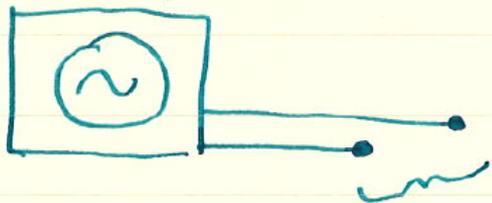


## AC circuits

We have AC power supplies

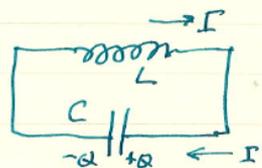


$$\text{EMF} = V_0 \cos \omega t$$

(Wednesday's lecture)

Now consider AC circuits

① An LC circuit is a harmonic oscillator



$$\mathcal{E}MF = \frac{Q}{C}$$

↳

$$\hookrightarrow -\frac{d\Phi}{dt} = -L \frac{dI}{dt}$$

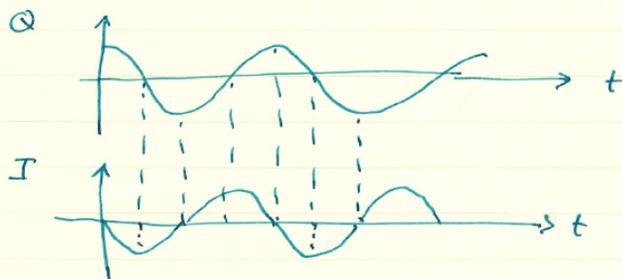
and  $\frac{dQ}{dt} = I$ .

$$L \frac{d^2Q}{dt^2} = -\frac{1}{C} Q$$

$$\frac{d^2Q}{dt^2} = -\omega_0^2 Q \quad \text{where } \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q(t) = Q_0 e^{i\omega_0 t} \quad (\text{Re implied})$$

$$I(t) = i\omega_0 Q_0 e^{i\omega_0 t} \quad (\text{Re implied})$$

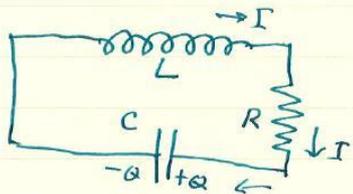


The period of oscillation is  $T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{LC}$ .

$Q$  and  $I$  are 90 degrees out of phase.

Energy oscillates between  $Q^2/2C$  (electric) and  $\frac{1}{2}LI^2$  (magnetic).

② An LRC circuit is a damped oscillator



$$\text{EMF} - IR = \frac{Q}{C}$$

$$\hookrightarrow -L \frac{dI}{dt}$$

and  $dQ/dt = I$ .

$$L \frac{d^2Q}{dt^2} + R \frac{dQ}{dt} + \frac{Q}{C} = 0$$

Try  $Q = e^{pt} \Rightarrow$

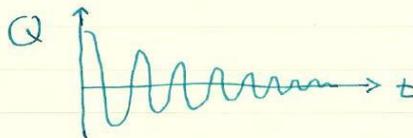
$$Lp^2 + Rp + \frac{Q}{C} = 0$$

$$p = -\frac{R}{2L} \pm i\sqrt{\omega_0^2 - (R/2L)^2}$$

So

$$Q = e^{-\gamma t} e^{i\omega t} \quad (\text{Re implied})$$

where  $\gamma = R/2L$  ;  $\omega = \sqrt{\omega_0^2 - \gamma^2}$  ;  $\omega_0 = \frac{1}{\sqrt{LC}}$



Energy is dissipated  
in the resistance.

Current  $I = \frac{dQ}{dt} = (-\gamma + i\omega) e^{-\gamma t} e^{i\omega t} \quad (\text{Re})$

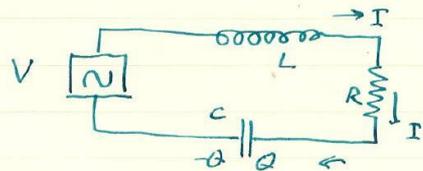
$$= -\gamma \cos \omega t e^{-\gamma t} + \omega \cos(\omega t + \pi/2) e^{-\gamma t}$$

$$= [-\gamma \cos \omega t + \omega \sin \omega t] e^{-\gamma t} \quad \leftarrow \boxed{i = e^{i\pi/2}}$$

$$= -\Omega \sin(\omega t + \phi) e^{-\gamma t} \quad \text{where } \begin{cases} \omega = \Omega \cos \phi \\ \gamma = \Omega \sin \phi \end{cases}$$

↑  
"phase shift"

### ③ The harmonically driven LRC circuit



$$\frac{dQ}{dt} = I$$

$$V = V_0 e^{i\omega t} \quad (\text{Re implied})$$
$$V = V_0 \cos \omega t$$

$$V - L \frac{dI}{dt} - IR = \frac{Q}{C}$$

$$V_0 e^{i\omega t} = L \frac{dI}{dt} + RI + \frac{Q}{C} \quad (\text{Re implied})$$

### Impedance

Write  $I = \frac{V}{Z}$  where  $Z$  = the "complex impedance"

(analogous to  $I = V/R$   
in a DC circuit.)

$$\text{So } I = \frac{V_0}{Z} e^{i\omega t} \quad \text{and } Q = \frac{V}{i\omega Z} e^{i\omega t}$$

$$1 = \frac{1}{Z} \left\{ i\omega L + R + \frac{1}{i\omega C} \right\}$$

$$Z = R + i\left(\omega L - \frac{1}{\omega C}\right)$$

$$\text{Or, } Z = |Z| e^{i\phi}$$

$$\text{where } |Z| = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$\text{and } \tan \phi = \frac{\omega L - 1/\omega C}{R}$$

## The current in the driven LRC circuit

$$I(t) = \frac{V_0}{Z} e^{i\omega t} \quad (\text{Re implied})$$

$$I(t) = \frac{V_0}{|Z|} \cos(\omega t - \varphi)$$

### Amplitude and RMS current

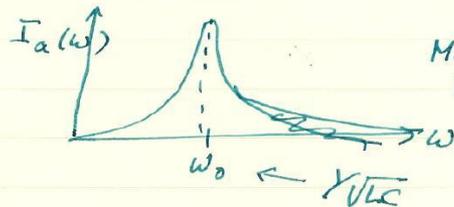
$$I(t) = I_a \cos(\omega t - \varphi)$$

$$I_{\text{RMS}}^2 = \frac{1}{T} \int_0^T I^2 dt = I_a^2 / 2$$

$$I_{\text{RMS}} = I_a / \sqrt{2}$$

$$\text{Resonance } I_a = V_0 / \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$\text{Resonance } I_a = V_0 / \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$



Max. amplitude occurs  
if  $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$

$\varphi < 0$     $\varphi = 0$     $\varphi > 0$   
"capacitive"   "inductive"

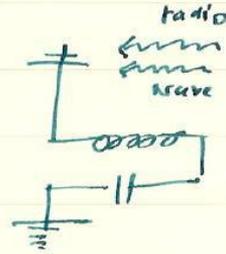
$$\tan \varphi = \frac{\omega L - 1/\omega C}{R}$$

This current  $\frac{V_0}{|Z|} \cos(\omega t - \varphi)$  is the steady state current. There is also a transient current which depends on the initial values of  $I$  and  $Q$ .

$$I(t) = \frac{V_0}{|Z|} \cos(\omega t - \varphi) + e^{-\gamma t} [c_1 \cos \omega t + c_2 \sin \omega t]$$

## Example: A radio receiver

- Replace  $V$  by an antenna.



- Tune  $C$  or  $L$  such that

$$\frac{1}{\sqrt{LC}} = \omega_B$$

where  $\omega_B$  is the broadcast frequency  
(carrier frequency of the transmitter; AM or FM)

- Amplify  $I(t)$  and use the amplified current to drive a loud speaker



- (• Also needs some circuitry to demodulate the AM or FM modulation.)