

## Magnetism and Matter (Chap. 9)

The long and interesting history of the science of magnetism...

### Pre-History

- Stone Age – Tools were made of stone
- Bronze Age (3500 to 1100 BC) – Tools of bronze (Cu – Sn alloy)
- Iron Age (1200 to 600 BC) – Tools of iron (Fe)

### Ancient History

- Thales of Miletus (624 – 546 BC) described a force in iron ores from Magnesia in Greece; the two main forms of iron ore are hematite and magnetite.



### Middle Ages

- Lodestones were used to magnetize compass needles. (Europe or China?)  
William Gilbert published *De Magnete* in the year 1600.

### 19th Century

- Michael Faraday found that most materials are diamagnetic (1845).

## Magnetic Materials Chapter 9

The atoms have magnetic dipole moments. There are 2 possibilities:

- permanent dipoles — paramagnetic materials
- induced dipoles — diamagnetic materials

### Notations

$\vec{m}$  = <sup>magnetic</sup><sub>1</sub> dipole moment of an atom

$\vec{M}(\vec{x})$  = "magnetization" = moment density

$\vec{M}(\vec{x}) = n(\vec{x}) \langle \vec{m} \rangle$

$n$   
atomic  
density

average moment in a  
small neighbourhood  
of  $\vec{x}$ .

*We will study three kinds of magnetism*

### *Ferromagnetism*

- Fe, Co, Ni; some alloys
- Microscopic domains have nonzero magnetization.
- A macroscopic sample of ferromagnetic material may have a permanent magnetization.

### *Paramagnetism*

- Placed in a magnetic field, the material has magnetization in the same direction as  $B$ .
- A paramagnetic material is attracted (weakly) to a magnet pole.

### *Diamagnetism*

- Placed in a magnetic field, the material has magnetization in the direction opposite to  $B$ .
- A diamagnetic material is repelled (weakly) from a magnet pole.

(There are two other, more subtle, kinds of magnetism in matter:  
antiferromagnetism and ferrimagnetism.)

H

■ Ferromagnetic   ■ Antiferromagnetic

He

3	4
Li	Be
11	12
Na	Mg

□ Paramagnetic   ■ Diamagnetic

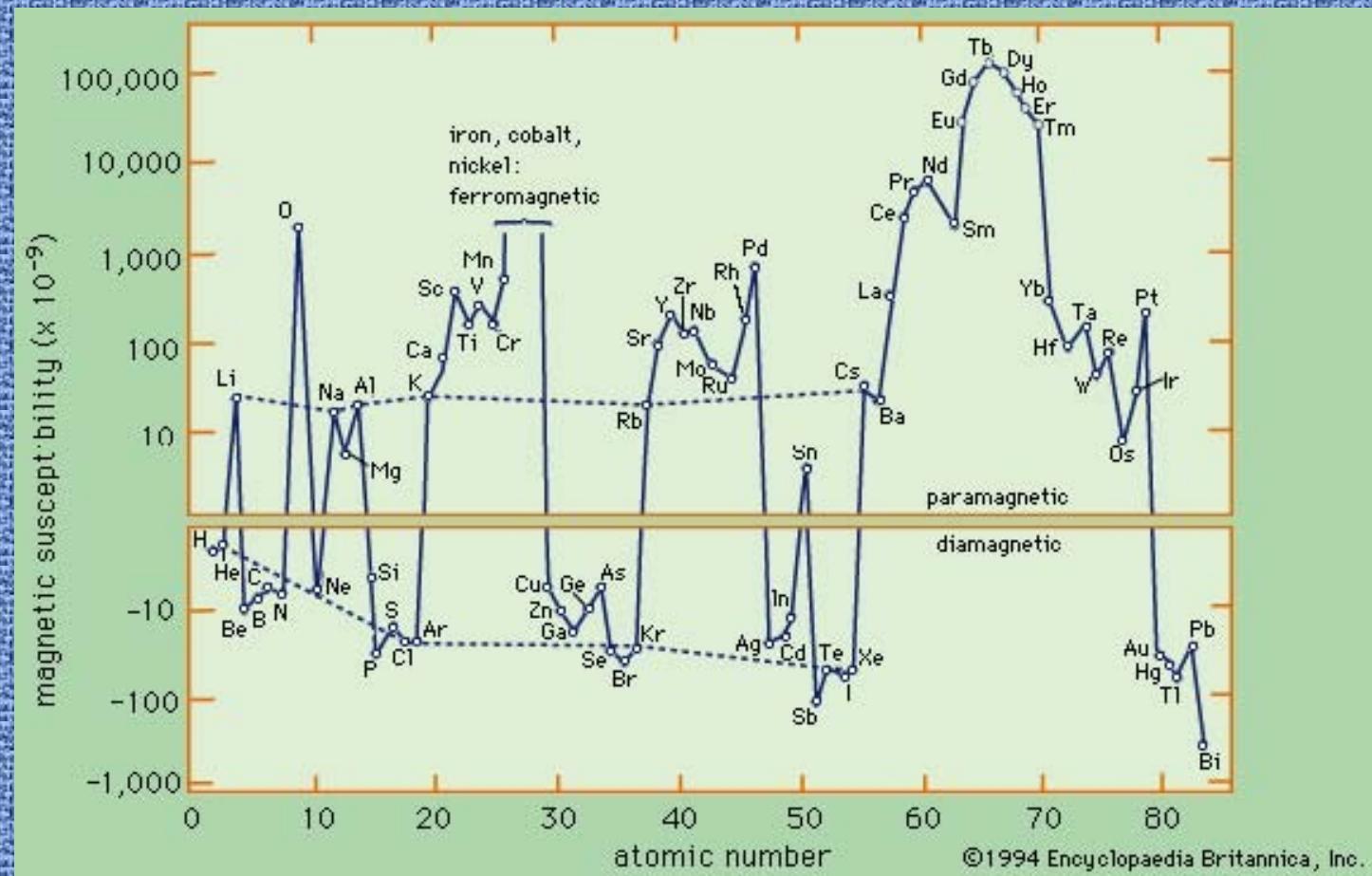
5	6	7	8	9	10
B	C	N	O	F	Ne
13	14	15	16	17	18
Al	Si	P	S	Cl	Ar

19	20
K	Ca
37	38

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe

55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	Ac															

58	59	60	61	62	63	64	65	66	67	68	69	70	71
Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu

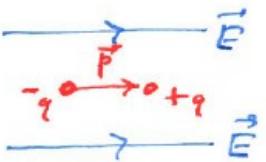


## Atomic dipoles

§ Electric dipole :

$$\vec{p} = q \vec{d}$$

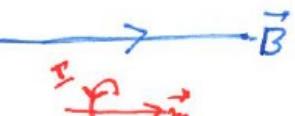
$$\text{torque} = \vec{p} \times \vec{E} ; \text{energy} = -\vec{p} \cdot \vec{E}$$



§ Magnetic dipole :

$$\vec{m} = I a \hat{n}$$

$$\text{torque} = \vec{m} \times \vec{B} ; \text{energy} = -\vec{m} \cdot \vec{B}$$

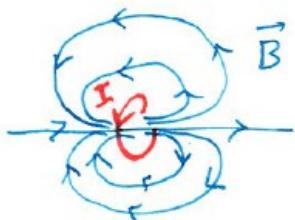
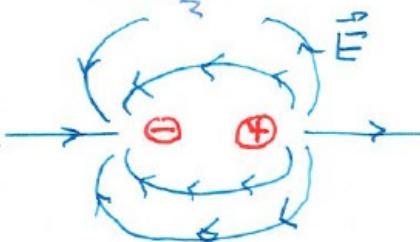


Similarities and differences

Electric dipole : two poles

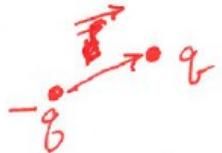
Magnetic dipole : current loop

The asymptotic fields have the same form



## Review of electric and magnetic dipoles

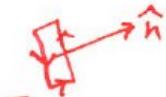
### Electric



$$\vec{p} = q \vec{r}$$

$$\vec{p} = \int \vec{x} \rho(\vec{x}) d^3x$$

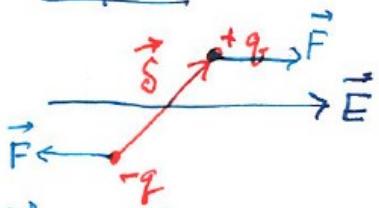
### Magnetic



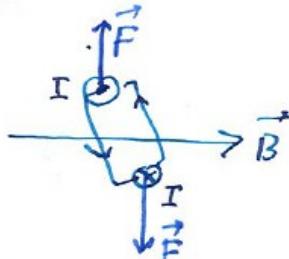
$$\vec{m} = IA \hat{n}$$

$$\vec{m} = \frac{1}{2} \int \vec{x} \times \vec{j}(\vec{x}) d^3x$$

### Torques



$$\begin{aligned}\vec{N} &= \frac{\vec{p}}{2} \times q\vec{E} + \left(-\frac{\vec{p}}{2}\right) \times (-q\vec{E}) \\ &= q\vec{p} \times \vec{E} = \vec{p} \times \vec{E}\end{aligned}$$

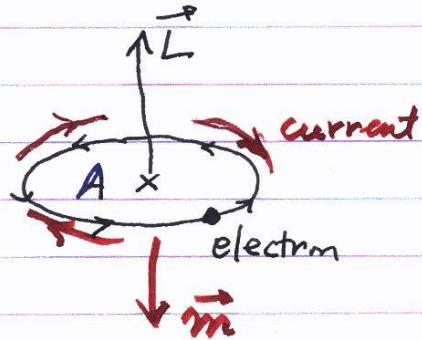


$$\begin{aligned}\vec{N} &= \frac{\vec{p}}{2} \times (I\vec{l} \times \vec{B}) + \left(-\frac{\vec{p}}{2}\right) \times (-I\vec{l} \times \vec{B}) \\ &= I\ell^2 \hat{n} \times \vec{B} = \vec{m} \times \vec{B}\end{aligned}$$

The torque acts in the direction toward alignment with the field.

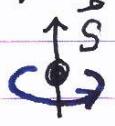
# Magnetization and Bound Current 9.2/1

Magnetic moment ( $\vec{m}$ ) and magnetization ( $\vec{M}$ )



$$\vec{m} = IA \hat{n}$$

$$\vec{m} = \frac{1}{2} \int \vec{x} \times \vec{j}(\vec{x}) d^3x$$

It's a bit more complicated, because an electron also has spin  and a

corresponding spin magnetic moment



$$\vec{m} = \frac{-e}{2m} (\vec{L} + 2\vec{S})$$

Check units:  $C \text{ kg m/s m/kg} = A \text{ m}^2$

## The essential idea for magnetism in matter

The magnetic field produced by a magnetized object is the same as that of a volume current density  $\vec{J}_{\text{Bound}}(\vec{x})$  and surface current density  $\vec{K}_{\text{Bound}}(\vec{x})$  given by

$$\vec{J}_B(\vec{x}) = \nabla \times \vec{M} \quad \text{and} \quad \vec{K}_B(\vec{x}) = \vec{M}(\vec{x}) \times \hat{n}$$

( $\vec{x}$  on the surface)

## Magnetization and Bound Current 9.2 / 2

Theorem A sample of matter with magnetization

$\vec{M}(\vec{x})$  has an associated magnetic field  $\vec{B}_{\text{Matter}}(\vec{x})$

which is the same as  $\vec{B}$  for a volume current density

$\vec{J}_{\text{Bound}}(\vec{x})$  and surface current density  $K_B(\vec{x})$ ,

$$\vec{J}_B(\vec{x}) = \nabla \times \vec{M} \quad \text{and} \quad \vec{K}_B(\vec{x}) = \vec{M}(\vec{x}) \times \hat{n}$$

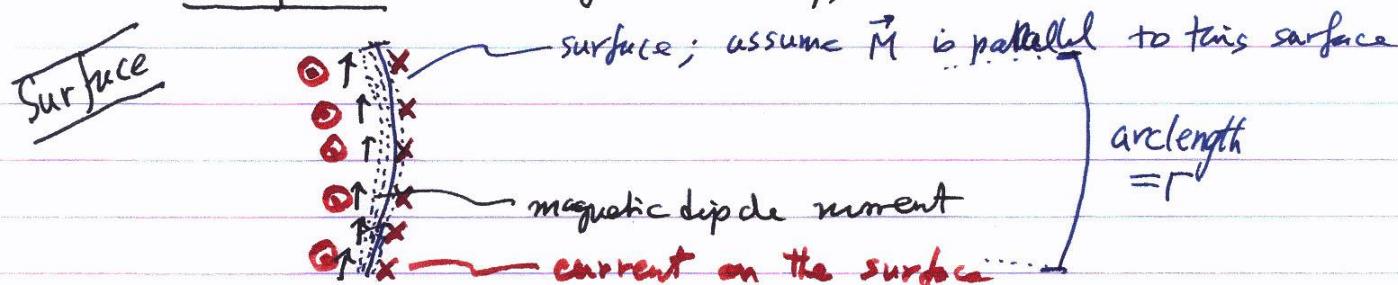
(in the volume)

(on the surface)

Proof #1 See Equations 9-7 to 9-14.

/ by integration /

Proof #2 (more geometrically)



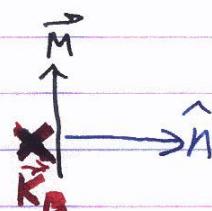
$$R_B \cdot \Gamma = \frac{\delta Q}{\delta t} = \frac{N \cdot I \delta t}{\delta t}$$

$$N = \# \text{ atoms} = n \cdot (A \Gamma)$$

$$\text{Thus } K_B = \frac{n A \Gamma I}{\Gamma} = n I A = M$$

Direction of  $\vec{K}_B$  is same as  $\vec{M} \times \hat{n}$

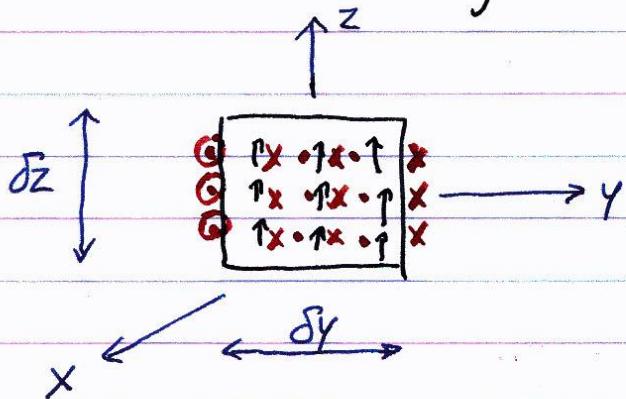
$$\vec{K}_B = \vec{M} \times \hat{n} \quad \underline{\text{Q.E.D.}}$$



$$\text{Check units : } \frac{A}{m} \text{ and } \frac{Am^2}{m^3} \quad \checkmark$$

Volume

Consider a small subvolume inside the material. Set up a coordinate system with the  $Z$  direction parallel to  $\vec{M}$



Current density  $\perp$  to the  $yz$  plane is  $J_{Bx} \hat{i}$

$$J_{Bx} \delta y \delta z = \frac{\delta Q}{\delta t}$$

The sign is tricky.

$$J_{Bx} \delta y \delta z = + (I_{atom N})_{y+\frac{\delta y}{2}} - (I_{atom N})_{y-\frac{\delta y}{2}}$$

$$= (I n A \delta z)_{y+\frac{\delta y}{2}} - (I n A \delta z)_{y-\frac{\delta y}{2}}$$

$$= M_z (y + \frac{\delta y}{2}) \delta z - M_z (y - \frac{\delta y}{2}) \delta z$$

$$= \frac{\partial M_z}{\partial y} \delta y \delta z$$

$$J_{Bx} = \frac{\partial M_z}{\partial y} \quad (\text{assuming } \vec{M} \text{ to be in the } z \text{ direction})$$

$$\vec{J}_B = \nabla \times \vec{M} \quad \underline{\text{Q.E.D.}}$$