What does an electromagnetic wave look like in a moving frame of reference?
I.e., given an electromagnetic wave in one inertial frame ***F***, what does it look like in another inertial frame ***F′***?

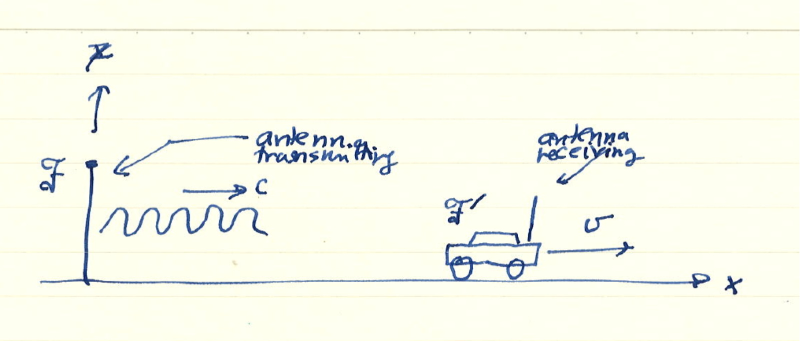
<u>Ref. frame F</u>

$\mathbf{E}(\mathbf{x},t) = E_0 \cos[\, kx - \omega t \,]\, \mathbf{e}_z$

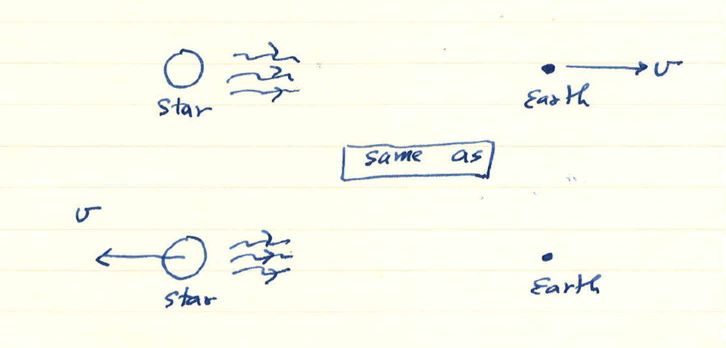
$\mathbf{B}(\mathbf{x},t) = (E_0/c) \cos[\, kx - \omega t \,]\, (-\mathbf{e}_Y)$

Now suppose ***F′*** moves with velocity v $\mathbf{e}_x$ with respect to ***F***. What are the fields as observed in the frame ***F′***?

<u>Example</u>



<u>Example</u>



<u>The red shift in astronomy</u>

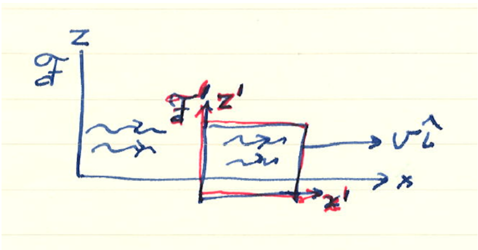
Ref. frame $\boldsymbol{F}$

$$\mathbf{E}(\mathbf{x},t) = E_0 \cos[\, kx - \omega t \,]\, \mathbf{e}_z$$

$$\mathbf{B}(\mathbf{x},t) = (E_0/c) \cos[\, kx - \omega t \,]\, (-\mathbf{e}_Y)$$

Ref. frame $\boldsymbol{F'}$

$\boldsymbol{F'}$ moves with velocity $v\, \mathbf{e}_x$ with respect to $\boldsymbol{F}$.



__coordinates_ _

$t' = \gamma\,(\, t - vx/c^2 \,)$ $\qquad$ $t = \gamma\,(\, t' + vx'/c^2 \,)$

$x' = \gamma\,(\, x - vt \,)$ $\qquad$ $x = \gamma\,(\, x' + vt' \,)$

$y' = y$ $\qquad$ $y = y'$

$z' = z$ $\qquad$ $z = z'$

__fields_ _

$E'_x = E_x \qquad \Rightarrow E'_x = 0$

$E'_y = \gamma\,(\, E_y - v\, B_z \,) \qquad \Rightarrow E'_y = 0$

$E'_z = \gamma\,(\, E_z + v\, B_y \,) \qquad \Rightarrow E'_z =$ (see below)

$$\begin{aligned} E'_z &= \gamma\,(\, E_0 - (v/c)\, E_0 \,) \cos[\, kx - \omega t \,] \\ &= \gamma\,(\, 1 - v/c \,)\, E_0 \\ &\quad .\cos[\, k\,\gamma\,(\, x' + vt' \,) - \omega\,\gamma\,(\, t' + vx'/c^2 \,) \,] \\ &= E'_0 \cos[\, k'x' - \omega' t' \,] \end{aligned}$$

where

$E'_0 = \gamma\,(\, 1 - v/c \,)\, E_0$

$k' = \gamma\,(\, k - \omega\, v/c^2 \,) = \gamma\,(\, 1 - v/c \,)\, k$

$\omega' = \gamma\,(\, \omega - v\, k \,) = \gamma\,(\, 1 - v/c \,)\, \omega$

Note:

$$\gamma\,(\, 1 - v/c \,) = \sqrt{\frac{1+v/c}{1-v/x}}$$

and _ _ _

$B'_x = B_x \quad \Rightarrow B'_x = 0$

$B'_y = \gamma\,(B_y + v\,E_z\,/c^2) \quad \Rightarrow B'_y = \text{(see below)}$

$B'_z = \gamma\,(B_z - v\,E_y\,/c^2) \quad \Rightarrow B'_z = 0$

$B'_y = \gamma\,(-E_0/c + (v/c^2)\,E_0\,)\cos[\,kx - \omega t\,]$

$$= \gamma\,(\,1 - v/c\,)\,(-E_0/c)$$
$$.\ \cos[\,k\,\gamma\,(\,x' + vt'\,) - \omega\,\gamma\,(\,t' + vx'/c^2\,)]$$

$$= -(E'_0/c)\cos[\,k'x' - \omega't'\,]$$

where E'_0 , k' and ω' were given before.

So here are the results:

$\mathbf{E}'(\mathbf{x}',t') = E'_0\cos[\,k'x' - \omega't'\,]\,\mathbf{e}_z$

$\mathbf{B}'(\mathbf{x}',t') = (E'_0/c)\cos[\,k'x' - \omega't'\,]\,(-\mathbf{e}_y)$

<u>What is e.m. field in the frame ***F'***?</u>

/1/ It's an electromagnetic wave polarized in the z direction and traveling in the x direction.

But wave parameters are different from their values in the ref. frame ***F*** ,

$$E_0' = \gamma\,(\,1 - v/c\,)\,E_0$$

$$k' = \gamma\,(\,1 - v/c\,)\,k$$

$$\omega' = \gamma\,(\,1 - v/c\,)\,\omega$$

/ii/ *The wavelength*

$$E_0' = \gamma\,(1 - v/c)\,E_0$$
$$k' = \gamma\,(1 - v/c)\,k$$
$$\omega' = \gamma\,(1 - v/c)\,\omega$$

λ = 2 pi /k

$$\lambda' = \sqrt{\frac{1+v/c}{1-v/c}}\;\lambda$$

If $v > 0$ then $\lambda' > \lambda$ (red shift);
if $v < 0$ then $\lambda' < \lambda$ (blue shift).

This is *similar* to the Doppler effect of sound, but also different: look up the formula for the Doppler effect .

/iii/ *The frequency*

$$\omega' = \sqrt{\frac{1-v/c}{1+v/c}}\;\omega$$

/iv/ *The phase velocity*

$v = \omega/k$

So, $v = c$ and $v' = c$; the speed of light is invariant.

/v/ *The intensity*

$$E_0' / E_0 = \sqrt{\frac{1-v/c}{1+v/c}}$$

Then intensity $\sim E_0^2$;
if $v > 0$ then the intensity is smaller;
if $v < 0$ then the intensity is larger.