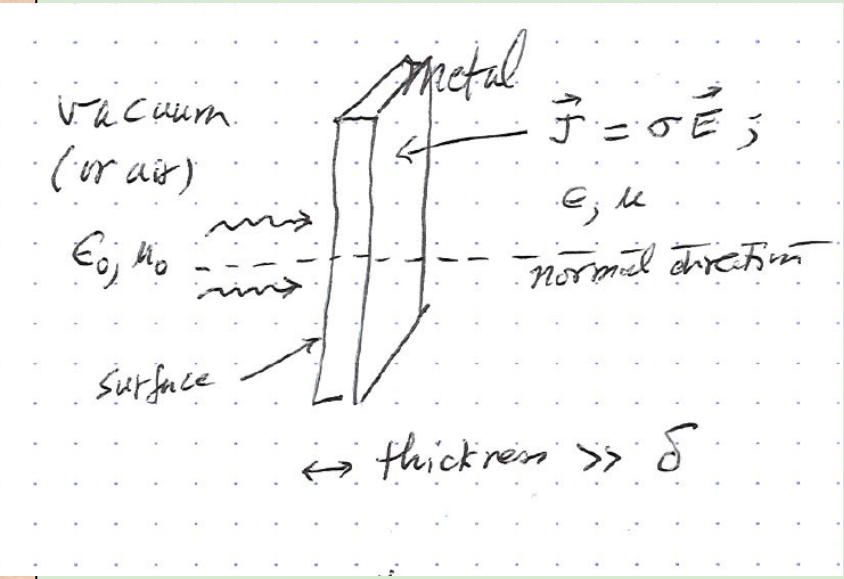
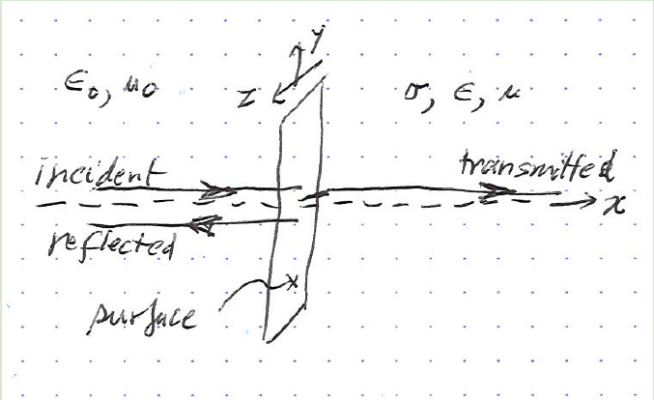
Reflection of light from a conducting surface



** Consider normal incidence.

** Assume thickness >> attenuation length.

Fields



incident ($x < 0$)

$$\mathbf{E} = \mathbf{e_y}\, E_0\, e^{i(kx - \omega t)}$$

$$\mathbf{B} = \mathbf{e_z}\, (E_0/c)\, e^{i(kx - \omega t)}$$

reflected ($x < 0$)

$$\mathbf{E''} = \mathbf{e_y}\, E''_0\, e^{i(-kx - \omega t)}$$

$$\mathbf{B''} = -\,\mathbf{e_z}\, (E''_0/c)\, e^{i(-kx - \omega t)}$$

transmitted ($x > 0$)

$$\mathbf{E'} = \mathbf{e_y}\, E'_0\, e^{i(k'x - \omega t)}$$

$$\mathbf{B'} = \mathbf{e_z}\, B'_0\, e^{i(k'x - \omega t)}$$

Fields

incident ($x < 0$)

$$\mathbf{E} = \mathbf{e}_y E_0 e^{i(kx - \omega t)}$$

$$\mathbf{B} = \mathbf{e}_z (E_0/c) e^{i(kx - \omega t)}$$

reflected ($x < 0$)

$$\mathbf{E}'' = \mathbf{e}_y E''_0 e^{i(-kx - \omega t)}$$

$$\mathbf{B}'' = -\mathbf{e}_z (E''_0/c) e^{i(-kx - \omega t)}$$

transmitted ($x > 0$)

$$\mathbf{E}' = \mathbf{e}_y E'_0 e^{i(k'x - \omega t)}$$

$$\mathbf{B}' = \mathbf{e}_z B'_0 e^{i(k'x - \omega t)}$$

Comments

(1) $\omega = \omega' = \omega''$; the frequencies must be equal so that the boundary conditions are satisfied for all t.

(2) $k = k'' = \omega/c$

(3) Recall *electromagnetic waves in a simple conductor*

$$k' = \kappa_1 + i\,\kappa_2$$

$$\kappa_{1,2} = \omega \sqrt{\mu\varepsilon/2}\,[\sqrt{1 + \sigma^2/(\varepsilon\,\omega)^2} \pm 1]^{1/2}$$

Attenuation length

$$\delta = 1/\kappa_2 \quad ,$$

because **E** or **B** ~ $\exp(-\kappa_2\, x)$ for $x > 0$.

Also, $B'_0 = k' E'_0/\omega$.

Boundary conditions

(1) $\mathbf{E}_{\text{tangential}}$ is continuous at x = 0

$$E_0 + E''_0 = E'_0$$

(2) $\mathbf{H}_{\text{tangential}}$ is continuous at x = 0

$$E_0 / (\mu_0 c) - E''_0 / (\mu_0 c) = k' E'_0 / (\mu\omega)$$

Reflection:

Solve for E''_0

$$E_0 + E_0'' = E_0'$$

$$= \frac{\mu\omega}{k'} \frac{1}{\mu_0 c} (E_0 - E_0'')$$

$$E_0'' = \frac{\mu\omega / k'\mu_0 c - 1}{\mu\omega / k'\mu_0 c + 1} E_0$$

$$E_0'' = \frac{\mu\omega - \mu_0 c k'}{\mu\omega + \mu_0 c k'} E_0$$

$$k' = k_1 + i k_2$$

Real Parts and Intensities

$$\vec{E} = \hat{e}_y E_0 \cos(kx - \omega t)$$

$$\vec{B} = \hat{e}_z \frac{E_0}{c} \cos(kx - \omega t)$$

$$I = \left\langle \frac{EB}{\mu_0} \right\rangle = \frac{E_0^2}{\mu_0 c} \langle \cos^2 \rangle = \frac{E_0^2}{2\mu_0 c}$$

$$\vec{E}'' = \hat{e}_y \{ \mathrm{Re}\, E_0'' \cos(-kx - \omega t) - \mathrm{Im}\, E_0'' \sin(-kx - \omega t) \}$$

$$\vec{B}'' = \frac{\hat{e}_z}{\mu_0 c} \{ \mathrm{Re}\, E_0'' \cos(-kx - \omega t) - \mathrm{Im}\, E_0'' \sin(-kx - \omega t) \}$$

$$I'' = \left\langle \frac{E'' B''}{\mu_0} \right\rangle = \frac{1}{\mu_0 c} \{ (\mathrm{Re}\, E_0'')^2 \langle \cos^2 \rangle + (\mathrm{Im}\, E_0'')^2 \langle \sin^2 \rangle - 2\, \mathrm{Re}\, \mathrm{Im} \langle \sin \cos \rangle \}$$

$$= \frac{(\mathrm{Re}\, E_0'')^2 + (\mathrm{Im}\, E_0'')^2}{2\mu_0 c}$$

Result: $I''/I = (E_0''/E_0)\,(E_0''/E_0)^*$

$$\frac{I''}{I} = \frac{(\mu\omega - \mu_0 c k_1)^2 + (\mu_0 c k_2)^2}{(\mu\omega + \mu_0 c k_1)^2 + (\mu_0 c k_2)^2}$$

For good conductors (i.e., metals)

$\sigma \gg \varepsilon\omega$

Then $\kappa_1 \approx \kappa_2 \approx \sqrt{\mu\sigma\omega/2}$

i.e., $c\,\kappa_1 \gg \sqrt{(\mu\varepsilon)/(\mu_0\varepsilon_0)}\;\omega \sim \omega$

Therefore we can make this approximation…

$$\frac{I''}{I} \approx \frac{\mu_0^2 c^2 k_1^2 + \mu_0^2 c^2 k_2^2 - 2\mu\mu_0 \omega c k_1}{\mu_0^2 c^2 k_1^2 + \mu_0^2 c^2 k_2^2 + 2\mu\mu_0 \omega c k_1}$$

$$\frac{I''}{I} = \frac{(\mu\omega - \mu_0 c k_1)^2 + (\mu_0 c k_2)^2}{(\mu\omega + \mu_0 c k_1)^2 + (\mu_0 c k_2)^2}$$

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$$\approx 1 - \frac{4\mu\mu_0 \omega c k_1}{\mu_0^2 c^2 (k_1^2 + k_2^2)}$$

$$\approx 1 - \frac{4\omega c \sqrt{\mu\sigma\omega/2}}{2c^2(\mu\sigma\omega/2)} \frac{\mu}{\mu_0}$$

$$\frac{I''}{I} \approx 1 - \sqrt{\frac{8\epsilon_0\omega}{\sigma}} \sqrt{\frac{\mu}{\mu_0}}$$

"Hagen-Rubens relation"

Most of the incident light energy is reflected. That's why metals are shiny.