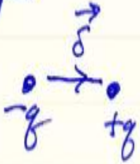


The dipole moment $\vec{p}$ ~~points~~ from − to +;
in $\vec{E}$, $\vec{p}$ experiences a torque toward alignment



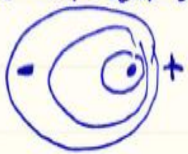
Define "dipole moment"

- For 2 charges



$$\vec{p} = q\vec{\delta}$$

- For a continuous distribution



$$\vec{p} = \int \vec{x}\, \rho(\vec{x})\, d^3x$$

In the equilibrium state, $\vec{p}$ <u>aligns</u> with the local field;

$$\langle \vec{p} \rangle = \alpha\, \vec{E}(\vec{x})$$

average moment in a neighborhood of $\vec{x}$.

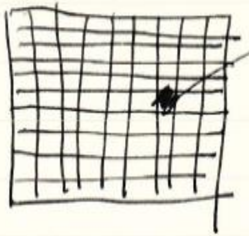
The Polarization Field $\vec{P}(\vec{x})$

Consider a sample of dielectric material.
Subdivide the sample into many small subvolumes.
But atoms are so tiny that each subvolume contains many atoms.

Define $\vec{P}(\vec{x}) = \dfrac{1}{\delta V} \displaystyle\sum_{i=1}^{\delta N} \vec{p}_i$;

or $\vec{P}(\vec{x}) = \dfrac{\delta N}{\delta V} \langle \vec{p} \rangle$;

i.e., $\vec{P}(\vec{x})$ = polarization = dipole moment density



subvolume δV at $\vec{x}$ contains δN atoms

"Free charge" and "Bound charge"

$$\vec{\rho}(\vec{x}) = \rho_{\text{Free}}(\vec{x}) + \rho_{\text{Bound}}(\vec{x})$$

Charge that has been added to the dielectric

charge that belongs to the atoms of the dielectric

We can relate $\rho_{\text{Bound}}(\vec{x})$ to $\vec{P}(\vec{x})$.

Theorem

$\rho_B(\vec{x}) = -\nabla\cdot\vec{P}$ for $\vec{x}$ inside the dielectric

$\sigma_B(\vec{x}) = \hat{n}\cdot\vec{P}$ for $\vec{x}$ on the surface of the dielectric

Proof

Recall the electrostatic potential $V_d(\vec{x})$ of an electric dipole $(\vec{E} = -\nabla V_d)$.

$\Rightarrow$

$$V(\vec{x}) = \int_{\text{sample}} \frac{d\vec{p}\cdot(\vec{x}-\vec{x}')}{4\pi\epsilon_0|\vec{x}-\vec{x}'|^3}$$ from the bound charges

$$d\vec{p} = \vec{P}(\vec{x}')\,d^3x'$$

$$(\vec{x}-\vec{x}')/|\vec{x}-\vec{x}'|^3 = \nabla'\frac{1}{|\vec{x}-\vec{x}'|}$$

now integrate by parts

$$\vec{P}\cdot\nabla'\frac{1}{R} = \nabla'\cdot\left(\frac{\vec{P}}{R}\right) - \frac{\nabla'\cdot\vec{P}}{R}$$

$$V(\vec{x}) = \oint_{\text{surface}} \frac{\hat{n}'\cdot\vec{P}(\vec{x}')}{|\vec{x}-\vec{x}'|}\,dA' + \int_{\text{sample}} \frac{-\nabla'\cdot\vec{P}(\vec{x}')}{|\vec{x}-\vec{x}'|}\,d^3x' \quad /4\pi\epsilon_0$$

$$= \oint_S \frac{\sigma_B\,dA'}{|\vec{x}-\vec{x}'|} + \int_V \frac{\rho_B(\vec{x}')\,d^3x'}{|\vec{x}-\vec{x}'|} \quad /4\pi\epsilon_0$$

I.e., V is exactly the same as for surface charge density $\sigma_B = \hat{n}'\cdot\vec{P}(\vec{x}')$ and volume charge density $\rho_B = -\nabla'\cdot\vec{P}(\vec{x}')$

Q.E.D

The Displacement Field $\vec{D}(\vec{x})$

Define $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$.

Then

$$\nabla\cdot\vec{D} = \epsilon_0 \nabla\cdot\vec{E} + \nabla\cdot\vec{P}$$
$$= \rho \quad - \rho_B$$
$$= \rho_F$$

$$\boxed{\nabla\cdot\vec{D} = \rho_{Free} \quad \text{and} \quad \vec{D} = \epsilon_0\vec{E} + \vec{P}.}$$

Consequences

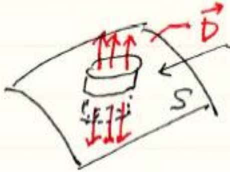
- Gauss's Law

$$\oint_S \vec{D}\cdot d\vec{A} = Q_{free}$$ /by Gauss's theorem/

- Boundary Conditions (important!)

D_n is continuous across a surface if $\sigma_{Free} = 0$; otherwise

$$D_n^{(+)} - D_n^{(-)} = \sigma_{Free}.$$



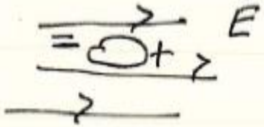
small cylindrical volume in limit height $\to 0$

$$\oint \vec{D}\cdot d\vec{A} = D_n^{(+)}A - D_n^{(-)}A = Q_{Free}$$
$$= \sigma_{Free}A$$

$$\therefore D_n^{(+)} - D_n^{(-)} = \sigma_{Free}$$

Isotropic Linear Dielectrics

Many isotropic insulators are "linear dielectrics"; i.e., there is a linear relationship between polarization and the electric field



$$\langle \vec{p} \rangle \propto \vec{E}(\vec{x})$$

Glass, water, gases and liquids, etc.

Material parameters for linear dielectrics

- χ_e electric susceptibility
  $\vec{P} = \chi_e \epsilon_0 \vec{E}$ ← Note: $\chi_e > 0$
- ϵ permittivity
  $\vec{D} = \epsilon \vec{E}$ ← Note: $\epsilon > \epsilon_0$

Note that $\epsilon = \epsilon_0 (1 + \chi_e)$ because

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = (1 + \chi_e) \epsilon_0 \vec{E} = \epsilon \vec{E}$$

- K dielectric constant
  $K = \epsilon / \epsilon_0$ ← Note: $K > 1$

$$K = \frac{\epsilon}{\epsilon_0} = 1 + \chi_e$$

Any of these material constants can be used to specify the dielectric properties of the material.