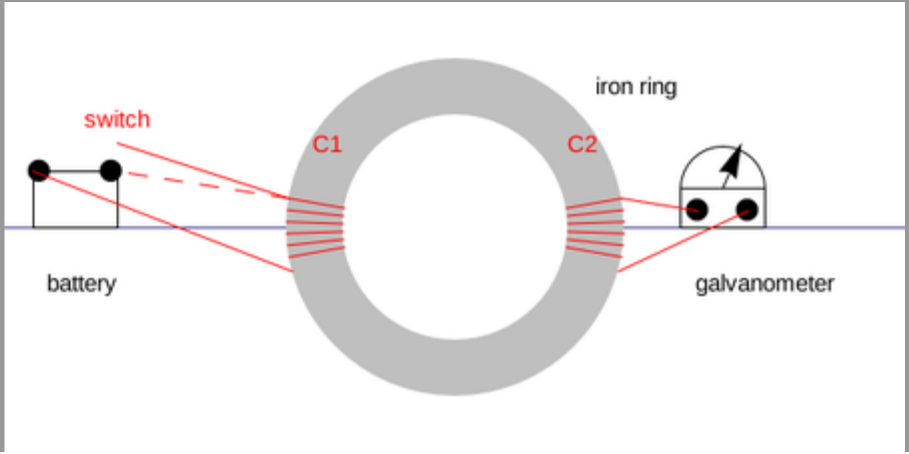
Electromagnetic Induction (Chap. 10)

E. M. induction was discovered in 1831,
by Michael Faraday (London, England)
and by Joseph Henry (Albany, NY), independently.

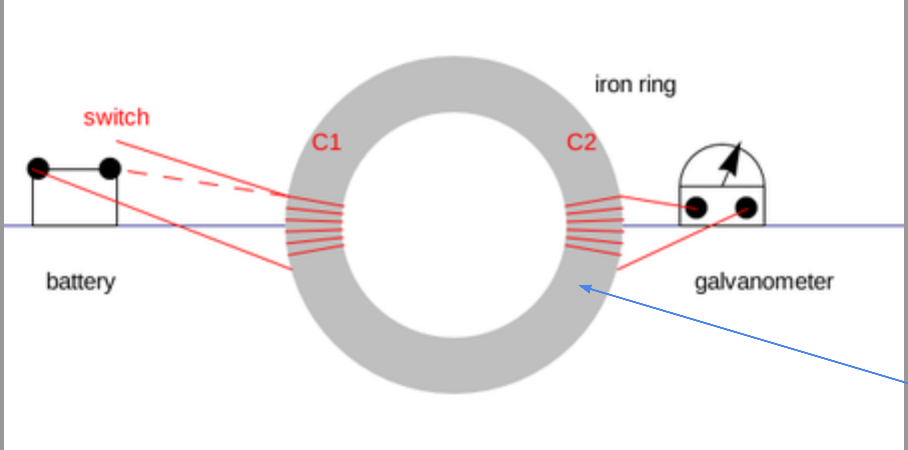
Faraday's discovery (one of his many demonstrations)



Close the switch ⟹ a temporary deflection of the galvanometer needle
Open the switch ⟹ a temporary deflection of the galvanometer needle
When the magnetic field CHANGES, there is an induced electric current in coil C2.

But the induced current is a secondary effect.

Electromagnetic Induction: When the magnetic field **B** *changes* (by closing or opening the switch) then there is an induced *electric field* **E** that curls around the change of **B**.



A time-dependent magnetic field **B**(t) exists around the ring. As it changes (in time) an electric field curls around the ring.

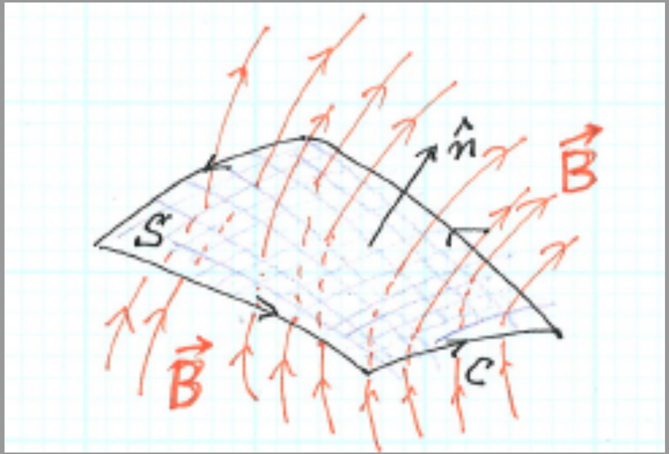
The induced electric field is the primary effect.
If there is a conductor present, then **E** will drive a current in the conductor; but the current is a secondary effect.

Two additional concepts that are used in the theory of Electromagnetic Induction

- Magnetic Flux
- Electromotive Force

Magnetic Flux

$$\Phi_M = \int_S \vec{B} \cdot d\vec{A} \qquad ; \; d\vec{A} = \hat{n}\, dA$$



The flux Φ_M can ***change*** in several ways …

- d**B** /dt ; electromagnetic induction; curl **E** = -d**B**/dt
- S could move or change shape ; motional e.m.f.

In either case, there is an e.m.f. around the boundary curve C.

E.M.F. = Electromotive Force

EMF is a tricky concept.

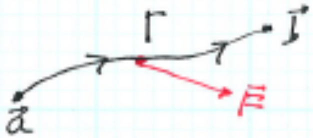
It does not have the units of force; it is Work/Charge = [N*m/C] = [Volts]

Three concepts that can be confused:

EMF, potential difference, voltage

EMF ($\mathcal{E}$)

In general, define $\mathcal{E}$ = work per unit charge done by a force $\vec{F}$ when a test charge moves along a curve Γ.



$$\mathcal{E}(\Gamma) = \int_\Gamma \frac{\vec{F}\cdot d\vec{l}}{q}$$

(✱) For an electrostatic force, $\vec{F} = q\vec{E}$; and $\nabla\times\vec{E} = 0 \Rightarrow \vec{E} = -\nabla V$ when $V(\vec{x})$ = electrostatic potential. Then

$$\mathcal{E}(\Gamma) = \int_\Gamma (-\nabla V)\cdot d\vec{l} = -\left[V(\vec{b}) - V(\vec{a})\right].$$

(Independent of path from $\vec{a}$ to $\vec{b}$)

For a closed curve C, $\mathcal{E}(C) = 0$.

(✱) For any conservative force (including all electrostatic forces) $\mathcal{E}(C) = 0$. for a closed curve.

(✱) For electromagnetic induction, $\nabla\times\vec{E} \neq 0$; then $\oint_C \vec{E}\cdot d\vec{l}$ = nonzero emf around the curve.

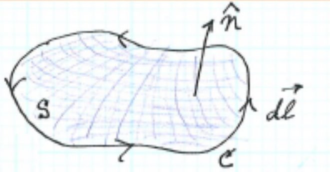
Faraday's Law

put into mathematical terms
by James Clerk Maxwell

(1) $$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Meaning: if **B** changes in time, then there must be an electric field **E** that curls around the change of **B**.

(2) Consider a surface *S* with boundary curve *C*. (The directions of the normal vector **n** of *S*, and the circulation around *C*, are related by the right hand rule.) Then,



$$\int_S \nabla \times \vec{E} \cdot d\vec{A} = \oint_C \vec{E} \cdot d\vec{\ell} \quad \text{(Stokes's theorem)}$$

$$\hookrightarrow -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{A} \quad \text{(Faraday's law)}$$

(3) EMF due to the electric field in electromagnetic induction

$$\mathcal{E} = -\frac{d\Phi_M}{dt}$$

$$\mathcal{E} = \text{emf} = \oint_C \vec{E} \cdot d\vec{\ell}$$

$$\Phi_M = \text{magnetic flux} = \int_S \vec{B} \cdot d\vec{A}$$