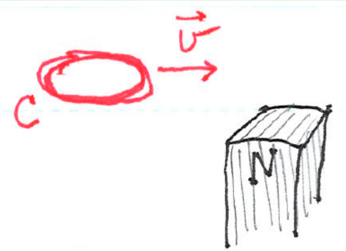
Electromagnetic Induction

(1) Motional EMF



- **B** is constant.
- The magnetic flux changes in time, because S moves.

$$\Phi_M = \int_S \vec{B} \cdot d\vec{A}$$

- There is EMF around C, because of the magnetic force q **v x B**.

$$\mathrm{EMF}(C) = -d\Phi/dt$$

I*n this frame of reference there is no electric field.*

(2) Electromagnetic Induction

- **B** changes in time.
- S does not move but the flux through S changes in time. By the principle of relativity, we should expect EMF around C; EMF = -dΦ/dt.

I*n this frame of reference there is an induced electric field;*

$$\begin{aligned} \mathrm{EMF} &= \oint_C \vec{E} \cdot d\vec{\ell} = \int_S \nabla \times \vec{E} \cdot d\vec{A} \\ &= -d\Phi/dt = \int (-\partial \vec{B}/\partial t) \cdot d\vec{A}; \end{aligned}$$

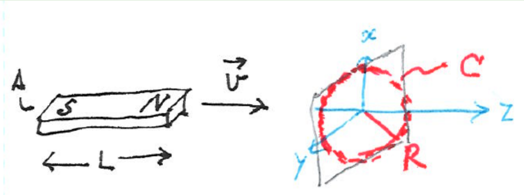
$$\nabla \times \vec{E} = -\partial \vec{B}/\partial t.$$

Faraday's Law

$$\nabla \times \vec{E} = -\partial \vec{B}/\partial t.$$

A magnetic field **B(x**,t) that changes in time produces an electric field **E(x**, t) that curls around the change of **B**.

Example (qualitative)

As the 

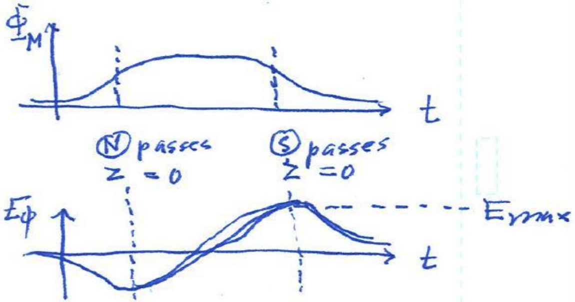
increasing z, what is the electric field at points on the circle C (at z = 0)?

By symmetry, $\vec{E} = E_\phi \hat{\phi}$

$$\oint_C \vec{E}\cdot d\vec{l} = E_\phi \cdot 2\pi R$$

$$= -\frac{d\Phi_m}{dt}$$

$$E_\phi(R,t) = \frac{-1}{2\pi R}\frac{d\Phi_m}{dt}$$



Suppose $B_{end} = 0.5$ T

and $L = 5$ cm, $v = 5$ cm/s, $A = 9$ cm^2

Estimate E_{max}

$$E_{max} \approx \frac{1}{2\pi R}\frac{BA}{\frac{1}{2}L/v}$$

$$= \frac{1.5\times10^{-4}\,V}{R}$$

1 T = 10^{-4} Vs/cm^2

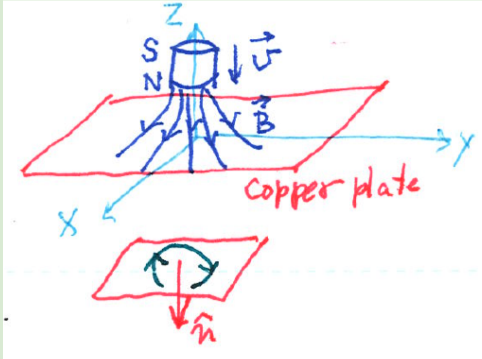
a rather small effect

Lenz's Law

If a conductor is present then the direction of the induced current *opposes the change of flux*.

Example

-- The magnet is above the conductor, moving *downward.*
-- Calculate the *clockwise* EMF.
-- Define **n** to be the downward normal vector.



Is the direction of the EMF in agreement with Lenz's Law?

clockwise EMF $= -\frac{d\Phi_M}{dt}$

$\frac{d\Phi_M}{dt}$ is positive (downward flux)

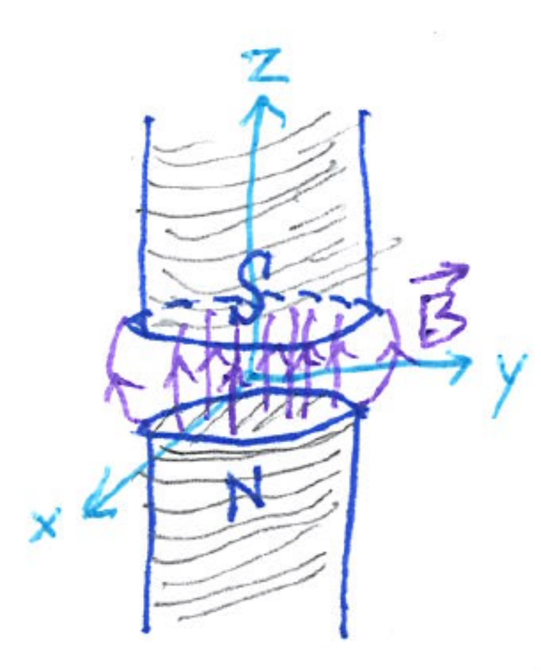
∴ clockwise EMF is negative.

The currents induced in the copper plate ("eddy currents") are ***counterclockwise***.

By Ampere's law, the eddy currents create a magnetic field directed upward (*right hand rule again!*). This opposes the change of flux.

<u>Example</u> (quantitative)

Consider two poles of an electromagnet. We can change the magnetic field by varying the current (not shown). Determine the electric field in the gap between the poles.



In the gap, $\vec{B}(\vec{x},t) \approx B_0(t)\hat{k}$.

By symmetry, $\vec{E}(\vec{x},t) = E_\phi(r,t)\hat{\phi}$.

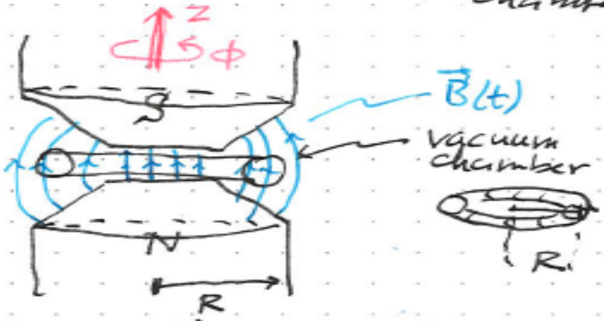
$$\oint_C \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi}{dt}$$

$$E_\phi \, 2\pi r = -\frac{dB_0}{dt}\,\pi r^2$$

$$E_\phi(r,t) = -\frac{r}{2}\frac{dB_0}{dt}$$

The Betatron

↳ an electron accelerator that uses the induced field $E_\phi \hat{\phi}$ to raise the electron energy in a circular vacuum chamber.



Increase $\vec{B}$ $\Rightarrow$ $E_\phi \hat{\phi}$ which raises the electron energy

BUT, we must keep the radius of the electron orbit constant! ($r = R$)

The betatron condition …

(1) The circular motion implies

$$\frac{mv^2}{R} = evB(R); \quad \text{or} \quad p_\phi = eRB(R)$$

(2) The electric force implies

$$\frac{dp_\phi}{dt} = eE_\phi(R) = \frac{e}{2\pi R}\frac{d\Phi_M}{dt}$$

$$\text{so} \quad p_\phi = \frac{e}{2\pi R}\Phi_M$$

Thus we must require

$$B(R) = \frac{\Phi_M}{2\pi R^2}; \quad \text{or} \quad B(R) = \frac{1}{2}B_{\text{avg.}}$$

$$\text{where} \quad B_{\text{avg.}} = \frac{1}{A}\int_S \vec{B}\cdot d\vec{A}$$

which is arranged by designing the correct shape of the magnet poles.

Donald Kerst and the first betatron (1940)

The second betatron at the University of Illinois (1950) produced 300 MeV electrons.