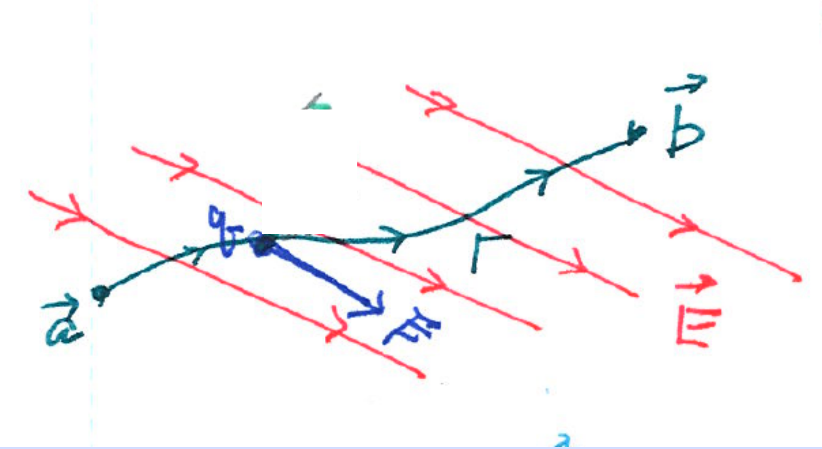
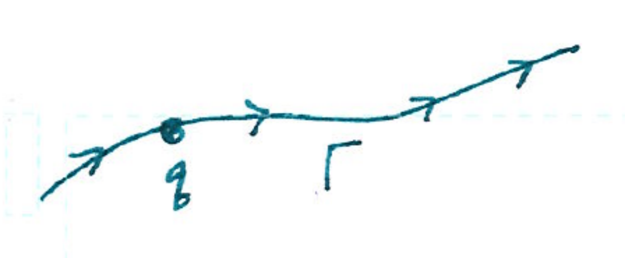
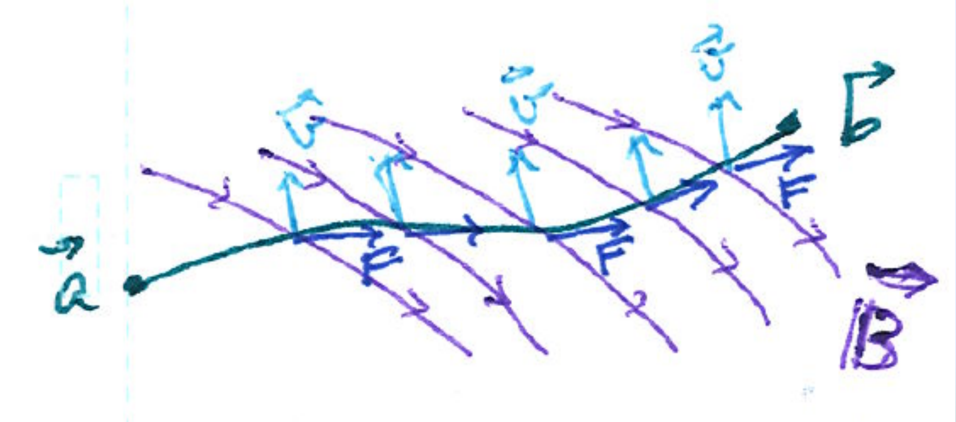
Recall the definition of EMF (Electromotive Force)





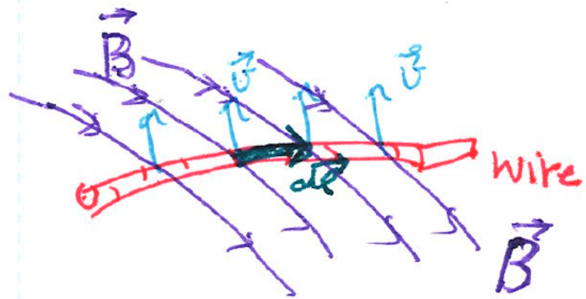


Motional EMF

If a conducting wire moves across a magnetic field, then there is an EMF along the wire,

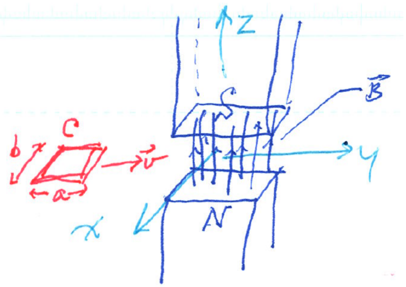
$$\text{Define EMF} = \int_{\text{wire}} \left(\vec{v} \times \vec{B}\right) \cdot d\vec{\ell}.$$

where **v** = the velocity of the wire segment **dl.**



The EMF will drive a current I = EMF /R if the wire is part of a complete conducting circuit (*just like a battery would*).

Example



C is a conducting loop, which will be pulled through the gap between the magnet poles. We have

$$\vec{B} = B_z \hat{k}; \qquad \text{and} \qquad \vec{v} = v_y \hat{j}$$

As the circuit ***C*** enters the field, the counterclockwise EMF is

$$\text{EMF}_1 = \int_{\text{length}\,b} \frac{q v_y B_z \hat{i}}{q} \cdot d\vec{\ell}$$

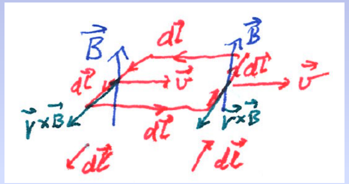
$$\text{EMF}_1 = -v_y B_z b \qquad \text{counterclockwise}$$

Check units: (m/s) T m = V ; OK

1/ Entering the field,
the counterclockwise EMF is

$$EMF_1 = - v_y B_z b$$

2/ While the circuit C is totally immersed in the magnetic field, the combined EMF is 0:

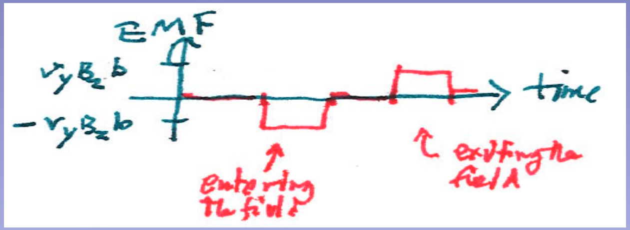


$$\int_{\text{left side}} \vec{v} \times \vec{B} \cdot d\ell + \int_{\text{right side}} \vec{v} \times \vec{B} \cdot d\ell = 0$$

3/ As the circuit exits the field, the counterclockwise EMF is

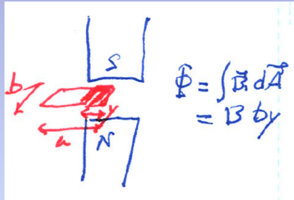
$$EMF_2 = + v_y B_z b$$

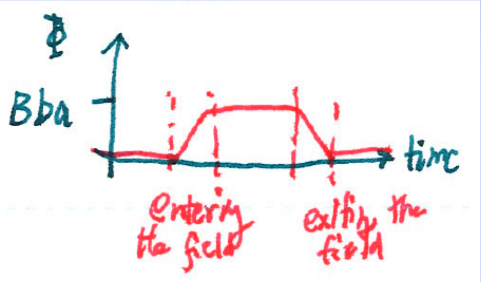
(from the EMF along the right hand side)



Now compare the EMF to the *magnetic flux* through the loop

$$\Phi = B_z A = B_z b y$$

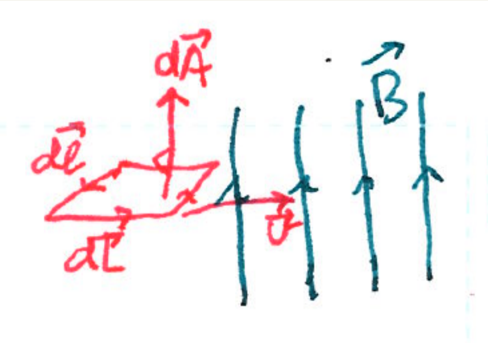




Comparing the two graphs, see that (dy/dt = v_y)

$$\text{EMF} = -\frac{d\Phi}{dt}$$

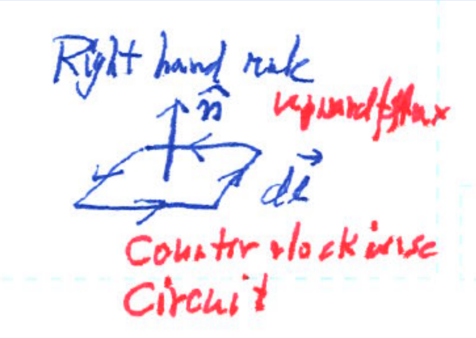
Summary



$$\text{EMF} = -\frac{d\Phi}{dt}$$

$$\oint_C \frac{\vec{F}}{q} \cdot d\vec{\ell} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A}$$

The directions of ***dl*** *and* ***dA*** *are related by the right-hand rule.*



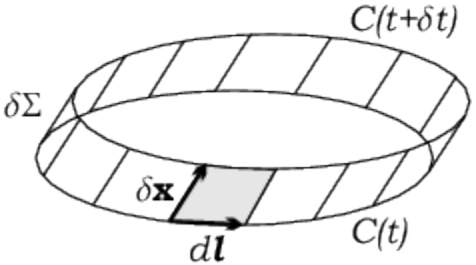
General Theorem

If a rigid loop C moves in a magnetic field **B,** then the EMF around C is

$$\mathrm{EMF} = -\frac{d\Phi}{dt}$$

where Φ = the flux of B through any surface bounded by C.

Proof



For the closed surface

S(t+dt) U S(t) U "ribbon" we have

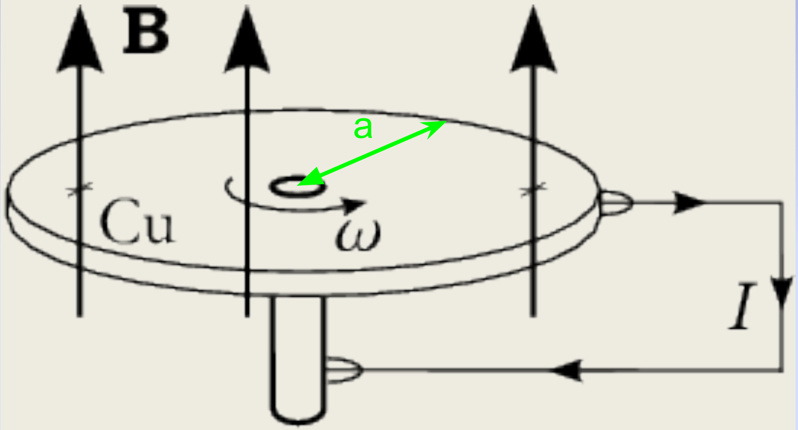
$$\oint_{\text{outward flux}} \vec{B} \cdot d\vec{A} = 0$$

$$\Phi_{\hat{n}}\left[S(t+dt)\right] - \Phi_{\hat{n}}\left[S(t)\right] + \int_{\text{ribbon}} \vec{B} \cdot d\vec{A} = 0$$

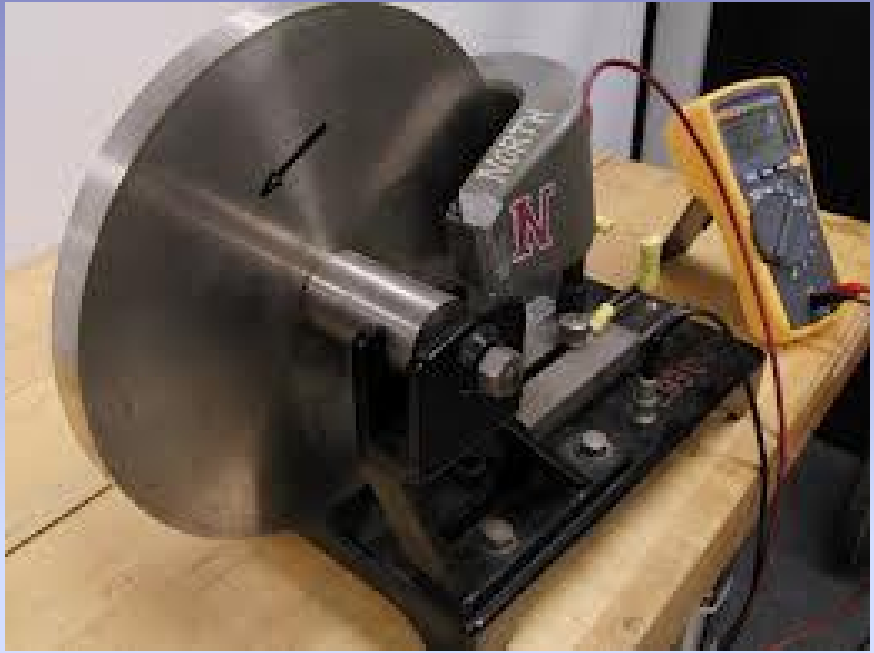
$$\begin{aligned} \frac{\delta\Phi}{\delta t} &= -\frac{1}{\delta t}\int_{\text{ribbon}} \vec{B} \cdot \left(d\vec{\ell} \times \vec{v}\delta t\right) \\ &= -\oint_C \left(\vec{v} \times \vec{B}\right) \cdot d\vec{\ell} \\ &= -\mathrm{EMF} \end{aligned}$$

Q. E. D.

The Faraday Disk Generator



The first electric generator (Faraday, 1831); it creates a DC voltage from center to edge.



$$\mathrm{EMF} = \frac{1}{2}\omega B a^2$$

Exercise : Put in some numbers.

Some crack-pot inventors have claimed that they can get more energy ***out*** of a disk generator than they put ***in***.

These “Free Energy Devices” are hoaxes.