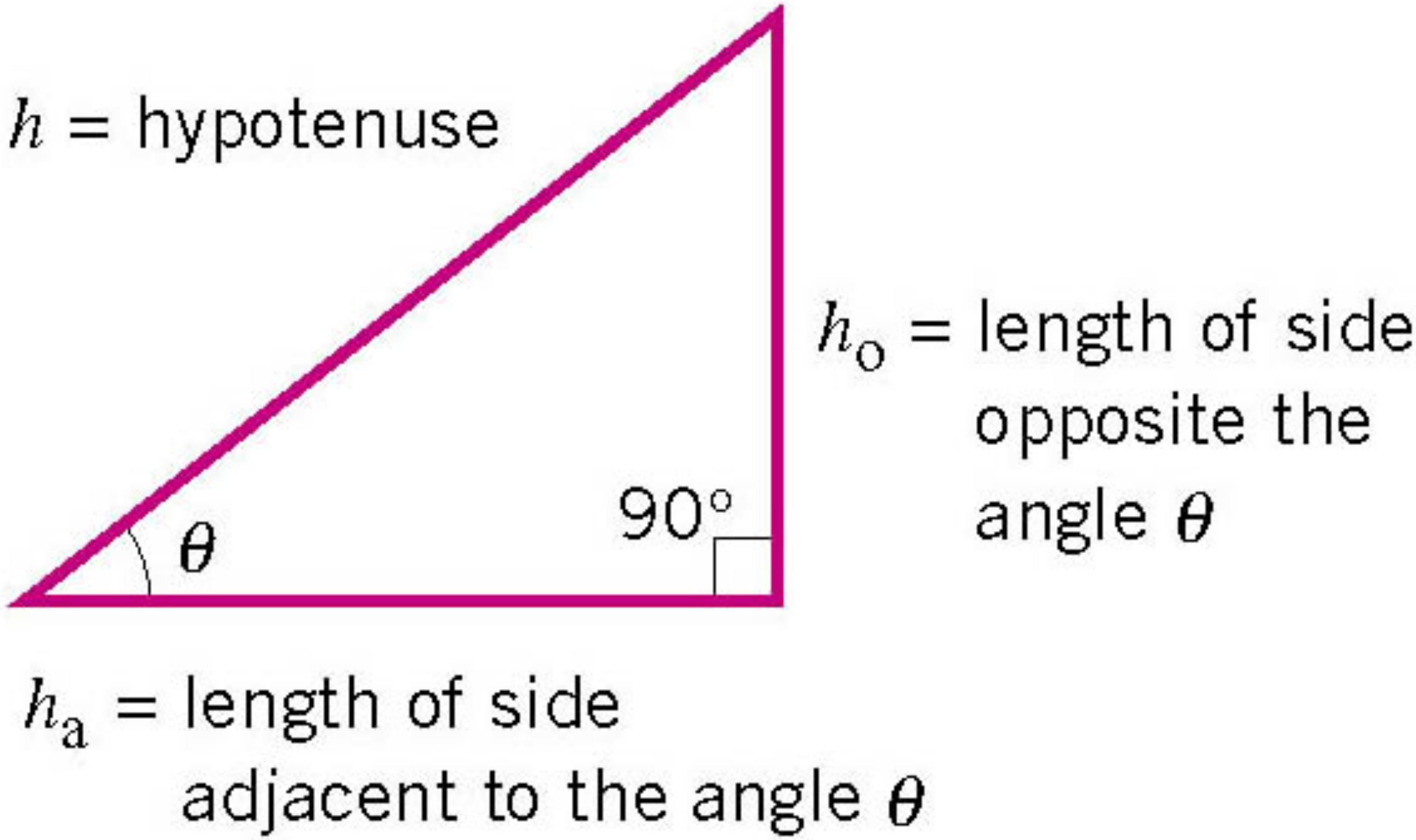
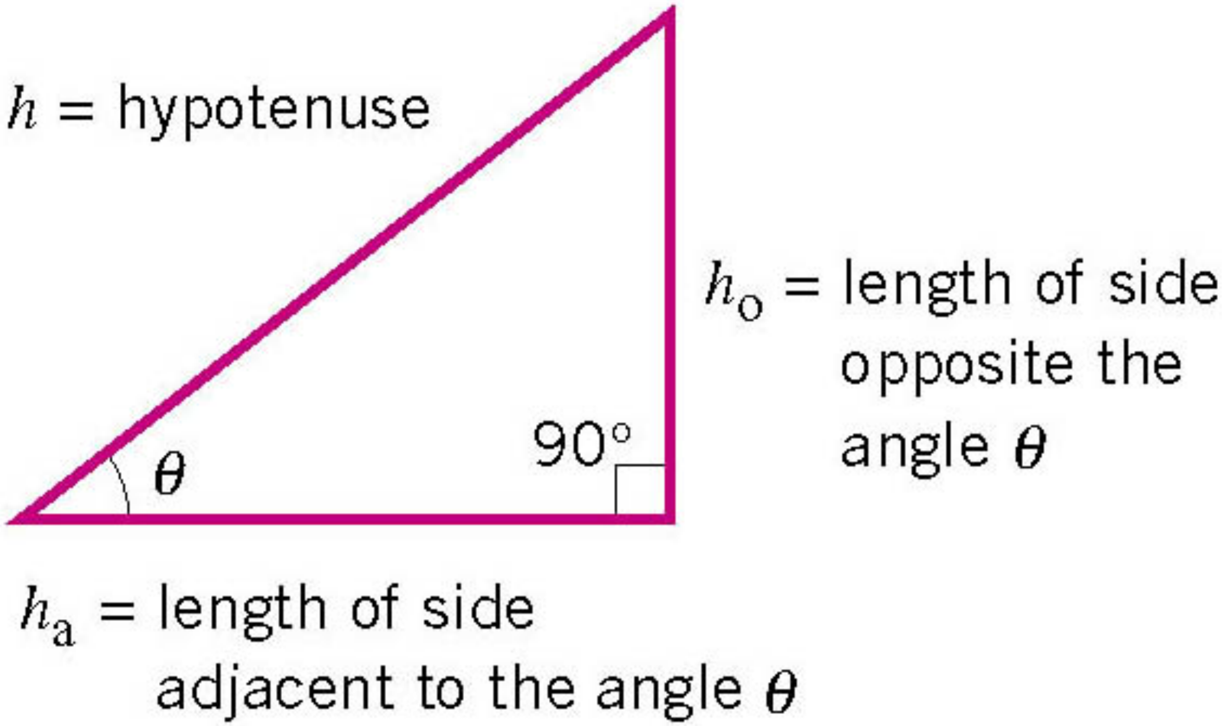
Chapter 3

Kinematics in Two Dimensions

3.1 Trigonometry



3.1 Trigonometry

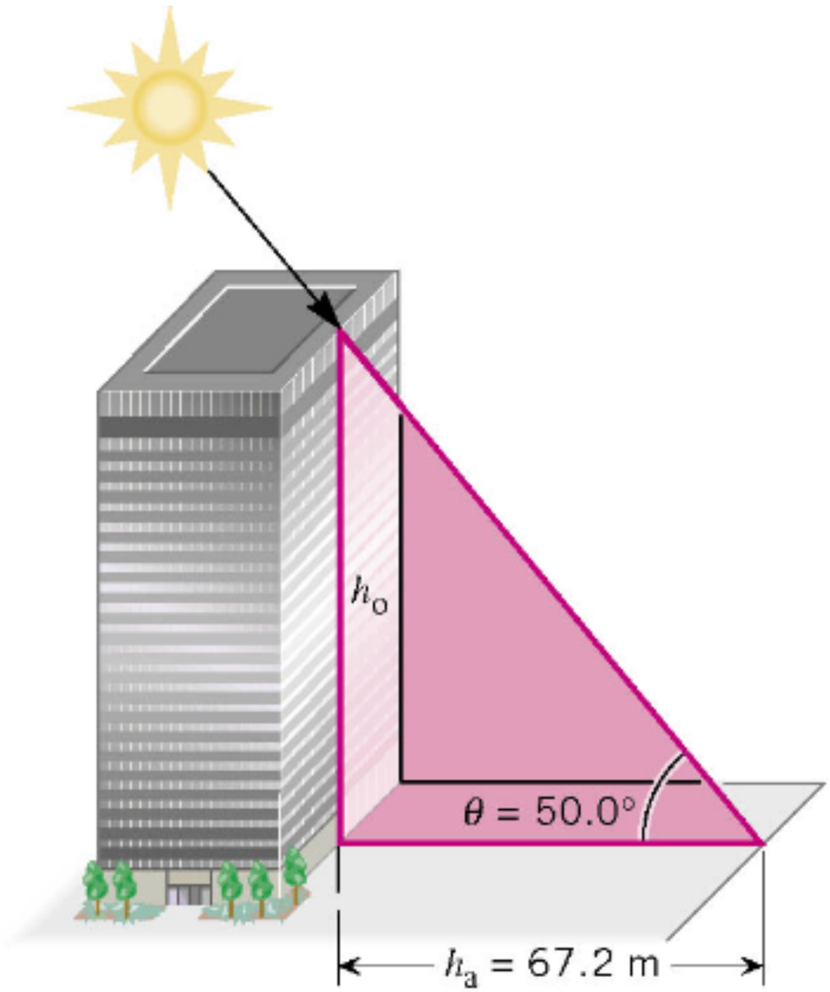


$$\sin\theta = \frac{h_o}{h}$$

$$\cos\theta = \frac{h_a}{h}$$

$$\tan\theta = \frac{h_o}{h_a}$$

3.1 Trigonometry

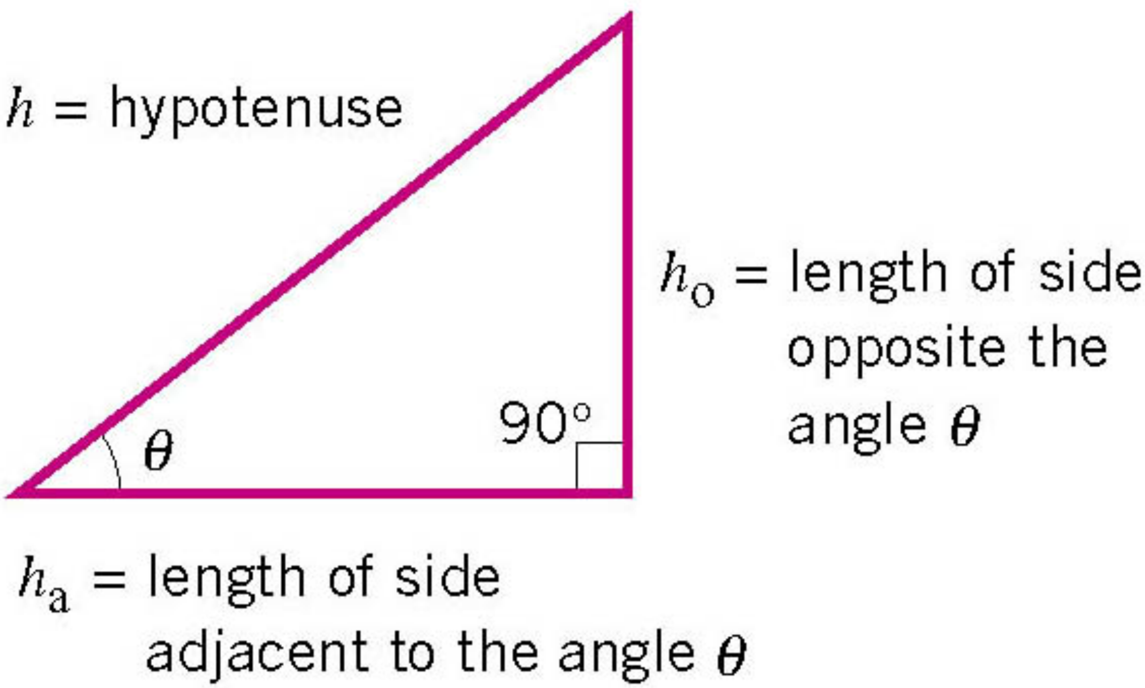


$$\tan\theta = \frac{h_o}{h_a}$$

$$\tan 50^\circ = \frac{h_o}{67.2\mathrm{m}}$$

$$h_o = \tan 50^\circ \left(67.2\mathrm{m}\right) = 80.0\mathrm{m}$$

3.1 Trigonometry

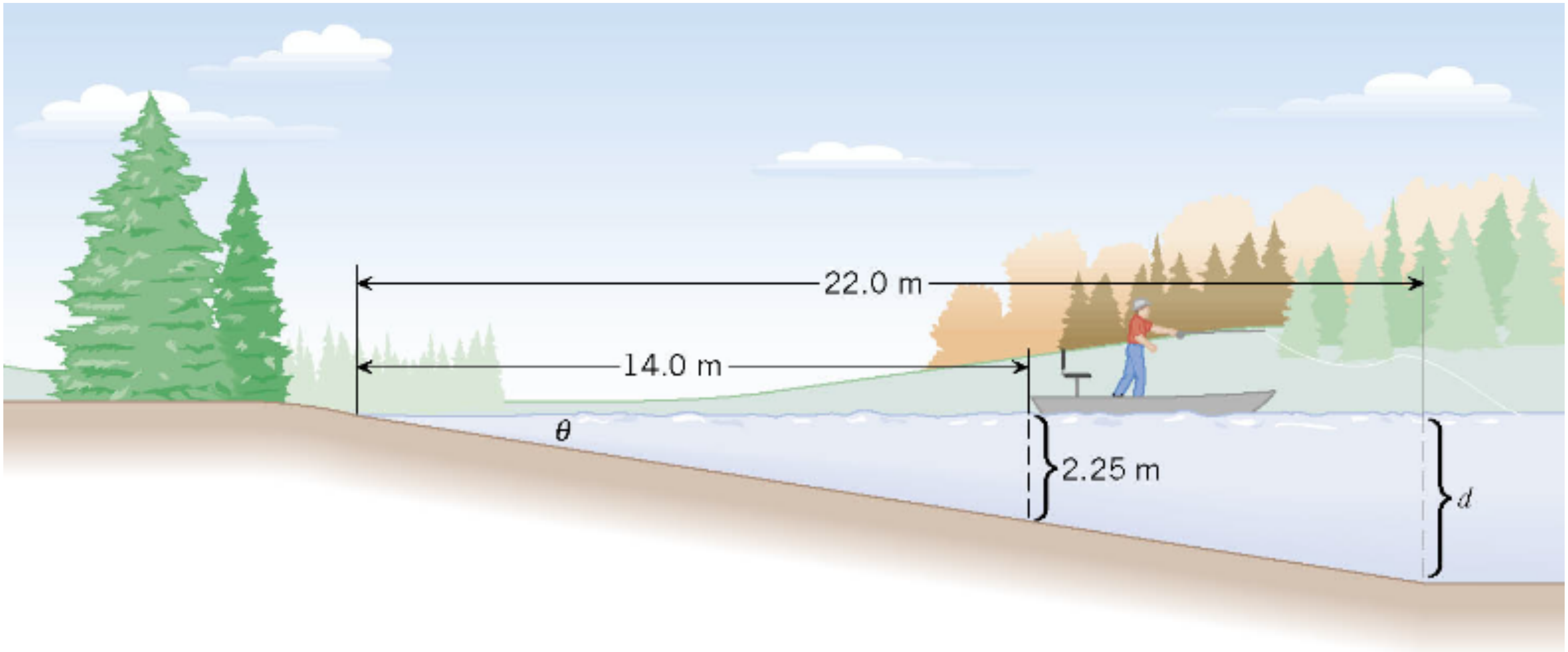


$$\theta = \sin^{-1}\left(\frac{h_o}{h}\right)$$

$$\theta = \cos^{-1}\left(\frac{h_a}{h}\right)$$

$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right)$$

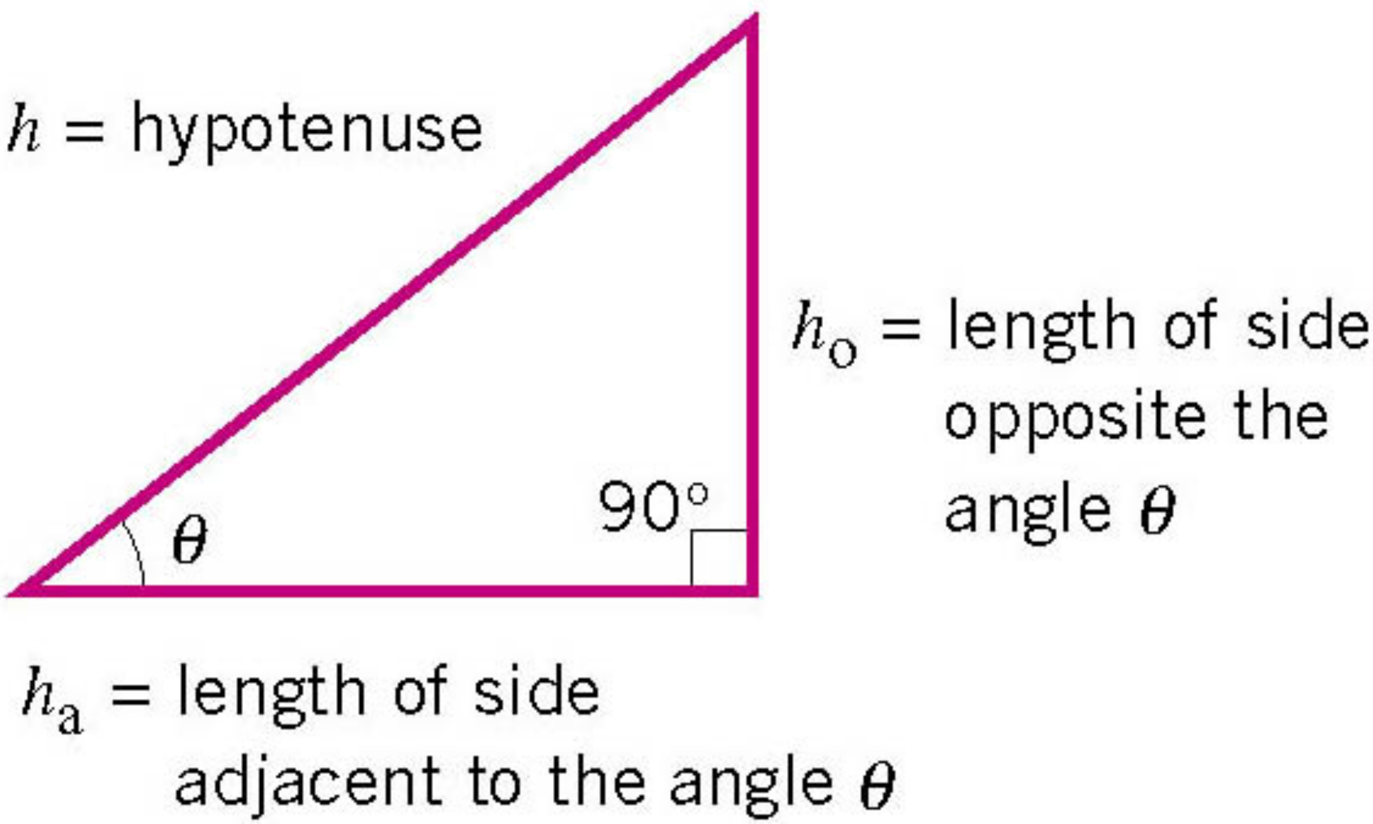
3.1 Trigonometry



$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right) \qquad \theta = \tan^{-1}\left(\frac{2.25\text{m}}{14.0\text{m}}\right) = 9.13^{\circ}$$

3.1 Trigonometry

Pythagorean theorem: $$h^2 = h_o^2 + h_a^2$$

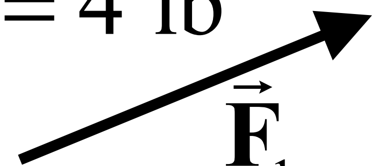


3.2 Scalars and Vectors

Directions of vectors $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ appear to be the same.

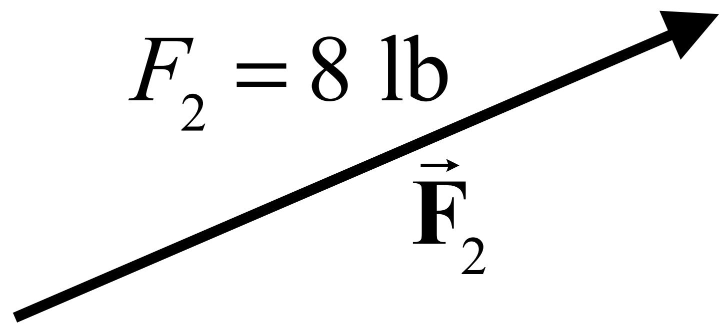
Vector $\vec{\mathbf{F}}_1$, (bold + arrow over it)

$$\text{has 2 parts:}\begin{cases}\text{magnitude} = F_1 \ \textit{(italics)} \\ \text{direction} = \text{up \& to the right}\end{cases}$$



Vector $\vec{\mathbf{F}}_2$, (bold + arrow over it)

$$\text{has 2 parts:}\begin{cases}\text{magnitude} = F_2 \ \textit{(italics)} \\ \text{direction} = \text{up \& to the right}\end{cases}$$

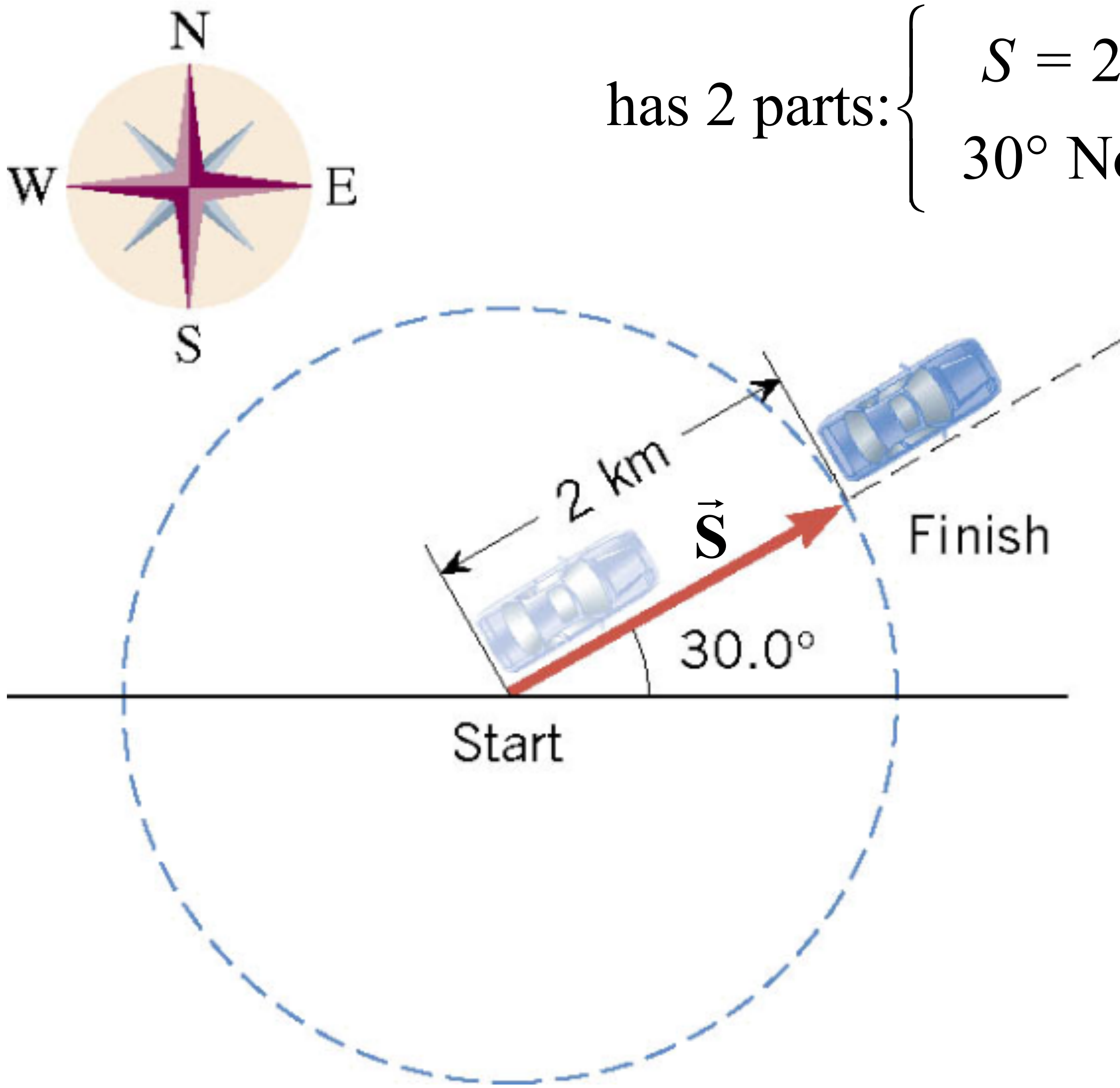


Directions of vectors $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ appear to be the same.

3.2 Scalars and Vectors

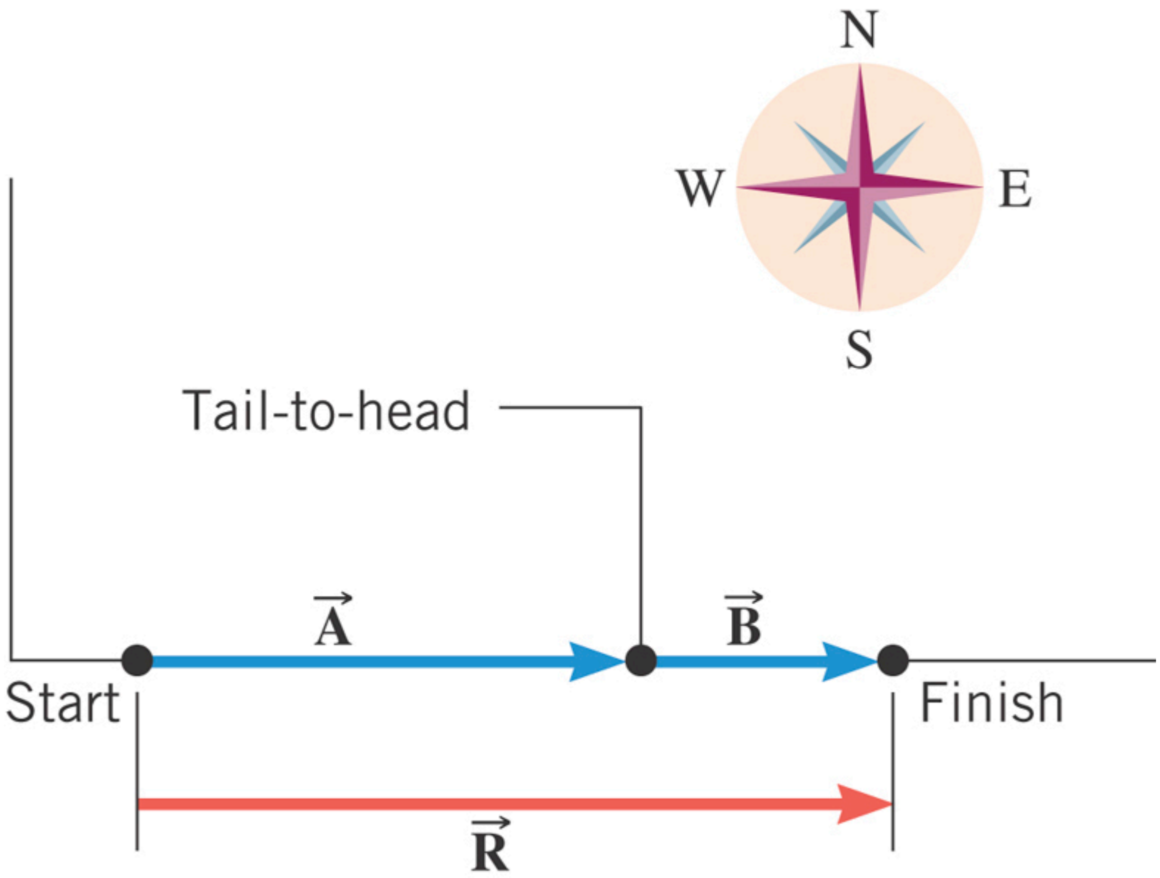
Displacement vector $\vec{\mathbf{S}}$

has 2 parts: $\begin{cases} S = 2 \text{ km (magnitude) , and} \\ 30° \text{ North of East (direction)} \end{cases}$



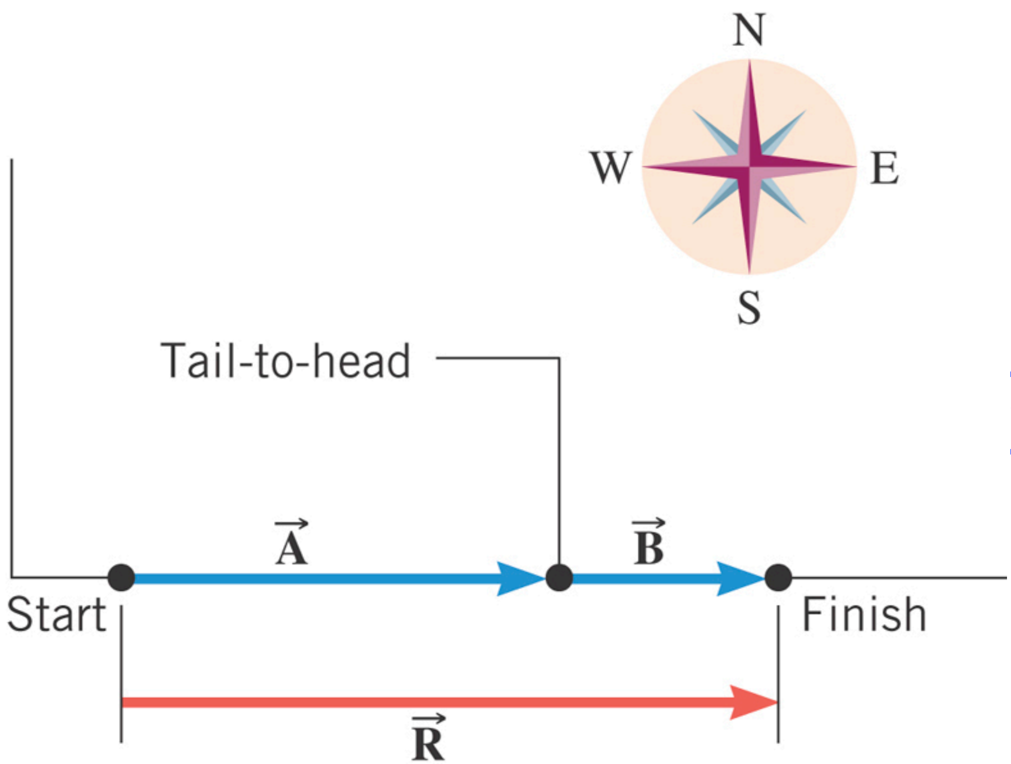
3.2 Scalars and Vectors (Vector Addition and Subtraction)

Often it is necessary to add one vector to another.



The "Resultant Vector"

3.2 Scalars and Vectors (Vector Addition and Subtraction)



Two displacement vectors with magnitudes *A* and *B*

$A = 5\text{m}$

$B = 3\text{m}$

Adding together

$A = 5\text{m}$ $B = 3\text{m}$

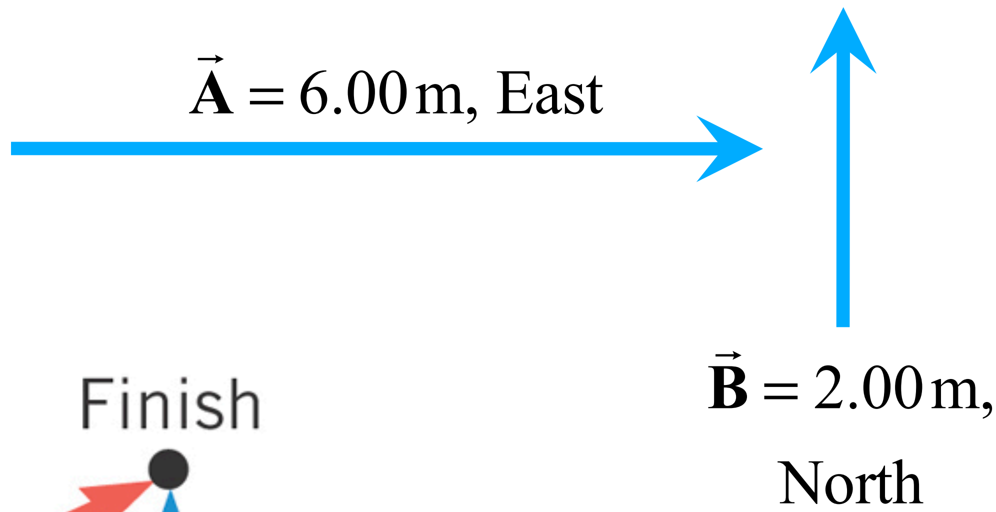
Resultant

$R = 8\text{m}$

Vector addition is "commutative" - order of the addition doesn't matter

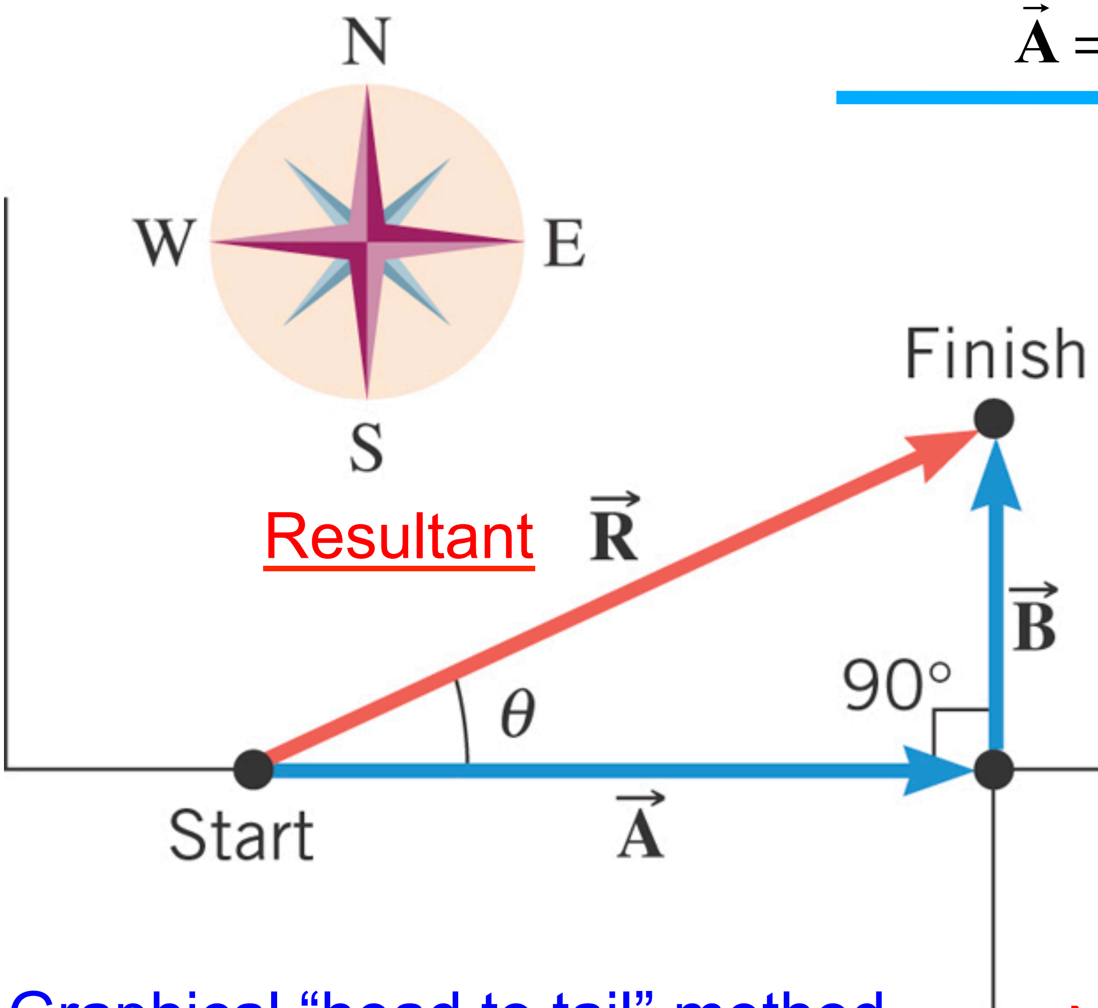
3.2 Scalars and Vectors (Vector Addition and Subtraction)

Two displacement vectors



Resultant

Adding vectors together

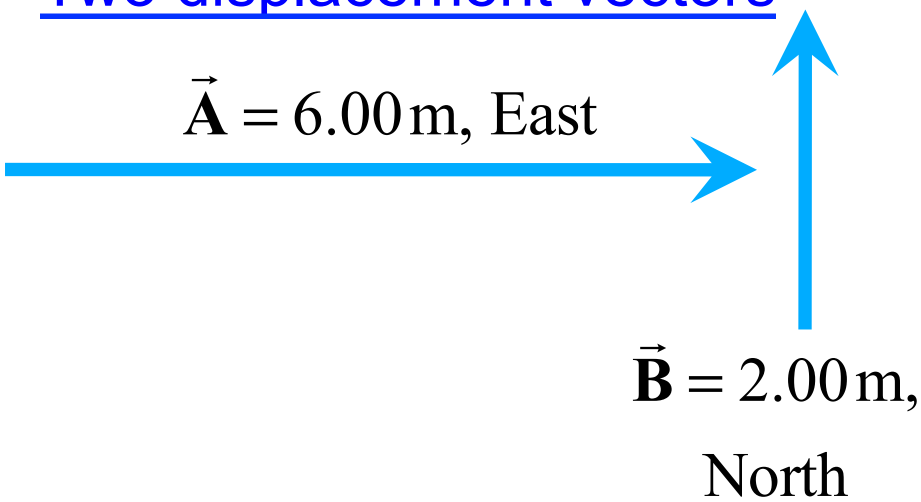


Graphical "head to tail" method

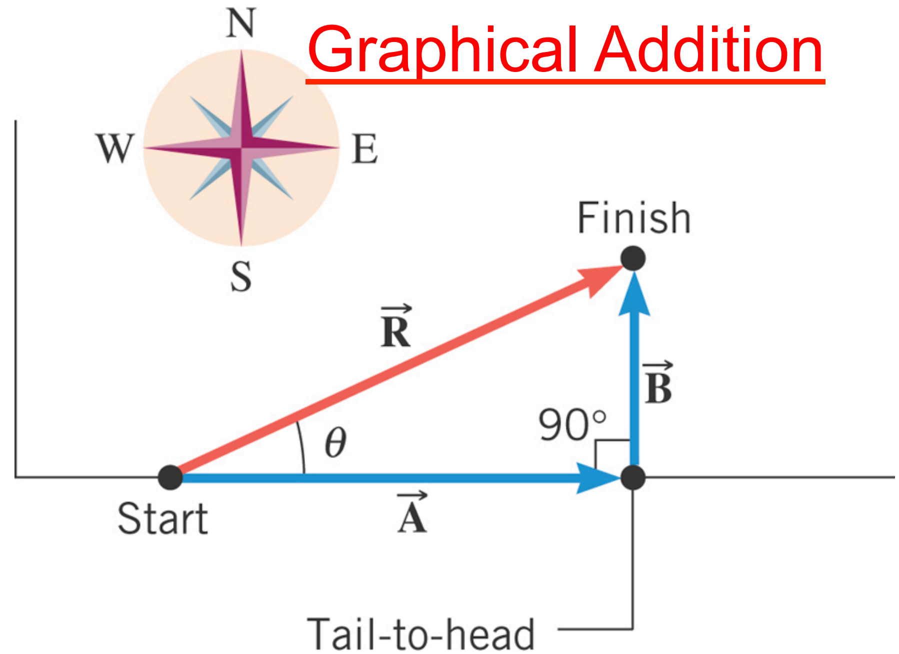
But only a visual aid
Not a substitute for math

3.2 Scalars and Vectors (Vector Addition and Subtraction)

Two displacement vectors

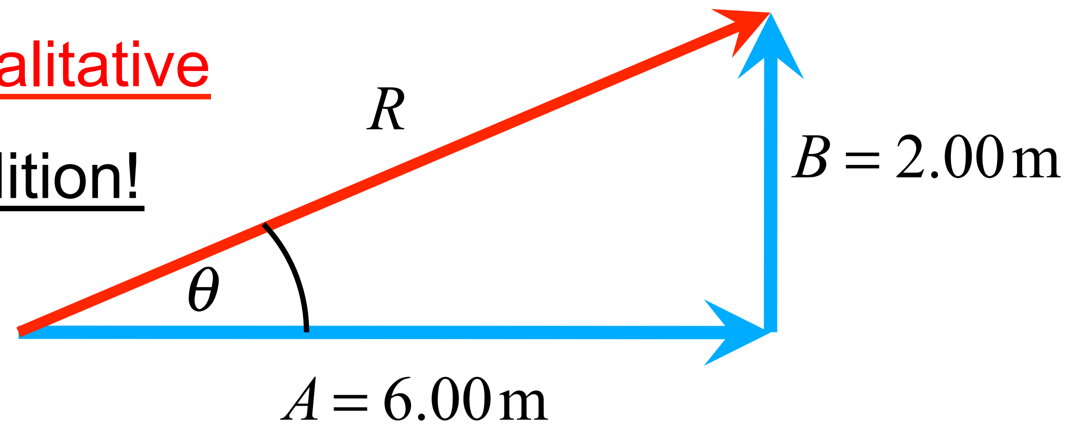


Graphical Addition



Graphical addition is only qualitative

YOU need quantitative addition!

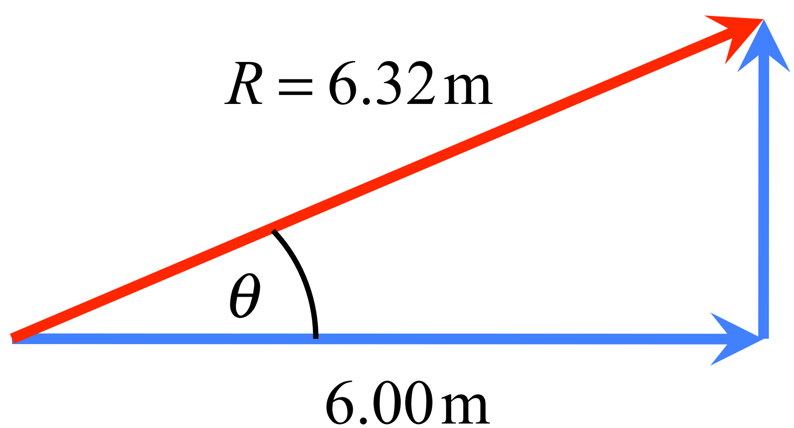


To do this addition of vectors requires trigonometry

Apply Pythagorean Theroem

$$R^2 = (2.00\,\mathrm{m})^2 + (6.00\,\mathrm{m})^2$$

$$R = \sqrt{(2.00\,\mathrm{m})^2 + (6.00\,\mathrm{m})^2} = 6.32\mathrm{m}$$

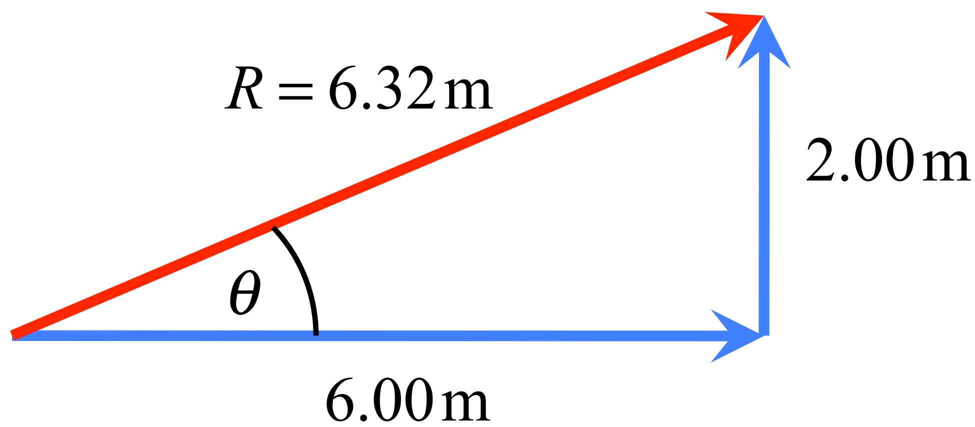


3.2 Scalars and Vectors (Vector Addition and Subtraction)

Use trigonometry to determine the angle

$$\tan\theta = 2.00/6.00 \qquad \text{tangent (angle)} = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\theta = \tan^{-1}\left(2.00/6.00\right) = 18.4^{\circ}$$



Also

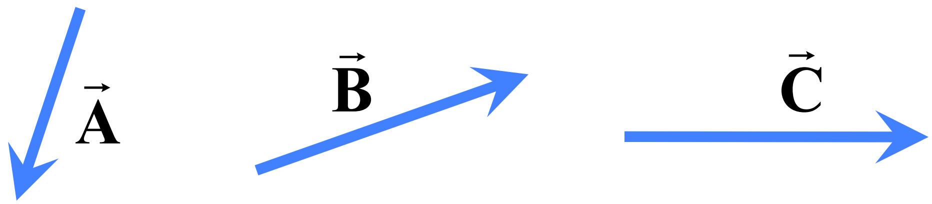
$$\theta = \sin^{-1}\left(2.00/6.32\right) = 18.4^{\circ}$$

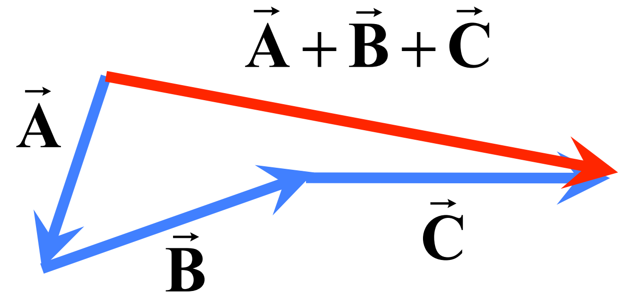
Also

$$\theta = \cos^{-1}\left(6.00/6.32\right) = 18.4^{\circ}$$

Clicker Question 3.1

Three vectors are given: $\vec{\mathbf{A}}, \vec{\mathbf{B}},$ and $\vec{\mathbf{C}}$.





Which of the choices below is equal to $\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}$?

A) $\vec{\mathbf{C}}+\vec{\mathbf{B}}+\vec{\mathbf{A}}$

B) $\vec{\mathbf{B}}+\vec{\mathbf{A}}+\vec{\mathbf{C}}$

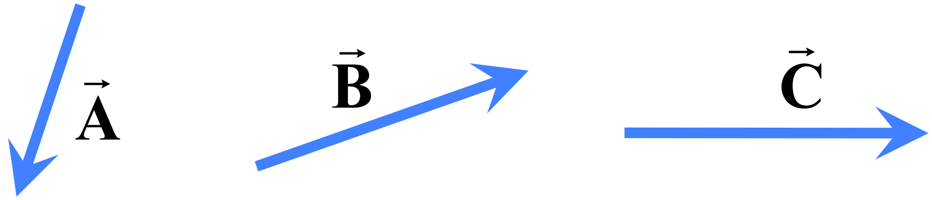
C) $\vec{\mathbf{C}}+\vec{\mathbf{A}}+\vec{\mathbf{B}}$

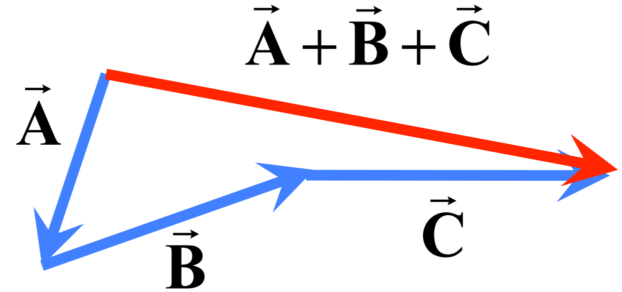
D) None of the above

E) All of the above

Clicker Question 3.1

Three vectors are given: $\vec{\mathbf{A}}, \vec{\mathbf{B}},$ and $\vec{\mathbf{C}}$.





Which of the choices below is equal to $\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}$?

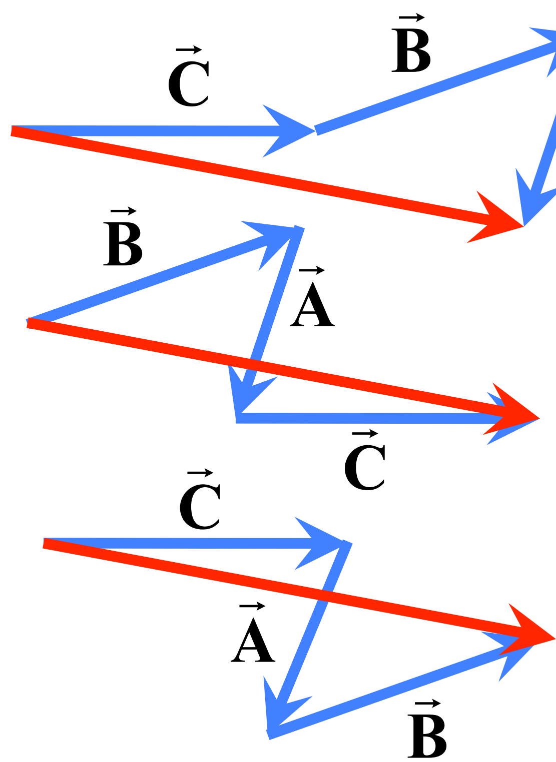
A) $\vec{\mathbf{C}}+\vec{\mathbf{B}}+\vec{\mathbf{A}}$

B) $\vec{\mathbf{B}}+\vec{\mathbf{A}}+\vec{\mathbf{C}}$

C) $\vec{\mathbf{C}}+\vec{\mathbf{A}}+\vec{\mathbf{B}}$

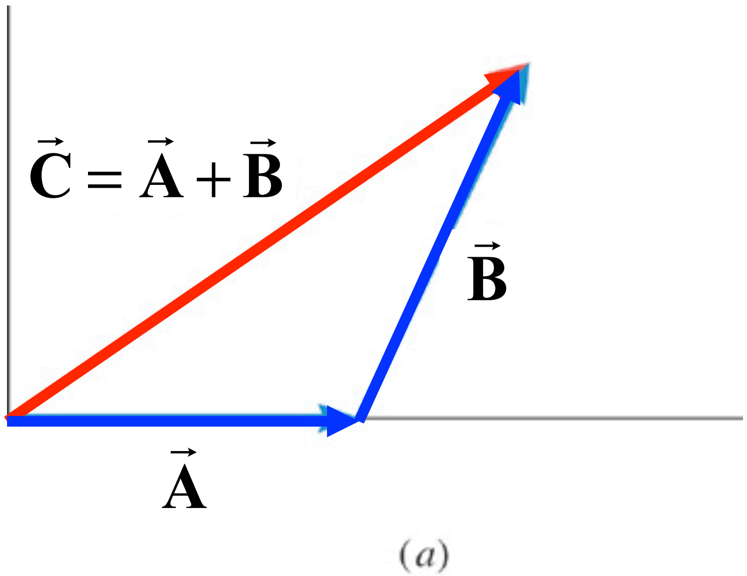
D) None of the above

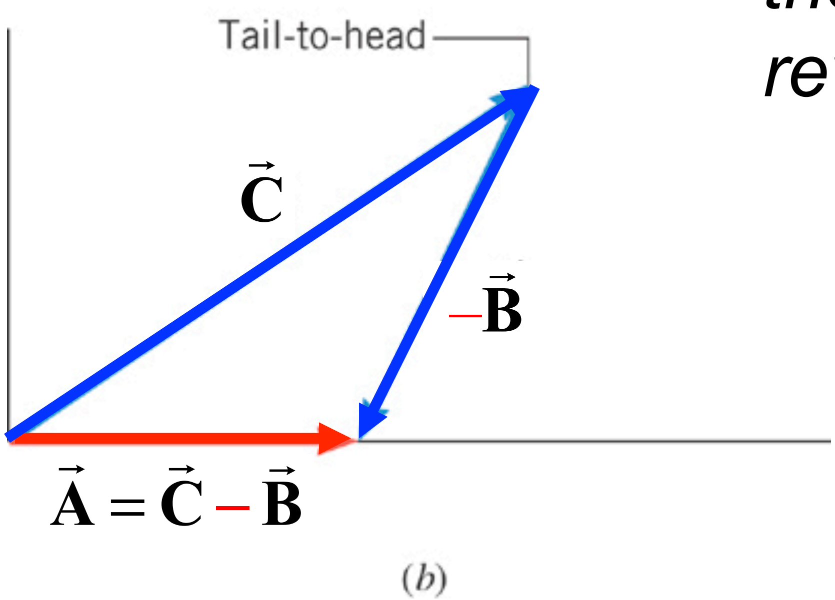
E) All of the above



Red vector is the same as $\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}$

3.2 Scalars and Vectors (Vector Addition and Subtraction)

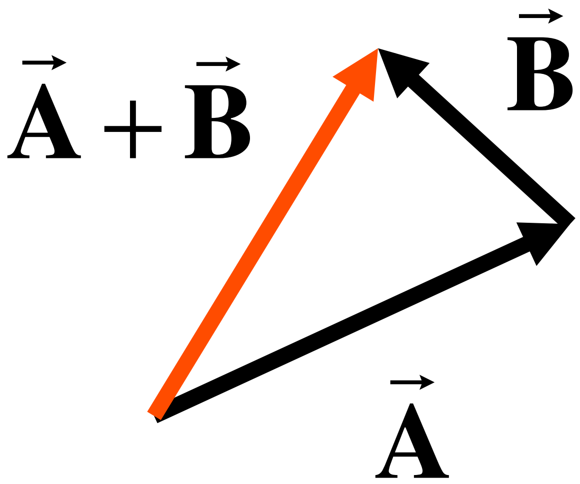




When a vector is multiplied by -1, the magnitude of the vector remains the same, but the direction of the vector is reversed.

3.2 Scalars and Vectors (Vector Addition and Subtraction)

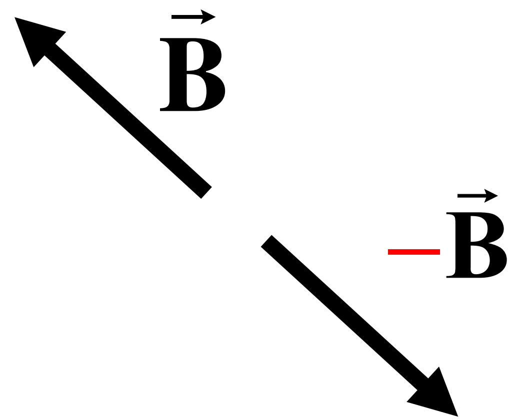
Add vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$



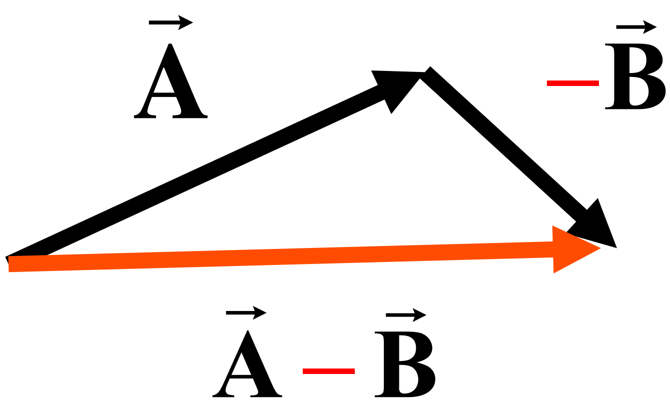
Now you are asked to find $\vec{\mathbf{A}}-\vec{\mathbf{B}}$

Instead of trying to do Vector Subtraction

add to vector $\vec{\mathbf{A}}$ the negative of the vector $\vec{\mathbf{B}}$

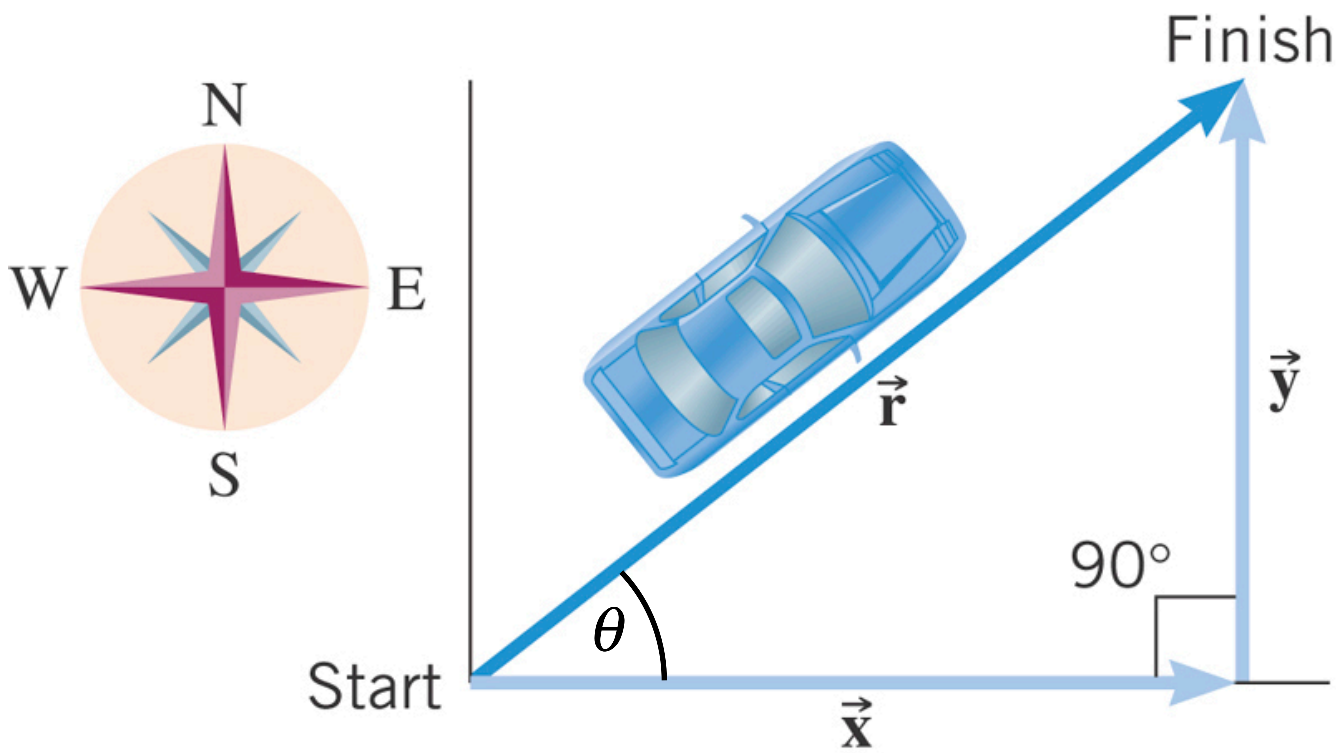


Subtracting vector $\vec{\mathbf{B}}$ from vector $\vec{\mathbf{A}}$



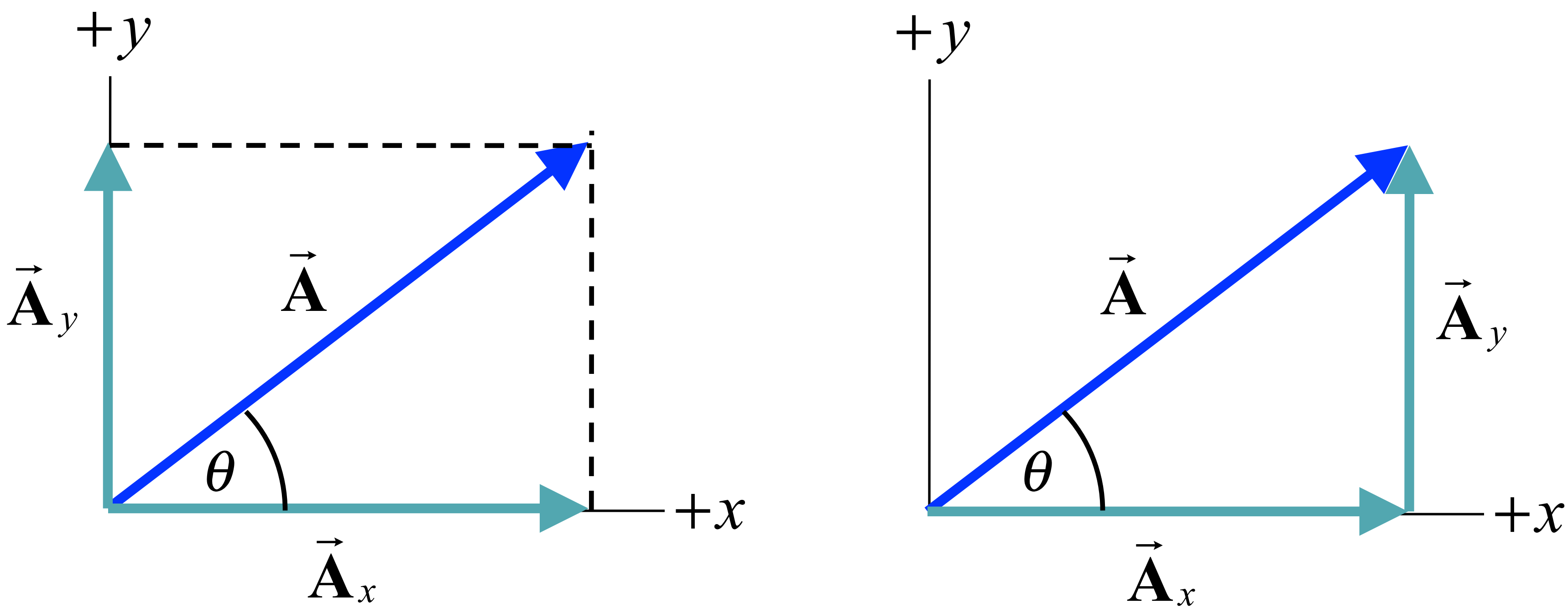
Adding vector $\vec{\mathbf{A}}$ to vector $-\vec{\mathbf{B}}$

3.2 Vector Addition and Subtraction (The Components of a Vector)



$\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$ are called the $x-$component vector and the $y-$component vector of $\vec{\mathbf{r}}$.

3.2 Vector Addition and Subtraction (The Components of a Vector)



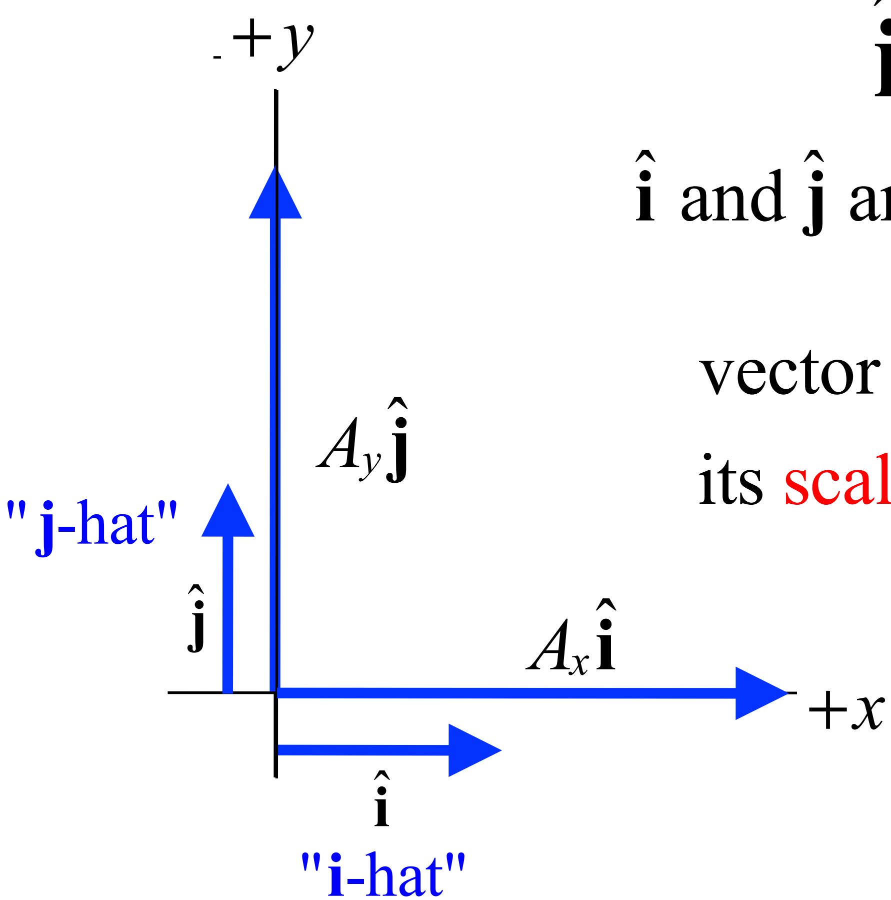
The vector components of $\vec{\mathbf{A}}$ are two perpendicular vectors $\vec{\mathbf{A}}_x$ and $\vec{\mathbf{A}}_y$ that are parallel to the x and y axes, and add together vectorially so that $\vec{\mathbf{A}} = \vec{\mathbf{A}}_\mathbf{x} + \vec{\mathbf{A}}_\mathbf{y}$.

3.2 Vector Addition and Subtraction (The Components of a Vector)

It is often easier to work with the **scalar components** of the vectors, rather than the component vectors themselves.

"**i**-hat" $\hat{\mathbf{i}}$ "**j**-hat" $\hat{\mathbf{j}}$

$\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are "unit vectors", magnitude is 1.

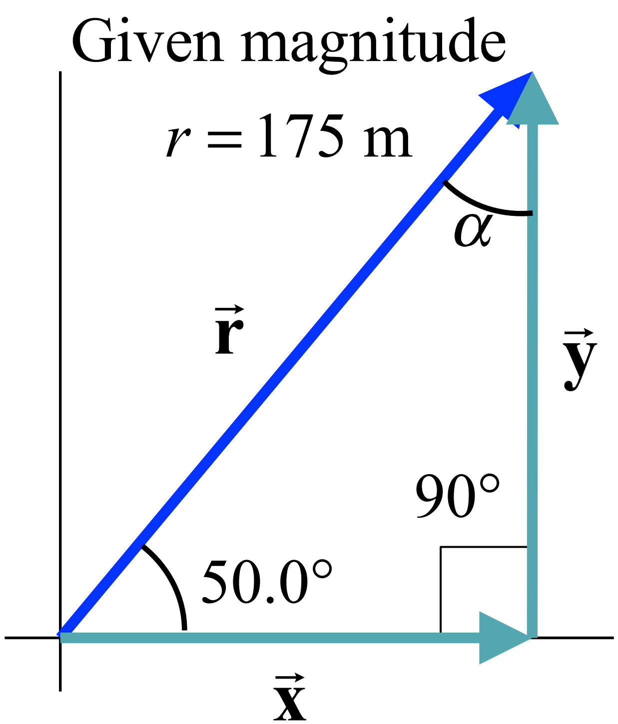


vector $\vec{\mathbf{A}}$ expressed in terms of its scalar components: $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$

A_x and A_y are the scalar components of the vector, $\vec{\mathbf{A}}$.

Example

A displacement vector has a magnitude of 175 m and points at an angle of 50.0 degrees relative to the *x* axis. Find the *x* and *y* components of this vector.



vector $\vec{\mathbf{x}}$ has magnitude x

vector $\vec{\mathbf{y}}$ has magnitude y

$$\sin\theta = y/r$$

y-component of the vector $\vec{\mathbf{r}}$

$$y = r\sin\theta = (175\text{ m})(\sin 50.0^\circ) = 134\text{ m}$$

$$\cos\theta = x/r$$

x-component of the vector $\vec{\mathbf{r}}$

$$x = r\cos\theta = (175\text{ m})(\cos 50.0^\circ) = 112\text{ m}$$

$$\boxed{\begin{aligned}\vec{\mathbf{r}} &= x\hat{\mathbf{i}} + y\hat{\mathbf{j}} \\ &= (112\text{ m})\hat{\mathbf{i}} + (134\text{ m})\hat{\mathbf{j}}\end{aligned}}$$

Check: $r = \sqrt{(112)^2 + (134)^2}\text{ m} = 175\text{m}$

Clicker Question 3.2

$\vec{\mathbf{A}} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}$, where $A_x = 3\,\mathrm{m}$, and $A_y = 4\,\mathrm{m}$.

What is the magnitude of the vector $\vec{\mathbf{A}}$?

A) $7\,\mathrm{m}$

B) $6\,\mathrm{m}$

C) $5\,\mathrm{m}$

D) $\sqrt{7}\,\mathrm{m}$

E) $25\,\mathrm{m}$

Clicker Question 3.2

$\vec{\mathbf{A}} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}$, where $A_x = 3\,\text{m}$, and $A_y = 4\,\text{m}$.

What is the magnitude of the vector $\vec{\mathbf{A}}$?

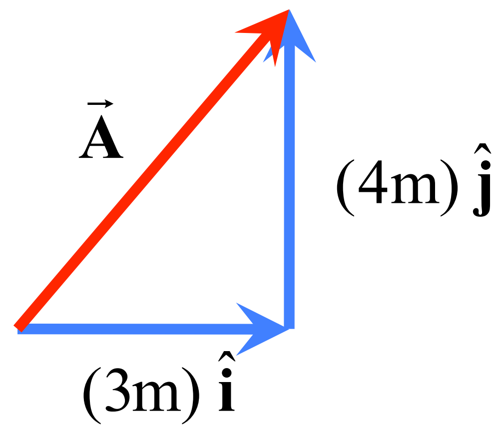
A) $7\,\text{m}$

B) $6\,\text{m}$

C) $\boxed{5\,\text{m}}$

D) $\sqrt{7}\,\text{m}$

E) $25\,\text{m}$

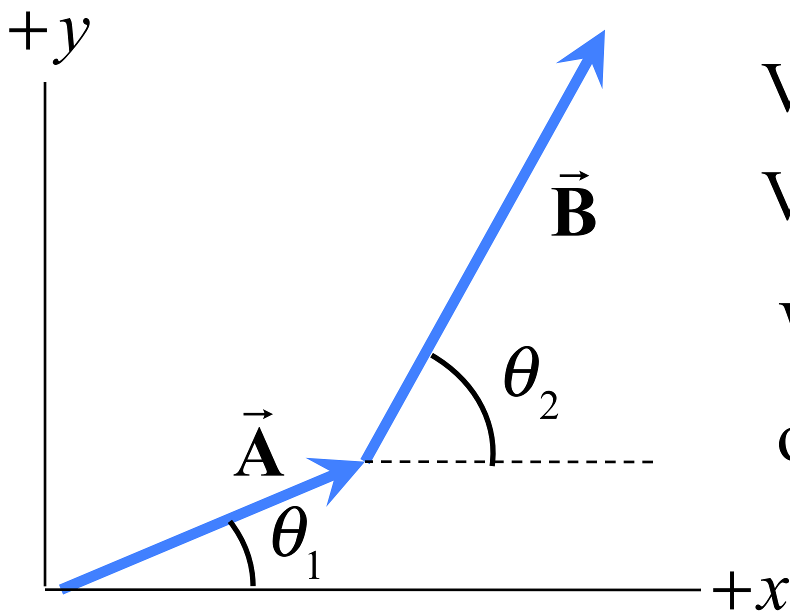


$$\vec{\mathbf{A}} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}$$

$$\text{magnitude of } \vec{\mathbf{A}},\; A = \sqrt{A_x^2 + A_y^2} = \sqrt{(3\text{m})^2 + (4\text{m})^2}$$

$$= \left(\sqrt{9+16}\right)\text{m} = \sqrt{25}\,\text{m} = 5\,\text{m}$$

3.2 Vector Addition and Subtraction (using Components)

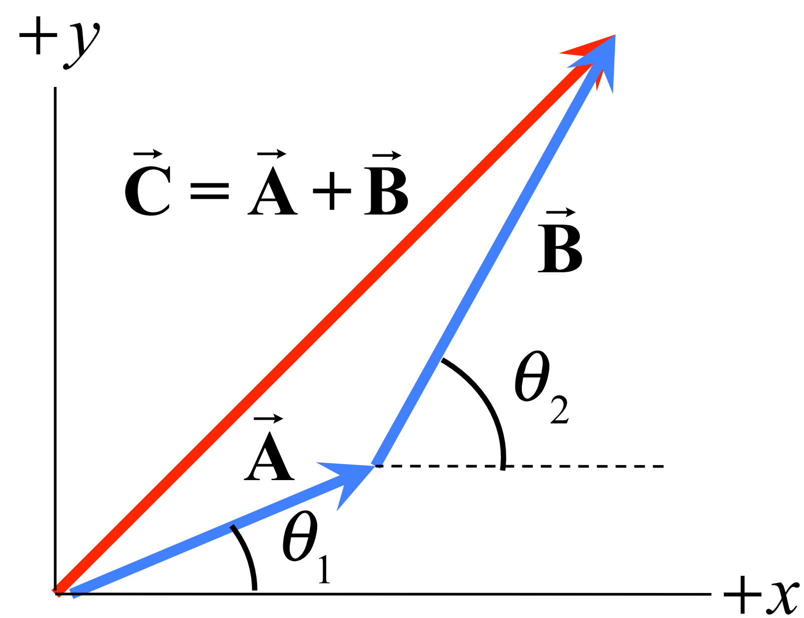


Vector $\vec{\mathbf{A}}$ has magnitude A and angle θ_1

Vector $\vec{\mathbf{B}}$ has magnitude B and angle θ_2

What is the magnitude and direction of vector $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$?

Graphically no PROBLEM



THIS IS A BIG PROBLEM

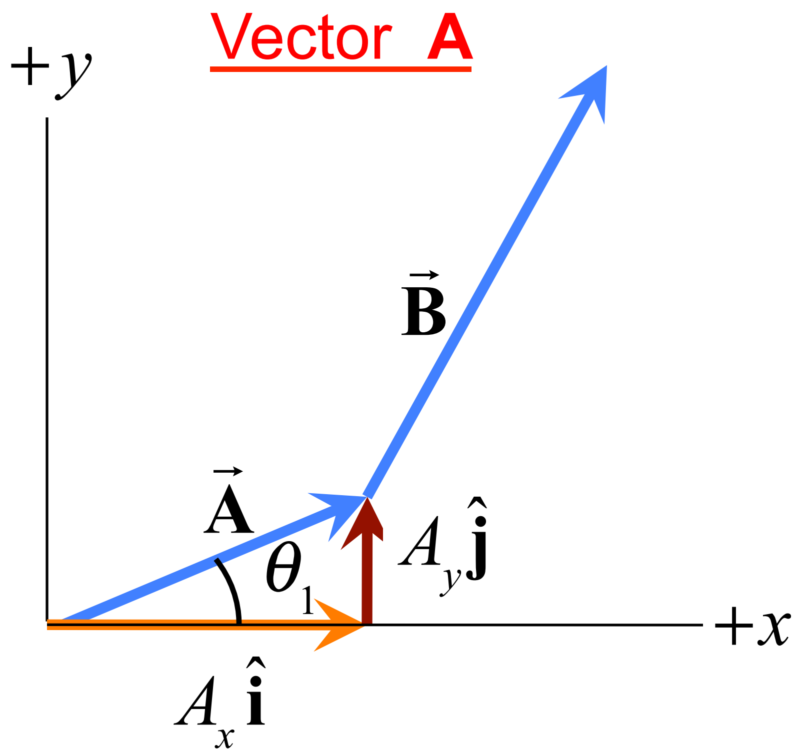
What is the magnitude of $\vec{\mathbf{C}}$, and what angle θ does it make relative to x-axis ?

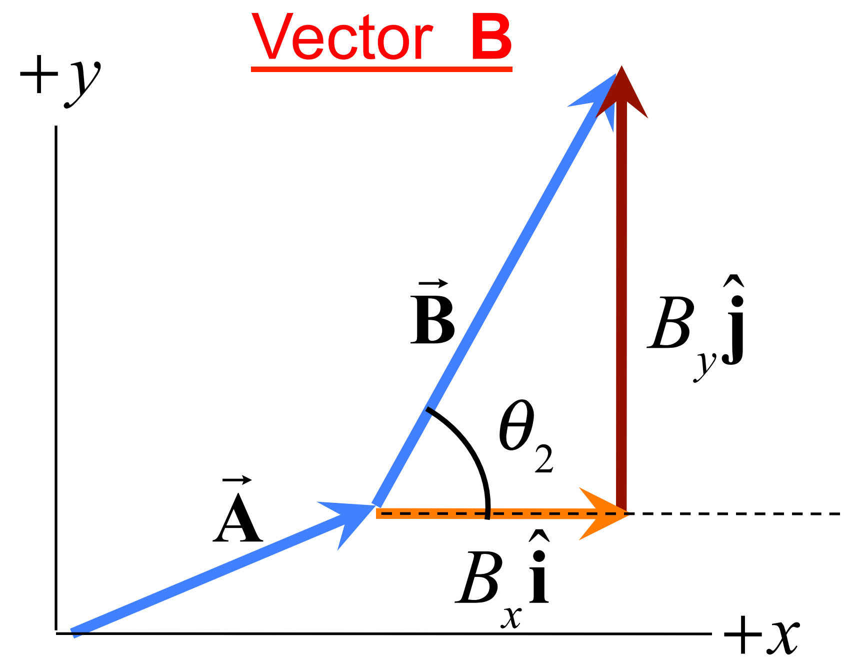
3.2 Vector Addition and Subtraction (using Components)

THIS IS A BIG PROBLEM

What is the magnitude of $\vec{\mathbf{C}}$, and what angle θ does it make relative to x-axis ?

The only way to solve this problem is to use vector components!





3.2 Vector Addition and Subtraction (using Components)

Get the components of the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

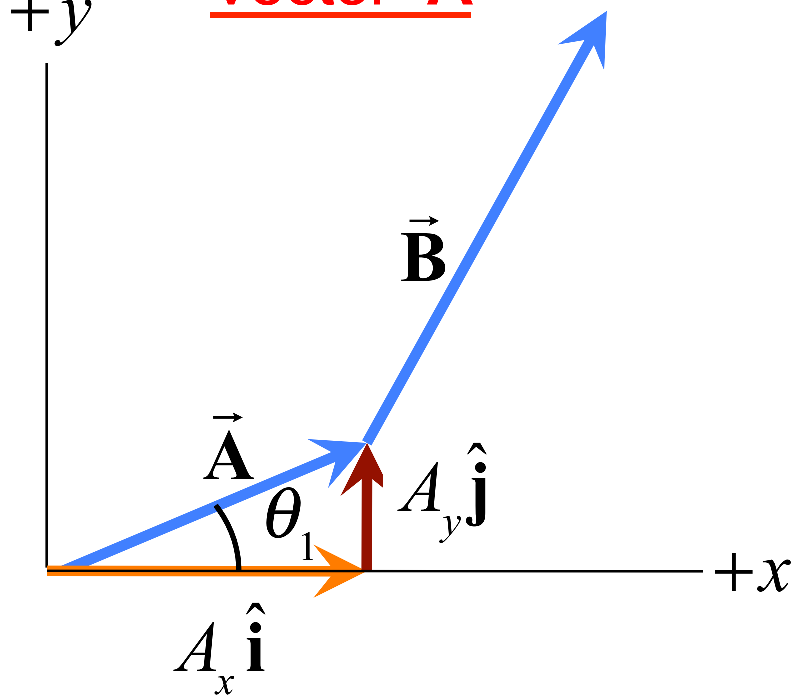
$\vec{\mathbf{A}}$: magnitude A and angle θ_1

$\vec{\mathbf{B}}$: magnitude B and angle θ_2

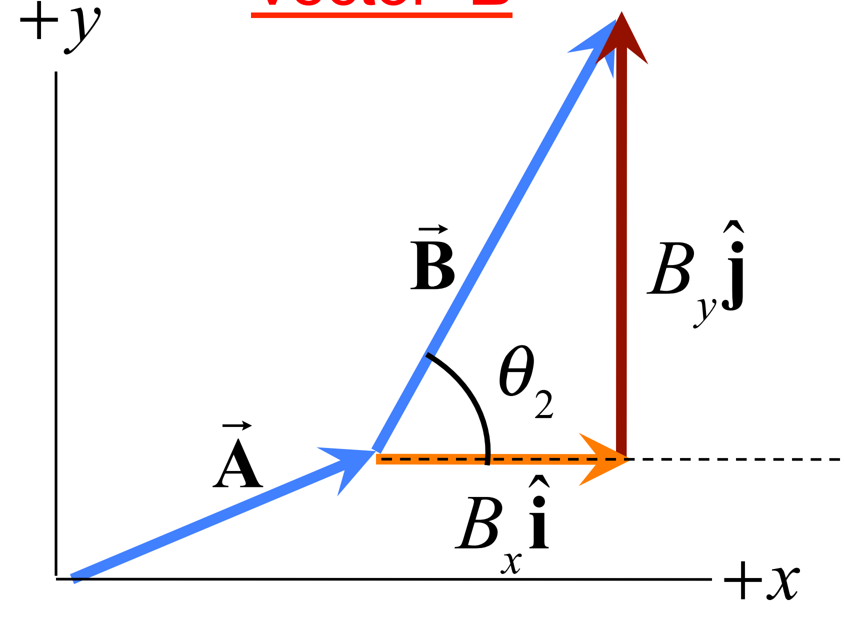
$$A_x = A\cos\theta_1$$
$$A_y = A\sin\theta_1$$

$$B_x = B\cos\theta_2$$
$$B_y = B\sin\theta_2$$

Vector A



Vector B

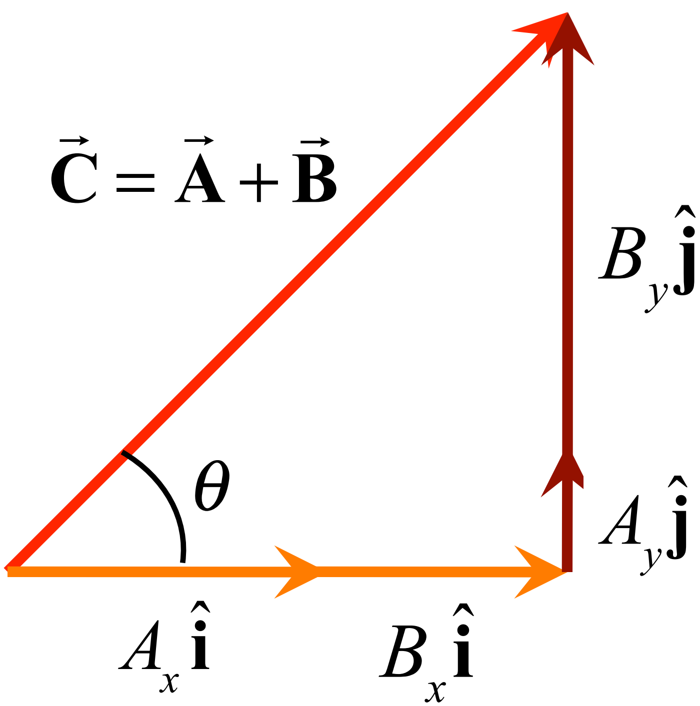


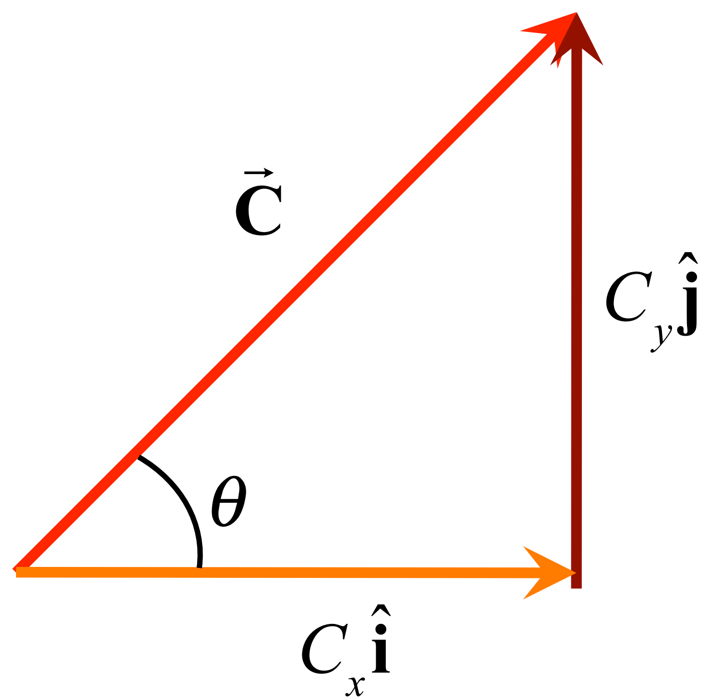
3.2 Vector Addition and Subtraction (using Components)

Get components of vector $\vec{\mathbf{C}}$ from components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

$$A_x = A\cos\theta_1 \qquad B_x = B\cos\theta_2 \quad \Rightarrow \quad C_x = A_x + B_x$$

$$A_y = A\sin\theta_1 \qquad B_y = B\sin\theta_2 \quad \Rightarrow \quad C_y = A_y + B_y$$

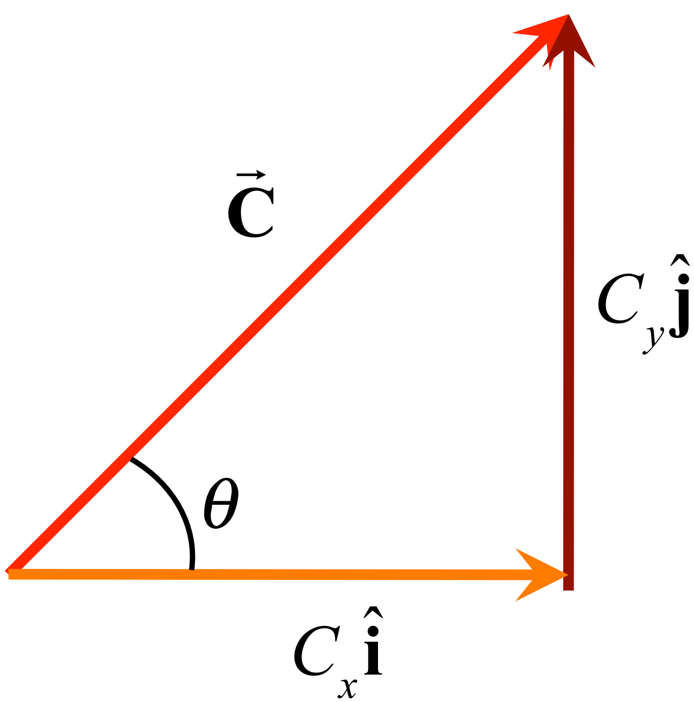




3.2 Vector Addition and Subtraction (using Components)

What is the magnitude of $\vec{\mathbf{C}}$, and what angle θ does it make relative to x-axis ?

$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$



PROBLEM IS SOLVED

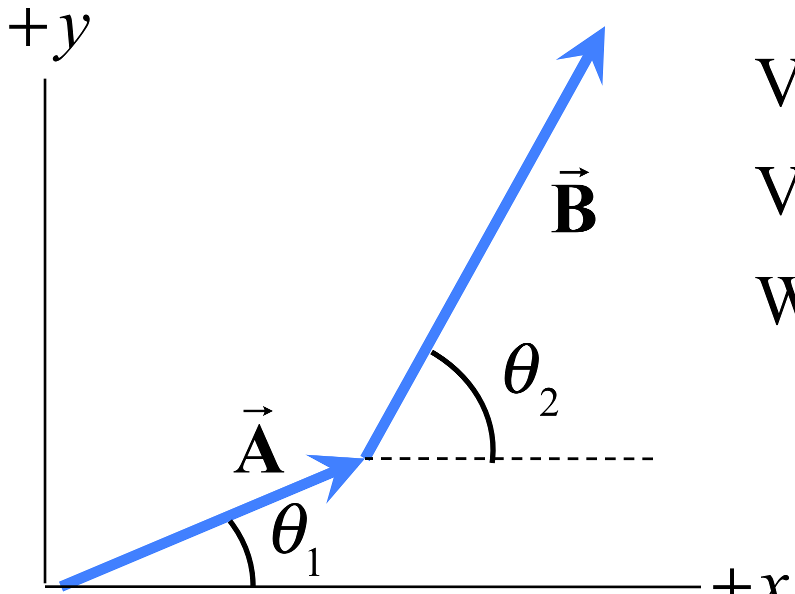
magnitude of $\vec{\mathbf{C}}$: $C = \sqrt{C_x^2 + C_y^2}$

Angle θ: $\tan\theta = \dfrac{C_y}{C_x}$;

$$\theta = \tan^{-1}(C_y / C_x)$$

3.2 Addition of Vectors by Means of Components

Summary of adding two vectors together



Vector $\vec{\mathbf{A}}$ has magnitude A and angle θ_1

Vector $\vec{\mathbf{B}}$ has magnitude B and angle θ_2

What is the vector $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$?

1) Determine components of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$: A_x, A_y and B_x, B_y

2) Add x-components to find $C_x = A_x + B_x$

3) Add y-components to find $C_y = A_y + B_y$

4) Determine the magnitude and angle of vector $\vec{\mathbf{C}}$

magnitude $C = \sqrt{C_x^2 + C_y^2}$; $\quad \theta = \tan^{-1}(C_y / C_x)$