Optics

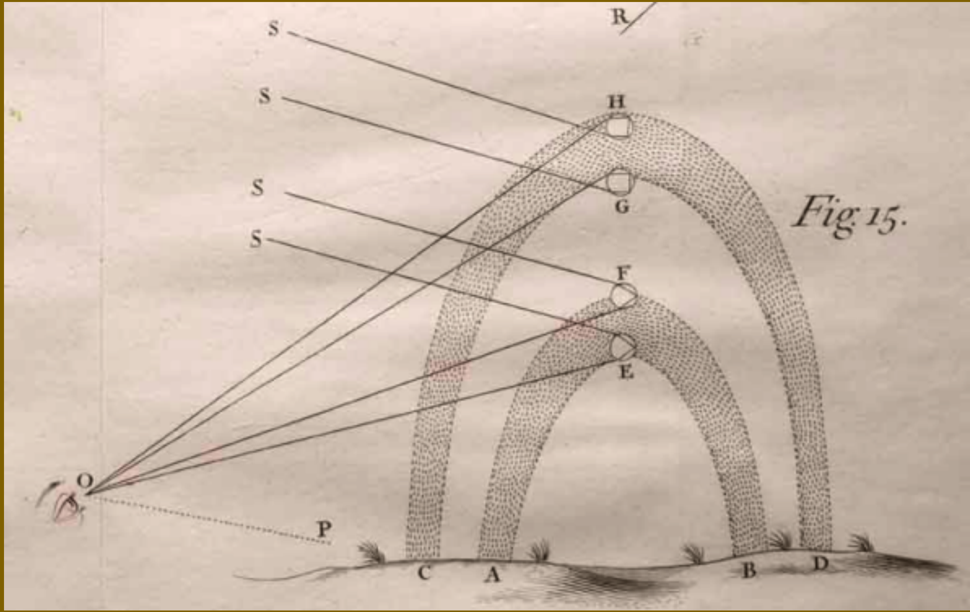
In 1672, as a young man, Isaac Newton made some interesting discoveries about light and colors, and published a short paper about his work. He showed that white light is a mixture of all the colors, opposing Aristotle's followers who assumed that white light is the purest form of light.

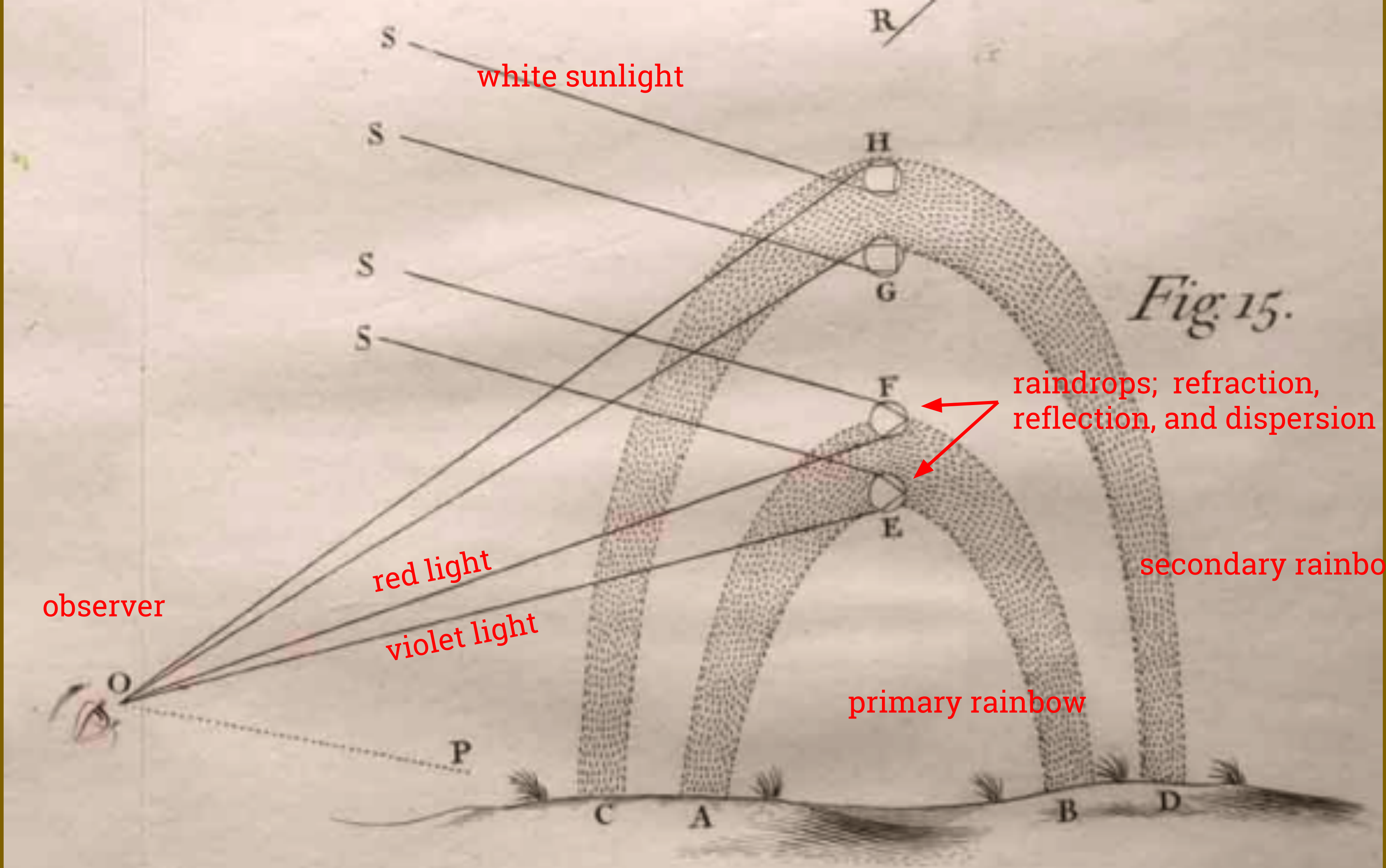
Some scientists criticized his work, which gave him a nervous breakdown. After that he decided that he should not publish his discoveries.

In 1704, now as a famous scientist, Newton published a book entitled ***Opticks: Or a Treatise of the Reflections, Refractions, Inflections & Colours of Light.*** He described his earlier discoveries and posed a number unanswered questions.

One question: does light consist of waves or particles? (Newton believed that light is a stream of particles; but his data were not accurate enough to answer the question.)

Figure 15 illustrated the theory of the rainbow.





white sunlight
S
S
S
S
R
H
G
F
E
Fig. 15.
raindrops; refraction,
reflection, and dispersion
secondary rainbow
primary rainbow
red light
violet light
observer
O
P
C
A
B
D

Newton did not know that light is an electromagnetic phenomenon.

Now we'll see how these properties of light -- reflection, refraction, dispersion, etc -- are explained in Maxwell's theory of electromagnetic waves.

Propagation of light* in a simple dielectric material

Review. Maxwell's equations in matter take this form ...

$$\nabla \cdot \mathbf{D} = \rho_{free}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$$
$$\nabla \times \mathbf{H} = \mathbf{J}_{free} + \partial \mathbf{D}/\partial t$$

where $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$ (polarization)
and $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$ (magnetization)

Light* : Visible light and other forms of electromagnetic waves.

Now consider electromagnetic fields with no free charges and no free currents ...

$$\nabla \cdot \mathbf{D} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$$
$$\nabla \times \mathbf{H} = \partial \mathbf{D}/\partial t$$

For "simple materials" we have
$\mathbf{D} = \varepsilon \mathbf{E}$ (dielectrics) and $\mathbf{B} = \mu \mathbf{H}$
(paramagnetic or diamagnetic materials).

The magnetic permeability is not so important; the dielectric effects are usually much larger. For completeness we'll use both parameters, but we'll call this a dielectric --- ε is more important.

Wave solutions

Try these functions to describe a polarized plane wave propagating in the dielectric material ...

$$\mathbf{E}(\mathbf{x},t) = \mathbf{E}_0 \sin(\mathbf{k}\cdot\mathbf{x} - \omega t)$$
$$\mathbf{B}(\mathbf{x},t) = \mathbf{B}_0 \sin(\mathbf{k}\cdot\mathbf{x} - \omega t)$$

Now demand that Maxwell's equations are satisfied. We've done this before, for EM waves in vacuum. Recall

$$\nabla \sin(\mathbf{k}\cdot\mathbf{x} - \omega t) = \mathbf{k} \cos(\mathbf{k}\cdot\mathbf{x} - \omega t)$$
$$\partial/\partial t \sin(\mathbf{k}\cdot\mathbf{x} - \omega t) = -\omega \cos(\mathbf{k}\cdot\mathbf{x} - \omega t)$$

$\nabla\cdot\mathbf{E} = 0$

implies $\mathbf{k}\cdot\mathbf{E}_0 = 0$; the oscillations of the electric field are perpendicular to **k**; light is a transverse wave.

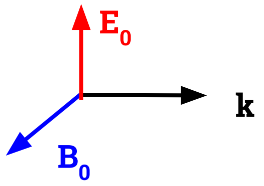
$\nabla\cdot\mathbf{B} = 0$

implies $\mathbf{k}\cdot\mathbf{B}_0 = 0$; the oscillations of the magnetic field are perpendicular to **k.**

$\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$

implies $\mathbf{k} \times \mathbf{E}_0 = +\omega\, \mathbf{B}_0$.

(i) **k**, $\mathbf{E}_0$, and $\mathbf{B}_0$ form an orthogonal triad.



(ii) Also, $B_0 = (k/\omega)\, E_0 = E_0 / v_{phase}$

(The phase velocity is ω/k.)

$\nabla \times (\mathbf{B}/\mu) = +\partial(\varepsilon\mathbf{E})/\partial t$

implies $\mathbf{k} \times \mathbf{B}_0 = -\mu\varepsilon\,\omega\,\mathbf{E}_0$;

(i) Note that $\mathbf{E}_0 \times \mathbf{B}_0$ is parallel to **k** ;

(ii) $k\,(k/\omega) = \mu\varepsilon\,\omega = \mu\varepsilon\, k\, v_{phase}$

$$v_{phase} = 1/(\mu\varepsilon)^{1/2}$$

The index of refraction

We define $n = c / v_{phase}$.

Maxwell's equations imply

$$n = \sqrt{\varepsilon\mu / \varepsilon_0\mu_0} \approx \sqrt{\varepsilon / \varepsilon_0}$$

Recall that $\varepsilon = \varepsilon_0 (1+\chi_e) > \varepsilon_0$;
therefore $n > 1$;
i.e., $c > v_{phase}$.

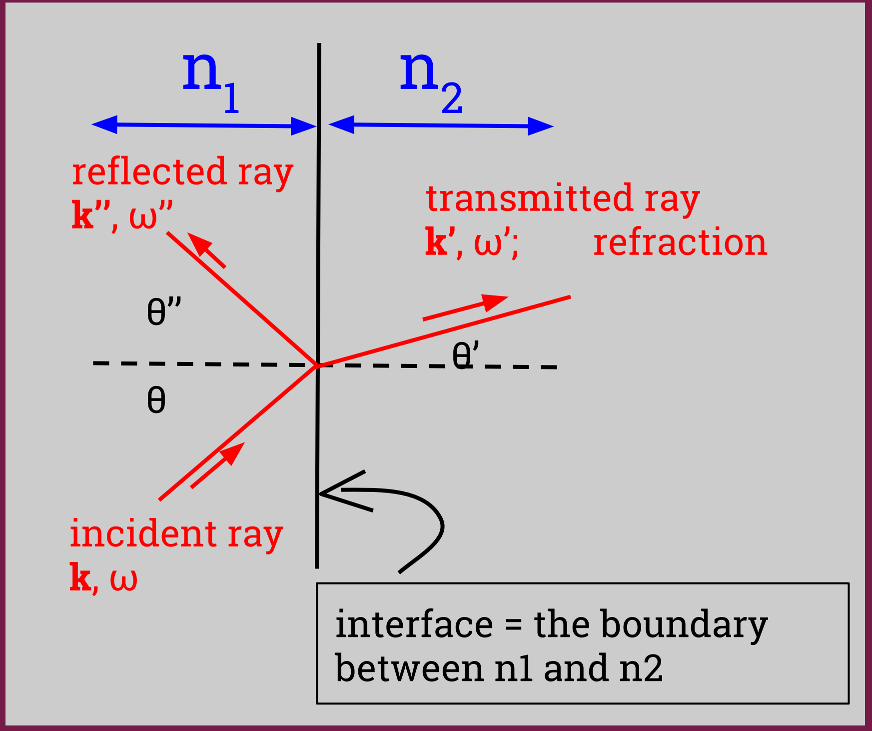
Light travels more slowly in a dielectric than in vacuum.*

Light* = visible light and other forms of electromagnetic waves

Reflection and Refraction

When light hits the boundary between two different dielectrics, there will be reflected waves and refracted waves.

Diagram for a planar surface.



Field Boundary Conditions and the laws of reflection and refraction

Recall the field boundary conditions...

- B_n is continuous; $B_{1n} = B_{2n}$
- E_t is continuous; $E_{1t} = E_{2t}$
- D_n is continuous ($D_{1n} = D_{2n}$) if there is no free surface charge.
- H_t is continuous ($H_{1t} = H_{2t}$) if there is no free surface current.

These equations must hold for all points **x** at the interface --- i.e., on the boundary surface between the two materials --- and for all times t.

Corollary

The frequencies of the field oscillations must be equal

$$\omega = \omega' = \omega'' \quad ;$$

otherwise the field boundary conditions would not hold for all t:

$$A \sin(\mathbf{k}\cdot\mathbf{x} - \omega t) + A'' \sin(\mathbf{k}''\cdot\mathbf{x} - \omega'' t)$$
$$= A' \sin(\mathbf{k}'\cdot\mathbf{x} - \omega' t) \; ;$$

...let t vary with **x** fixed.

Corollary

The parallel components of the wave vectors must be equal

$$k_{||} = k'_{||} = k''_{||} \quad ;$$

by the same reasoning -- let **x** vary *along the surface* with t fixed;

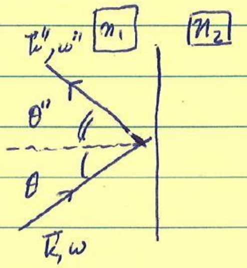
$$\mathbf{k}\cdot\mathbf{x} = k_{||} \; x_{||} \quad \text{if} \quad x_{perp} = 0.$$

Reflection and refraction

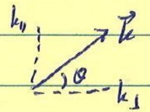
The law of reflection

$\theta'' = \theta$;   the law of equal angles.

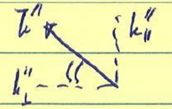
proof..



Law of Reflection



$$\sin\theta = \frac{k_\parallel}{|\vec{k}|} = \frac{k_\parallel}{\omega/v_1} = \frac{k_\parallel}{\omega}\frac{c}{n_1}$$



$$\sin\theta'' = \frac{k''_\parallel}{|\vec{k}''|} = \frac{k''_\parallel}{\omega''/v_1} = \frac{k''_\parallel}{\omega''}\frac{c}{n_1}$$

But $k''_\parallel = k_\parallel$ and $\omega'' = \omega$.

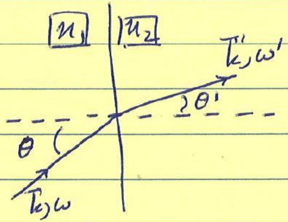
Thus $\sin\theta'' = \sin\theta$; $\theta'' = \theta$

The law of refraction

$$n_2 \sin\theta' = n_1 \sin\theta \; ;$$

Snell's law.

proof..



Law of Refraction

$$\sin\theta = \frac{k_\parallel}{\omega}\frac{c}{n_1}$$

$$\sin\theta' = \frac{k'_\parallel}{\omega'}\frac{c}{n_2}$$

We have $k'_\parallel = k_\parallel$ and $\omega' = \omega$;

then $n_1 \sin\theta = n_2 \sin\theta'$ (Snell's law)

Next time: use the field boundary conditions to determine the ***intensities*** of the reflected and transmitted waves.