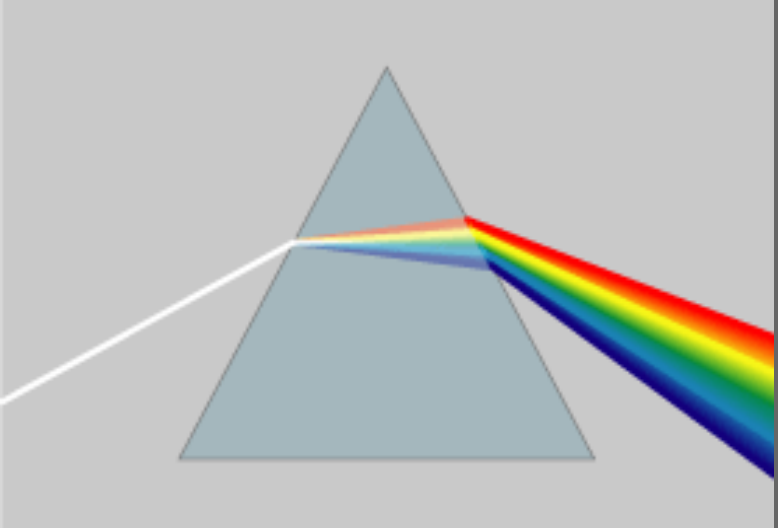
Dispersion --- frequency dependence of optical properties --- for dielectrics



Review of the results from the classical electron model (H. Lorentz)

Electron dynamics

$$m\ddot{x} = -kx - \gamma\dot{x} - eE_0 e^{-i\omega t}$$

$$\Rightarrow \quad x = \frac{eE_0}{m(\omega_0^2-\omega^2) - i\omega\gamma}\, e^{-i\omega t}$$

$$\alpha = \frac{p_0}{E_0} = \frac{-ex_0}{E_0} = \frac{e^2}{m(\omega_0^2-\omega^2) - i\omega\gamma}$$

Permittivity

$$\frac{\epsilon}{\epsilon_0} = 1 + \frac{\nu\alpha}{\epsilon_0} \qquad \text{assuming } \nu\alpha < \epsilon_0$$

$$\epsilon = \epsilon_1 + i\epsilon_2$$

$$\frac{\epsilon_1}{\epsilon_0} = 1 + \frac{\nu e^2}{m\epsilon_0}\,\frac{\omega_0^2-\omega^2}{(\omega_0^2-\omega^2)^2 + (\omega\gamma/m)^2}$$

$$\frac{\epsilon_2}{\epsilon} = \frac{\nu e^2}{m\epsilon_0}\,\frac{\omega\gamma/m}{(\omega_0^2-\omega^2)^2 + (\omega\gamma/m)^2}$$

ϵ_1 and ϵ_2 depend on ω;

or, on Λ where $\omega = 2\pi c/\Lambda$.

Dispersion relation

$$K^2 = \mu_0\omega^2\epsilon \quad ; \quad \epsilon = \epsilon_1 + i\epsilon_2$$

So $K = K_1 + iK_2$

$$K_1^2 = \tfrac{1}{2}\mu_0\omega^2\left[\epsilon_1 + \sqrt{\epsilon_1^2+\epsilon_2^2}\right]$$

$$K_1^2 \approx \mu_0\omega^2\epsilon_0$$

$$K_2 = \frac{\mu_0\omega^2\epsilon_2}{2K_1} \qquad K_2 \approx \frac{\omega}{2c}\,\frac{\epsilon_2}{\epsilon_0}$$

Index of refraction

$$\vec{E} \propto e^{i(K_1 x - \omega t)} \quad \text{implies} \quad v_{phase} = \frac{\omega}{K_1}$$

$$n = \frac{c}{v_{phase}} = \frac{cK_1}{\omega}$$

$$n = \left\{\frac{\epsilon_1 + \sqrt{\epsilon_1^2+\epsilon_2^2}}{2\epsilon_0}\right\}^{1/2}$$

Energy absorption length

Energy absorption length

$$u \propto e^{-2K_2 x} = \frac{1}{e} \text{ at } x = 1/2K_2$$

$$\delta = \frac{1}{2K_2} = \frac{c}{\omega}\,\frac{\epsilon_0}{\epsilon_2}$$

Graphical analysis

Pick some characteristic values for the parameters ("guesstimates")

UV ⇒ electron transitions

Λ = wavelength in vacuum < 100 nm = 0.1 micron

$\omega_0 = 2\pi c / \Lambda \quad > 2 \times 10^{16}$ /s

$\Delta E = h\omega / 2\pi \quad > 15$ eV

mass = m_e

IR ⇒ molecular vibrations

Λ = wavelength in vacuum > 1000 nm = 1.0 micron

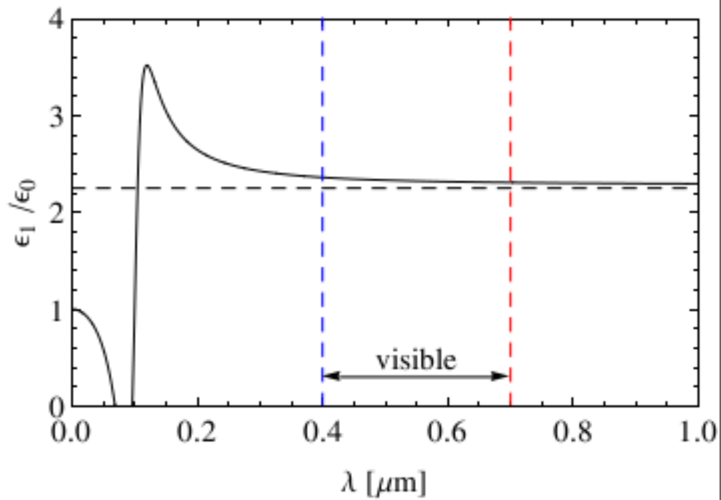
$\omega_0 = 2\pi c / \Lambda \quad < 2 \times 10^{15}$ /s

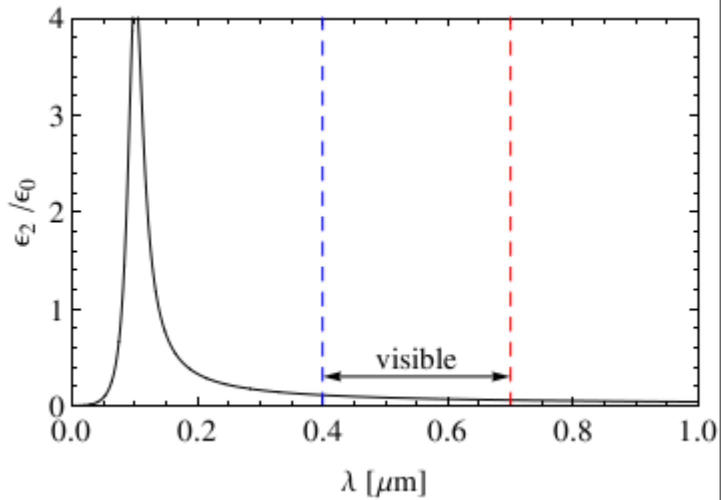
$\Delta E = h\omega / 2\pi \quad < 1.5$ eV

mass > 2000 m_e

Then make plots...

permittivity





Glass ; UV and Optical wavelengths

Classical electron dynamics; Lorentz model

```
(* units *)
{Ukg, Um, Us, UC}
Umicron = 1.0*^-6 * Um
UJ = Ukg * Um^2 / Us^2;
UV = UJ / UC;
(* universal constants *)
cl = 3.0*^8 * Um / Us;
ee = 1.6*^-19 * UC;
me = 9.11*^-31 * Ukg;
eps0 = 8.85*^-12 * UC / UV / Um;
```

{Ukg, Um, Us, UC}

$1. \times 10^{-6}$ Um

```
(* Model paramater values *)
atomicradius = 1.0*^-10 * Um;
volume = 1.33 * Pi * atomicradius^3;
nu = 1.0 / volume;
ω0 = 2 * Pi * cl / (0.1 * Umicron) (* UV *)
nu * ee^2 / (2 * me * eps0) / ω0^2   (* should be a small *)
gamma = 0.3 * me * ω0 (* damping parameter *)
```

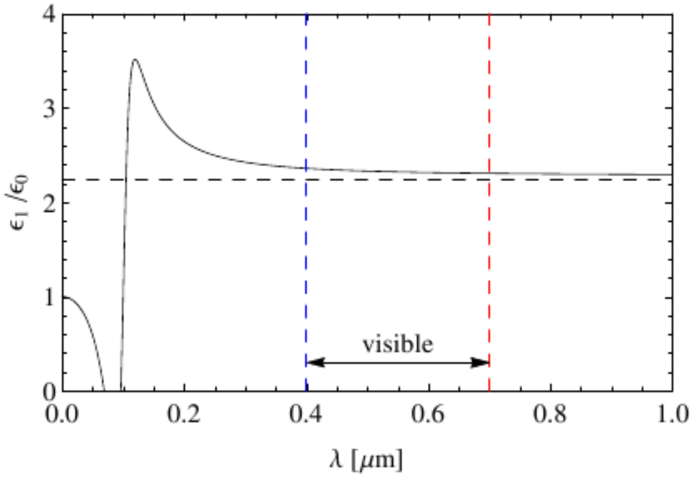
$\frac{1.88496 \times 10^{16}}{\text{Us}}$

1.06941

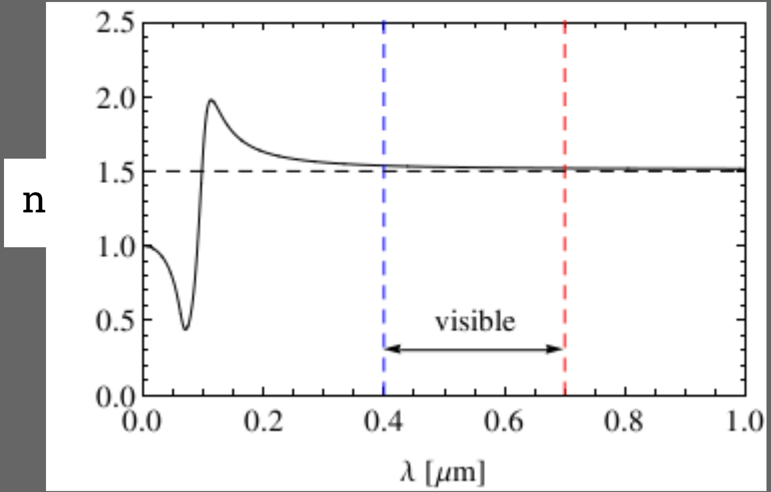
$\frac{5.15158 \times 10^{-15}\ \text{Ukg}}{\text{Us}}$

```
ffac = 0.6; (* fudge factor *)
eps1[ω_] := 1 + ffac * nu * ee^2 / (me * eps0) *
    (ω0^2 - ω^2) / ((ω0^2 - ω^2)^2 + (ω * gamma / me)^2)
eps1[ {ω0 / 2, ω0, ω0 * 2} ]
UV1 = Plot[eps1[2 * Pi * cl / (λ * Umicron)], {λ, 0, 1.0},
  Evaluate[fancy], FrameLabel → fl1, PlotStyle → Black,
  PlotRange → {{0, 1.0}, {0, 4}},
  Epilog → {lred, lvio, vis, l15sq}]
```

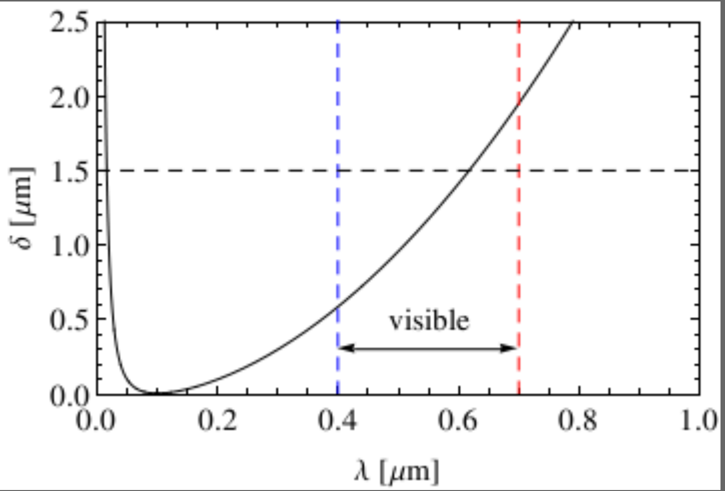
{2.64525, 1., 0.588688}



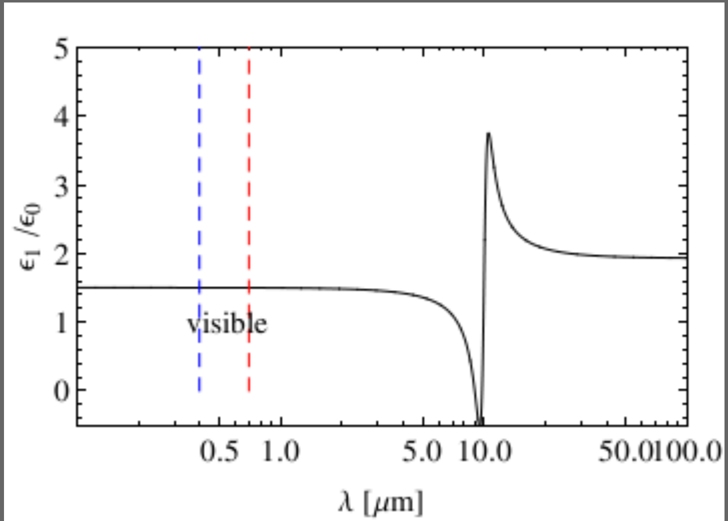
Index of refraction produced by UV natural frequency (Lorentz model)



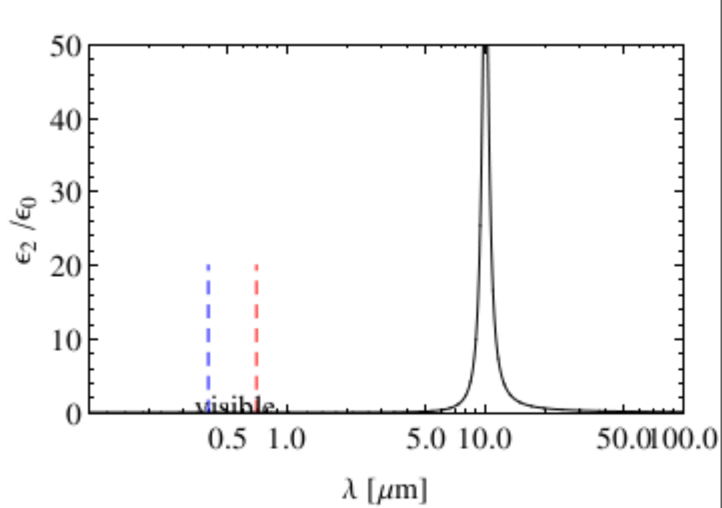
Energy absorption length produced by UV natural frequency in the Lorentz model (obviously not very realistic!)



Real part of ε produced by IR natural frequency in the Lorentz model



Imaginary part of ε produced by IR natural frequency in the Lorentz model



Optical constants of silica glass from extreme ultraviolet to far infrared at near room temperature

Rei Kitamura,[1] Laurent Pilon,[1,*] and Miroslaw Jonasz[2]

[1]Department of Mechanical and Aerospace Engineering, Henry Samueli School of Engineering and Applied Science, University of California, Los Angeles, Los Angeles, California 90095, USA
[2]MJC Optical Technology, 217 Cadillac Street, Beaconsfield, QC H9W 2W7, Canada

*Corresponding author: pilon@seas.ucla.edu

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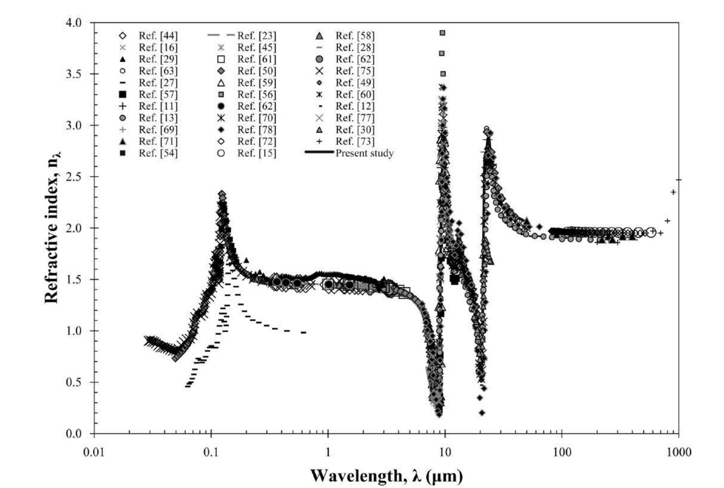
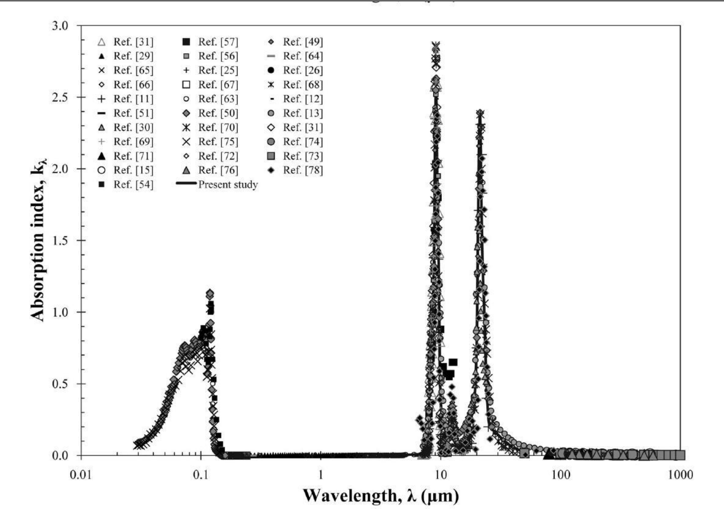
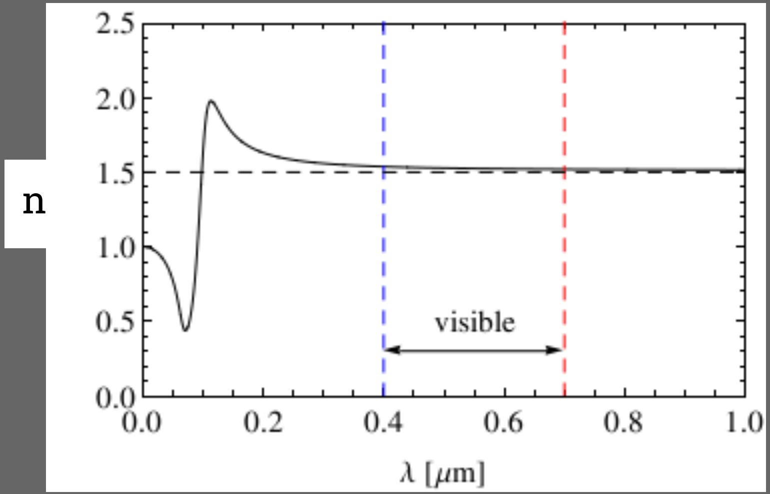
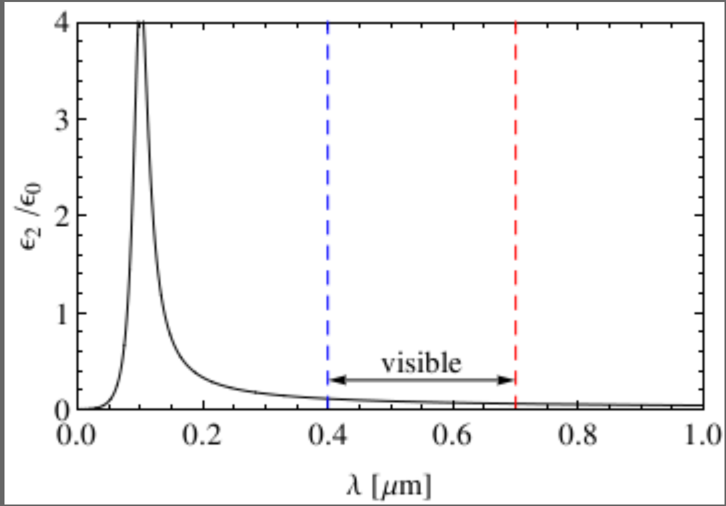


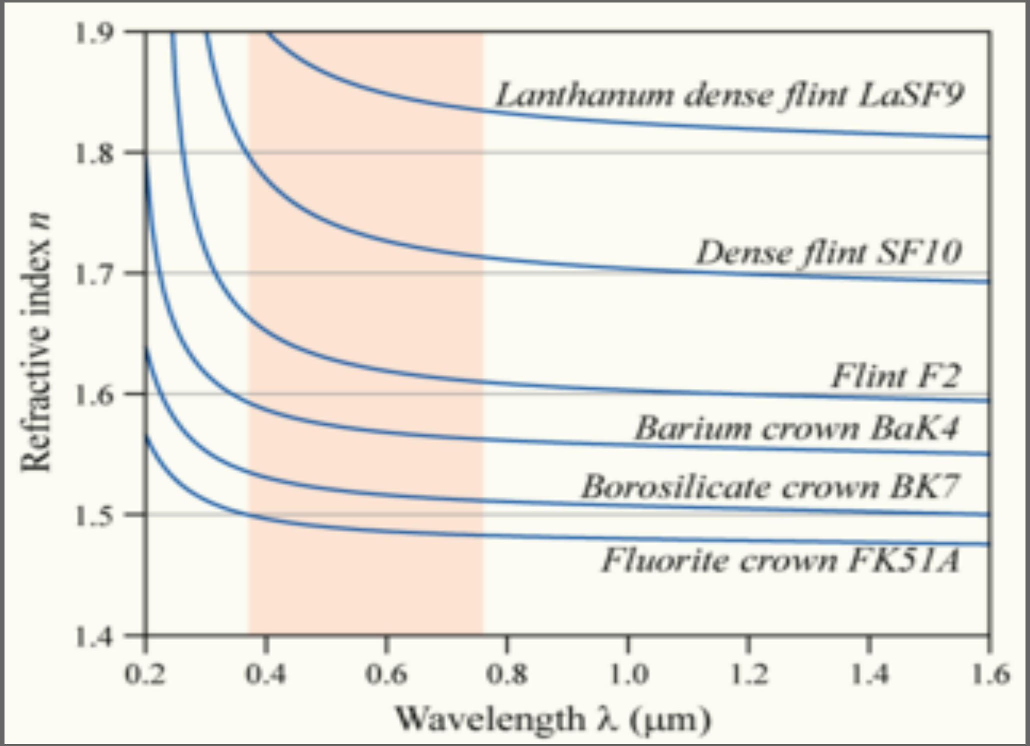


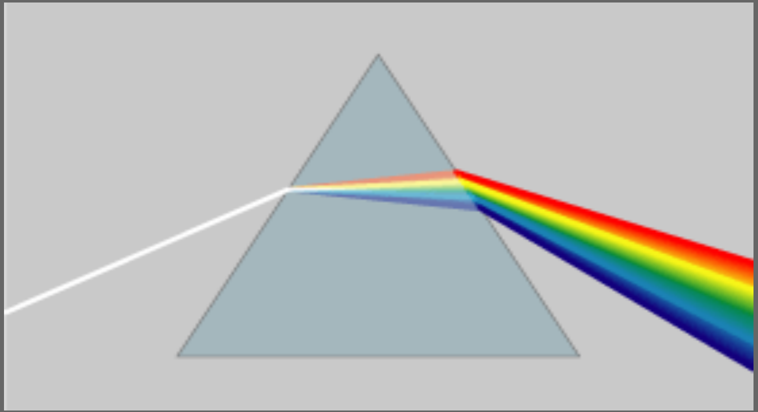




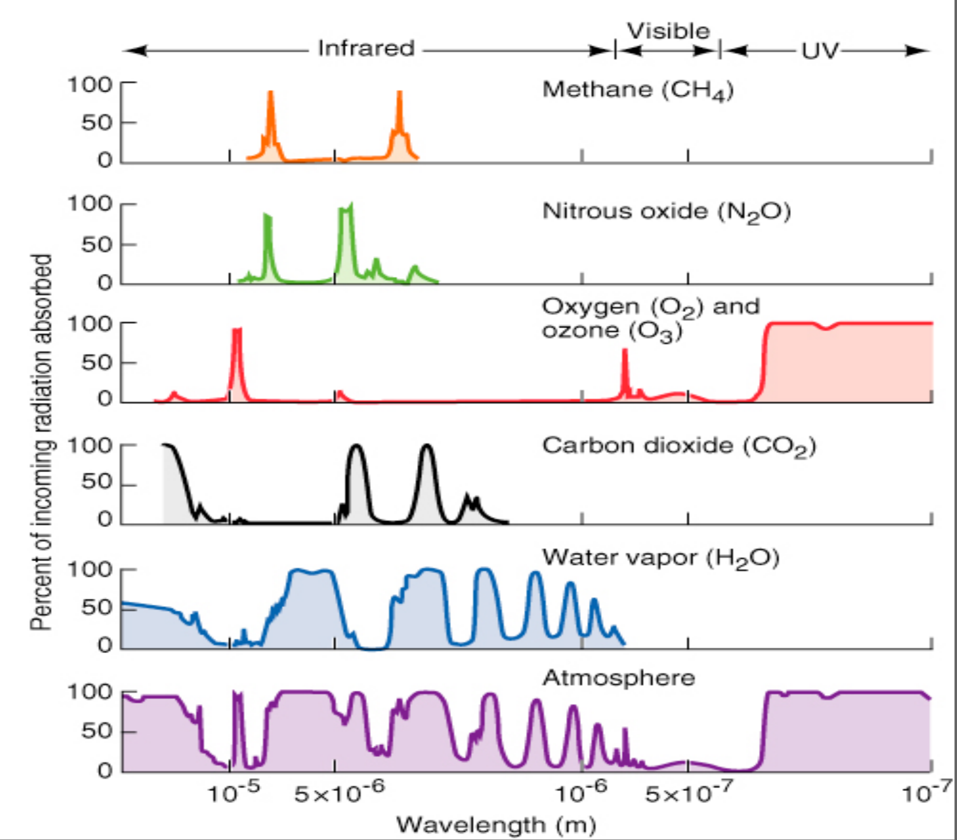
Comments

- At frequencies corresponding to visible light,
  n increases as wavelength decreases;
  that explains prisms and the rainbow.
- "anomalous dispersion" at the resonance
- Absorption of light occurs strongly at the resonance;
  absorption length ~ $1/\varepsilon_2$.

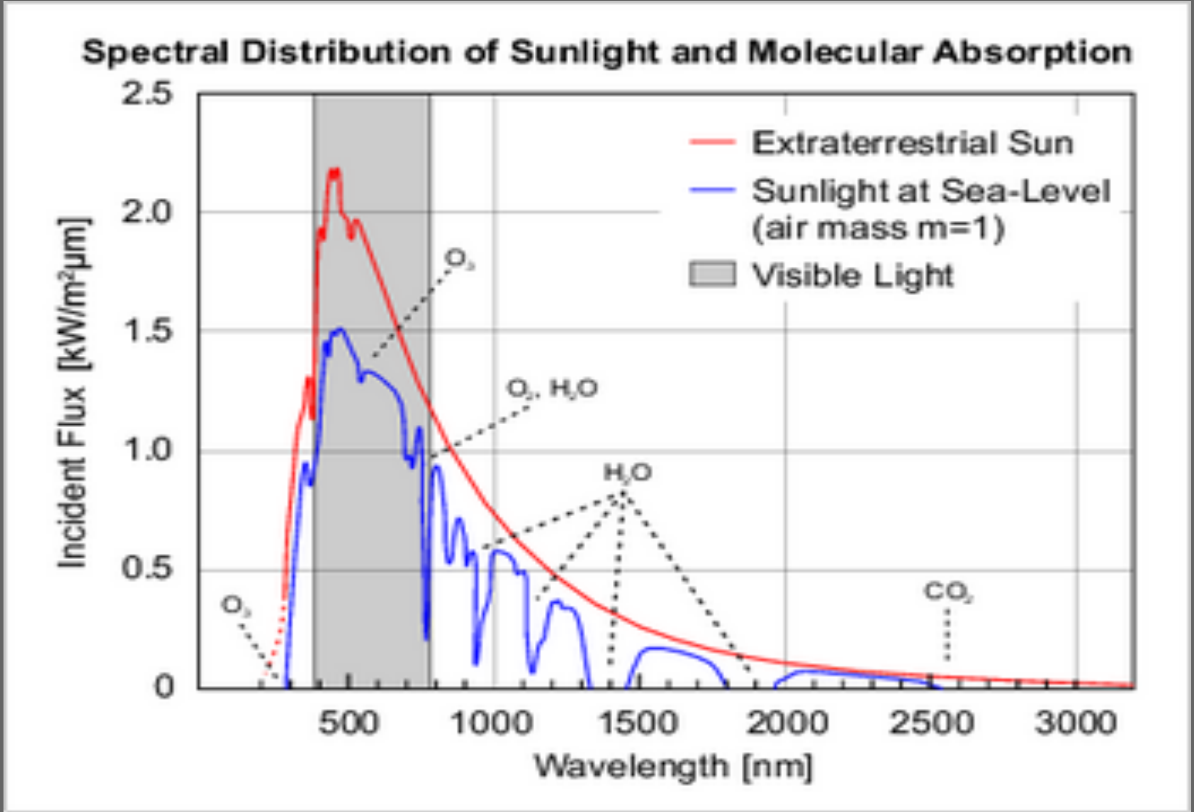

Lanthanum dense flint LaSF9
Dense flint SF10
Flint F2
Barium crown BaK4
Borosilicate crown BK7
Fluorite crown FK51A
Refractive index n
Wavelength λ (μm)
1.9
1.8
1.7
1.6
1.5
1.4
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6



Resonant absorption of infrared waves by molecules



Absorption of sunlight by atmospheric molecules



Exam Questions

/A/ Why do molecules absorb *infrared* light?

/B/ Why is nitrogen (N_2) ***not*** a greenhouse gas?