

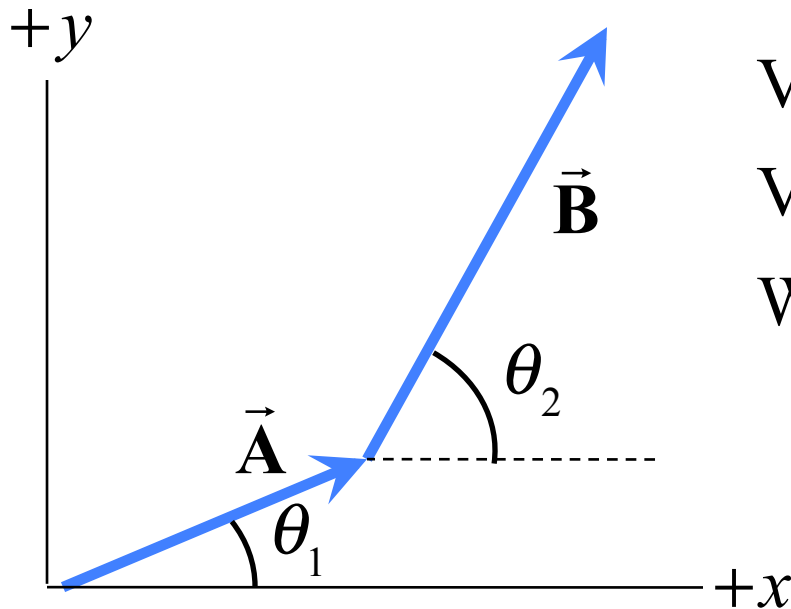
Chapter 3

Kinematics in Two Dimensions

continued

3.2 Addition of Vectors by Means of Components

Summary of adding two vectors together



Vector $\vec{A}$ has magnitude A and angle θ_1

Vector $\vec{B}$ has magnitude B and angle θ_2

What is the vector $\vec{C} = \vec{A} + \vec{B}$?

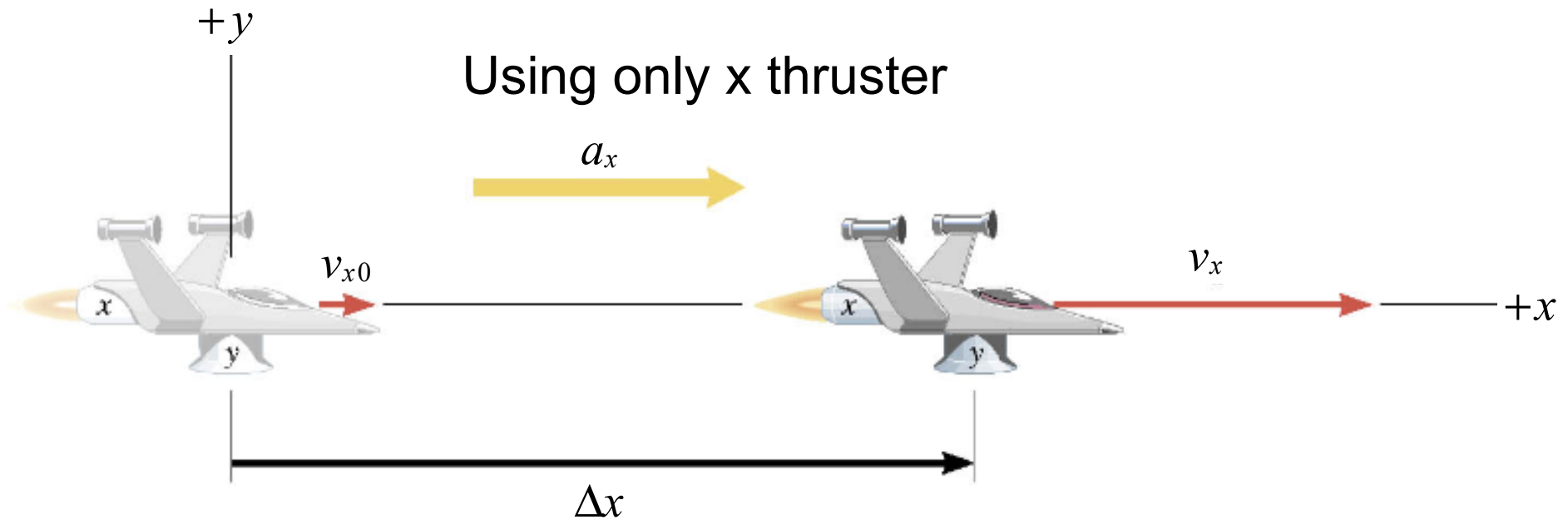
$$\vec{A} = A_x \hat{i} + A_y \hat{j} \quad \vec{B} = B_x \hat{i} + B_y \hat{j}$$

- 1) Determine components of vectors $\vec{A}$ and $\vec{B}$: A_x, A_y and B_x, B_y
- 2) Add x-components to find $C_x = A_x + B_x$
- 3) Add y-components to find $C_y = A_y + B_y$
- 4) Determine the magnitude and angle of vector $\vec{C}$

$$\text{magnitude } C = \sqrt{C_x^2 + C_y^2}; \quad \theta = \tan^{-1}(C_y/C_x)$$

3.2 Equations of Kinematics in Two Dimensions

Except for time, motion in x and y directions are INDEPENDENT.



Motion in x direction with constant acceleration.

$$v_x = v_{x0} + a_x t$$

$$\Delta x = \frac{1}{2} (v_{x0} + v_x) t$$

$$\Delta x = v_{x0} t + \frac{1}{2} a_x t^2$$

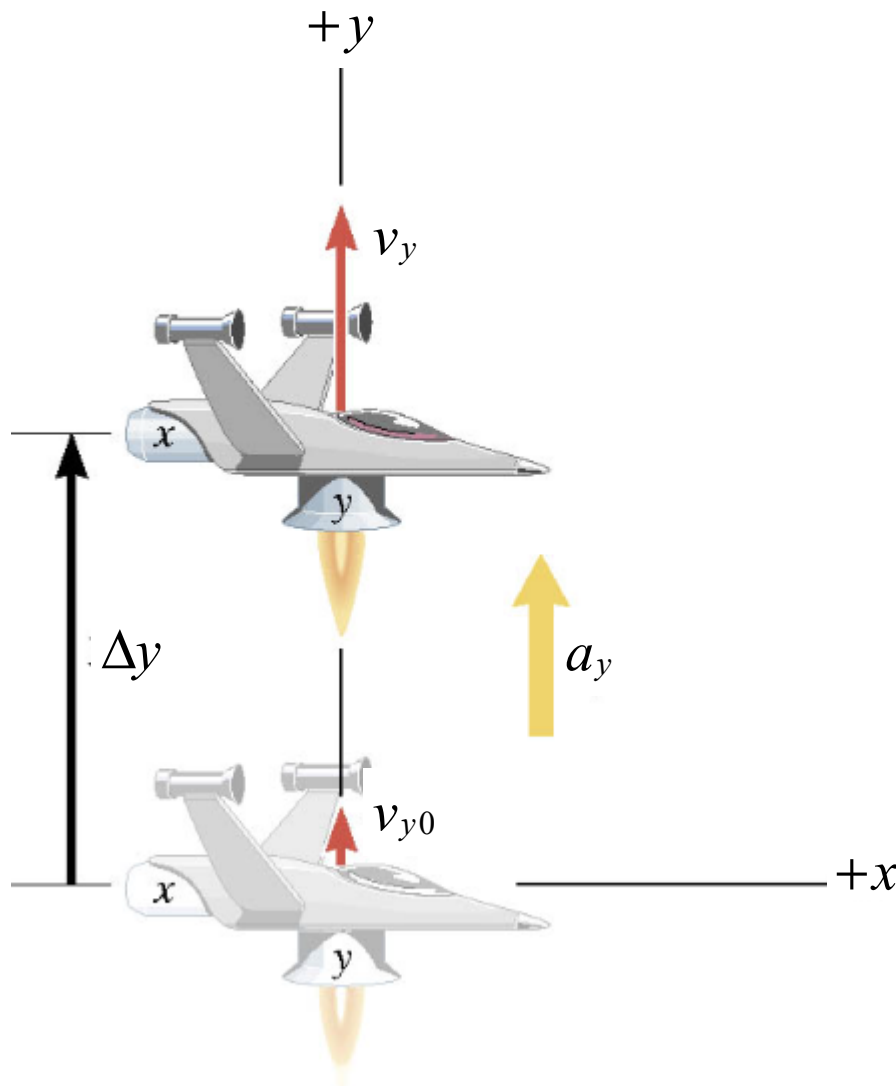
$$v_x^2 = v_{x0}^2 + 2a_x \Delta x$$

3.2 Equations of Kinematics in Two Dimensions

Except for time, motion in x and y directions are INDEPENDENT.

Constant acceleration
motion in y direction.

Using only y thruster



$$v_y = v_{y0} + a_y t$$

$$\Delta y = v_{y0} t + \frac{1}{2} a_y t^2$$

$$\Delta y = \frac{1}{2} (v_{y0} + v_y) t$$

$$v_y^2 = v_{y0}^2 + 2a_y \Delta y$$

3.2 *Equations of Kinematics in Two Dimensions*

Reasoning Strategy

1. Make a drawing.
2. Decide which directions are to be called positive (+) and negative (-).
3. Write down the values that are given for any of the five kinematic variables associated with each direction.

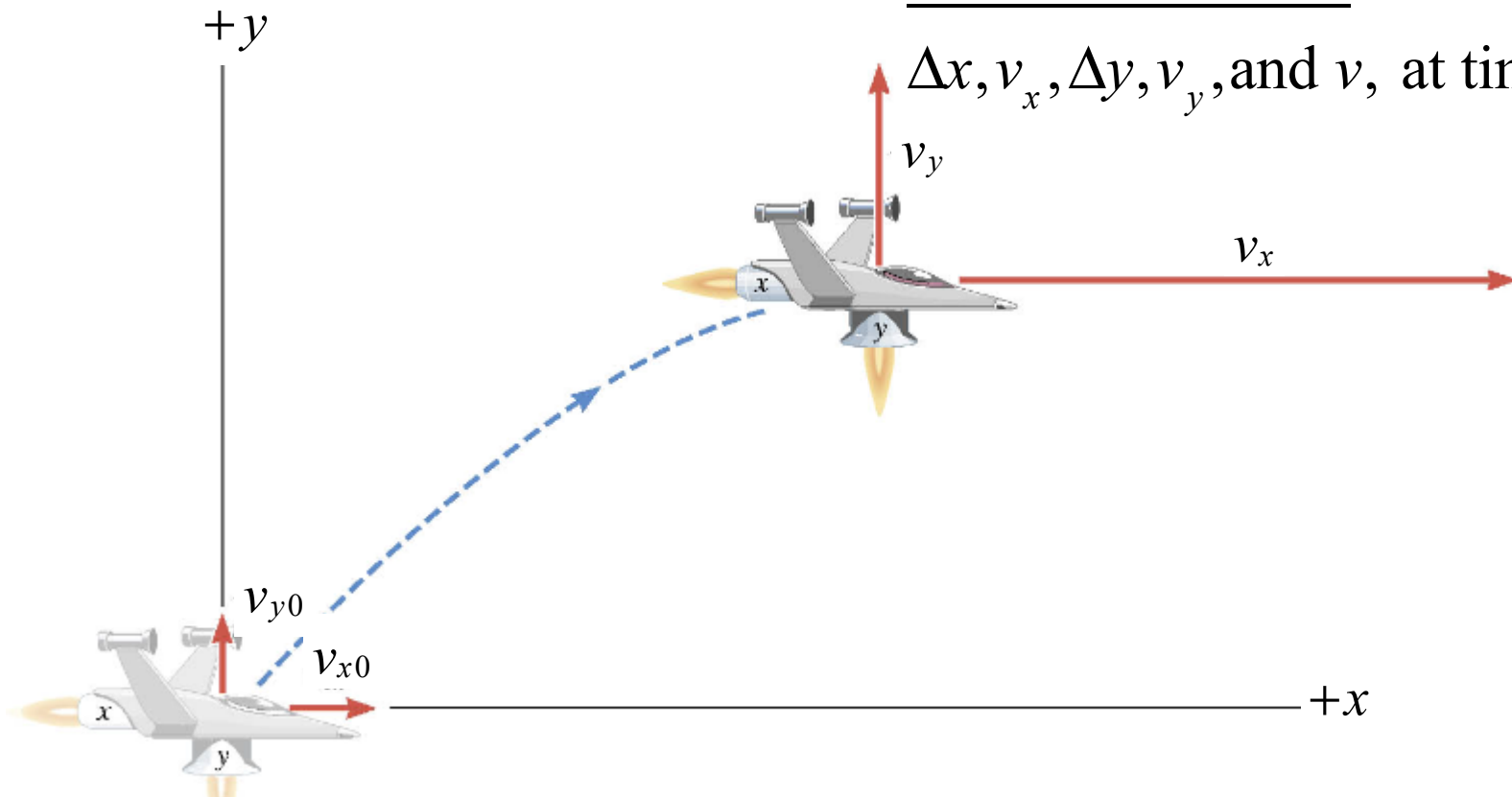
3.2 Equations of Kinematics in Two Dimensions

Example: A Moving Spacecraft

In the x direction, the spacecraft has an initial velocity component of $+22$ m/s and an acceleration of $+24$ m/s². In the y direction, the analogous quantities are $+14$ m/s and an acceleration of $+12$ m/s². Find (a) Δx and v_x , (b) Δy and v_y , and (c) the final velocity of the spacecraft at a time 7.0 s later.

Want final values:

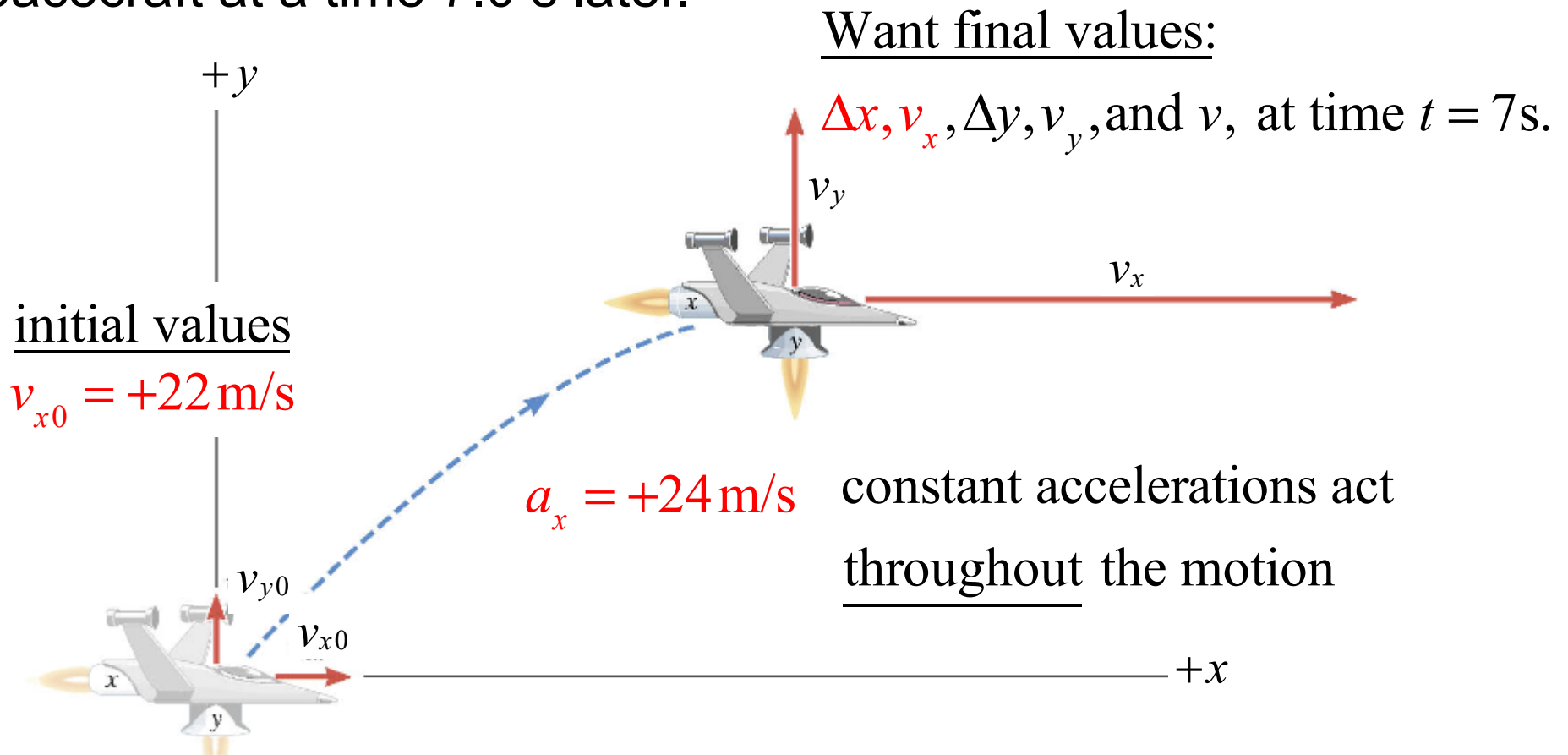
$\Delta x, v_x, \Delta y, v_y$, and v , at time $t = 7$ s.



3.2 Equations of Kinematics in Two Dimensions

Example: A Moving Spacecraft

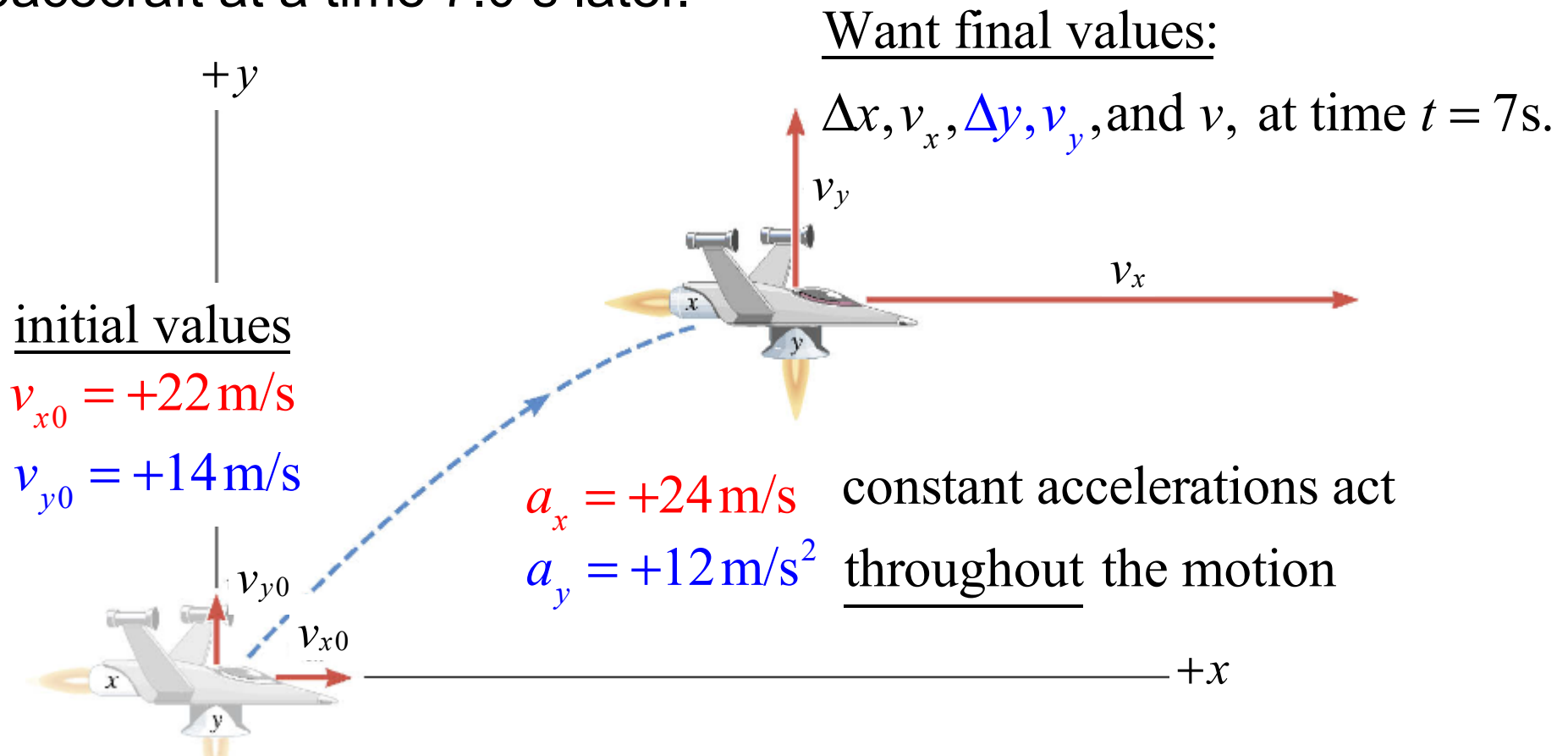
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3.2 Equations of Kinematics in Two Dimensions

Example: A Moving Spacecraft

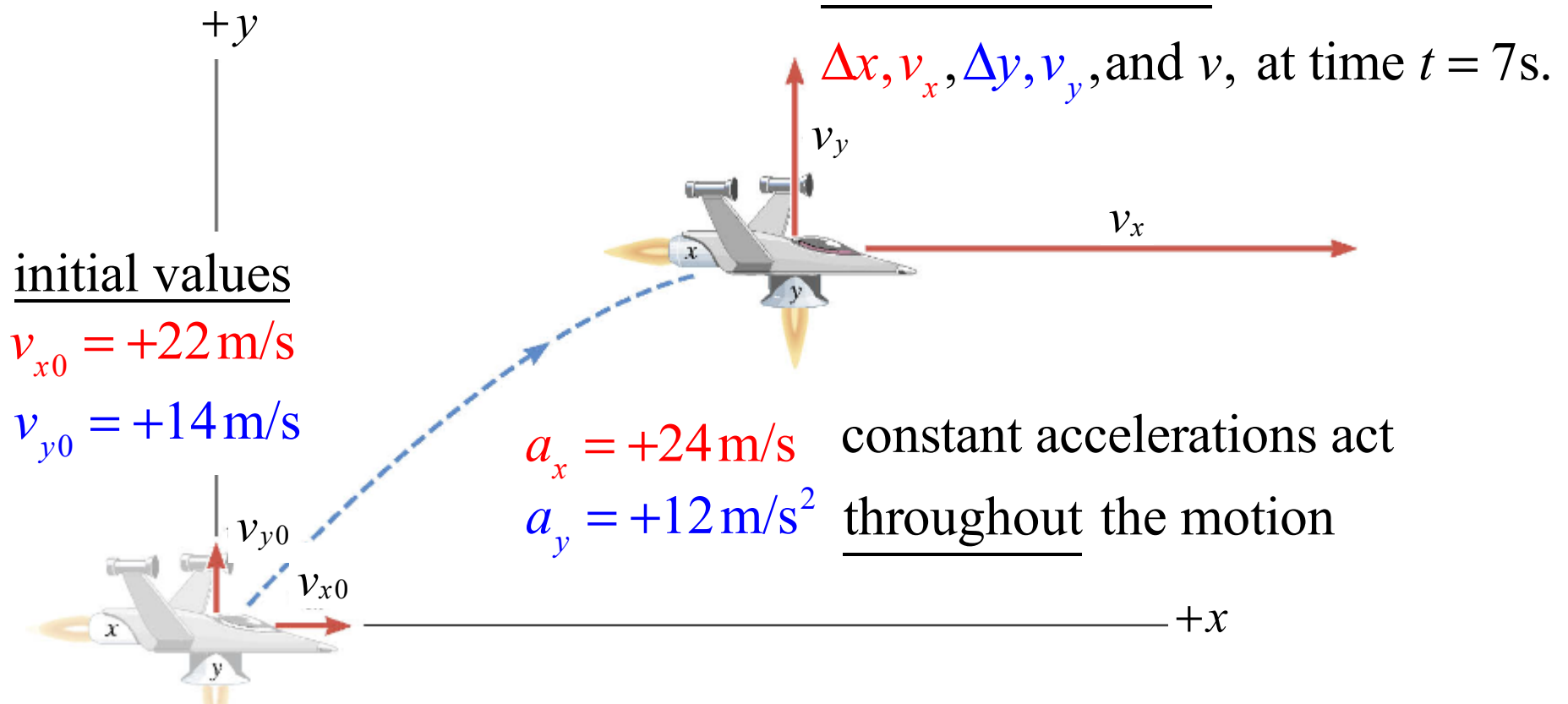
In the x direction, the spacecraft has an initial velocity component of $+22 \text{ m/s}$ and an acceleration of $+24 \text{ m/s}^2$. In the y direction, the analogous quantities are $+14 \text{ m/s}$ and an acceleration of $+12 \text{ m/s}^2$. Find (a) Δx and v_x , (b) Δy and v_y , and (c) the final velocity of the spacecraft at a time 7.0 s later.



3.2 Equations of Kinematics in Two Dimensions

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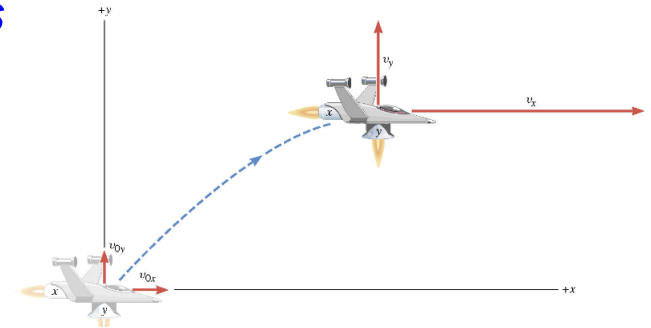
3.2 *Equations of Kinematics in Two Dimensions*

Reasoning Strategy

1. Make a drawing.
2. Decide which directions are to be called positive (+) and negative (-).
3. Write down the values that are given for any of the five kinematic variables associated with each direction.
4. Verify that the information contains values for at least three of the kinematic variables. Do this for x and y . Select the appropriate equation.
5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.
6. Keep in mind that there may be two possible answers to a kinematics problem.

3.2 Equations of Kinematics in Two Dimensions

Example: A Moving Spacecraft:



x direction motion

Δx	a_x	v_x	v_{x0}	t
?	+24.0 m/s ²	?	+22 m/s	7.0 s

$$\begin{aligned}\Delta x &= v_{x0}t + \frac{1}{2}a_x t^2 \\ &= (22 \text{ m/s})(7.0 \text{ s}) + \frac{1}{2}(24 \text{ m/s}^2)(7.0 \text{ s})^2 = +740 \text{ m}\end{aligned}$$

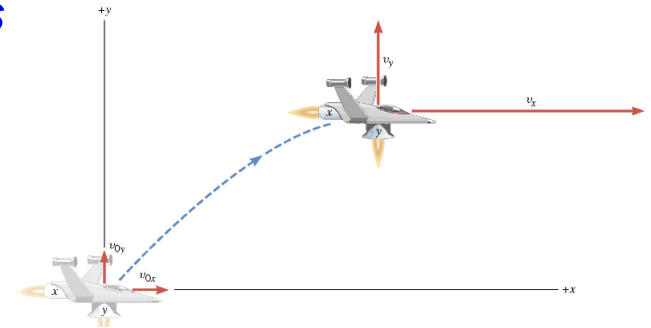
x displacement

$$\begin{aligned}v_x &= v_{x0} + a_x t \\ &= (22 \text{ m/s}) + (24 \text{ m/s}^2)(7.0 \text{ s}) = +190 \text{ m/s}\end{aligned}$$

x component of velocity

3.2 Equations of Kinematics in Two Dimensions

Example: A Moving Spacecraft:



y direction motion

Δy	a_y	v_y	v_{y0}	t
?	+12.0 m/s ²	?	+14 m/s	7.0 s

$$\Delta y = v_{y0}t + \frac{1}{2}a_y t^2$$

y displacement

$$= (14 \text{ m/s})(7.0 \text{ s}) + \frac{1}{2}(12 \text{ m/s}^2)(7.0 \text{ s})^2 = +390 \text{ m}$$

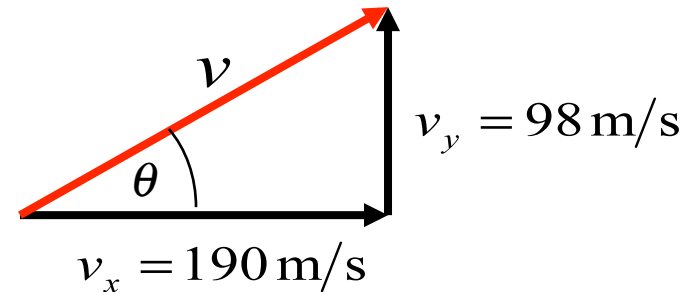
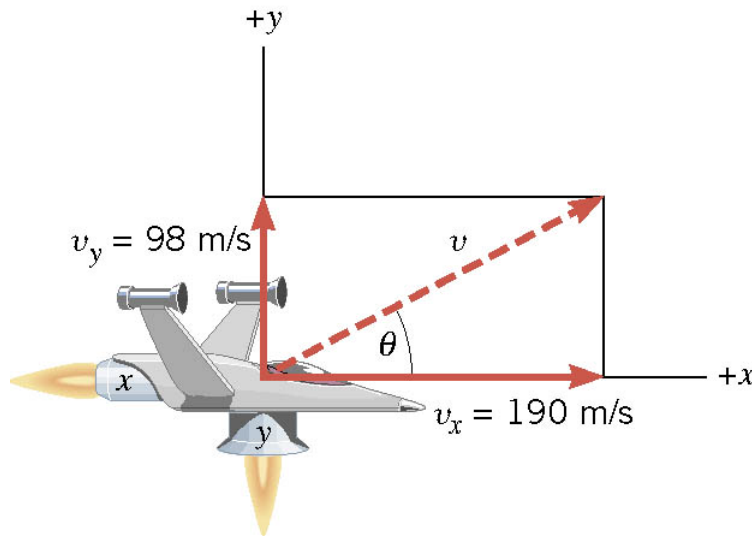
$$v_y = v_{y0} + a_y t$$

y component of velocity

$$= (14 \text{ m/s}) + (12 \text{ m/s}^2)(7.0 \text{ s}) = +98 \text{ m/s}$$

3.2 Equations of Kinematics in Two Dimensions

Can also find final speed and direction (angle) at $t = 7\text{s}$.



$$\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{(190 \text{ m/s})^2 + (98 \text{ m/s})^2} = 210 \text{ m/s} \end{aligned}$$

$$\theta = \tan^{-1}(98/190) = 27^\circ$$

3.3 *Projectile Motion*

Under the influence of gravity alone, an object near the surface of the Earth will accelerate downwards at 9.81m/s^2 .

Great simplification for projectiles !

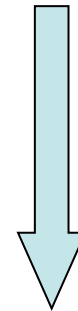
y direction

up is positive so

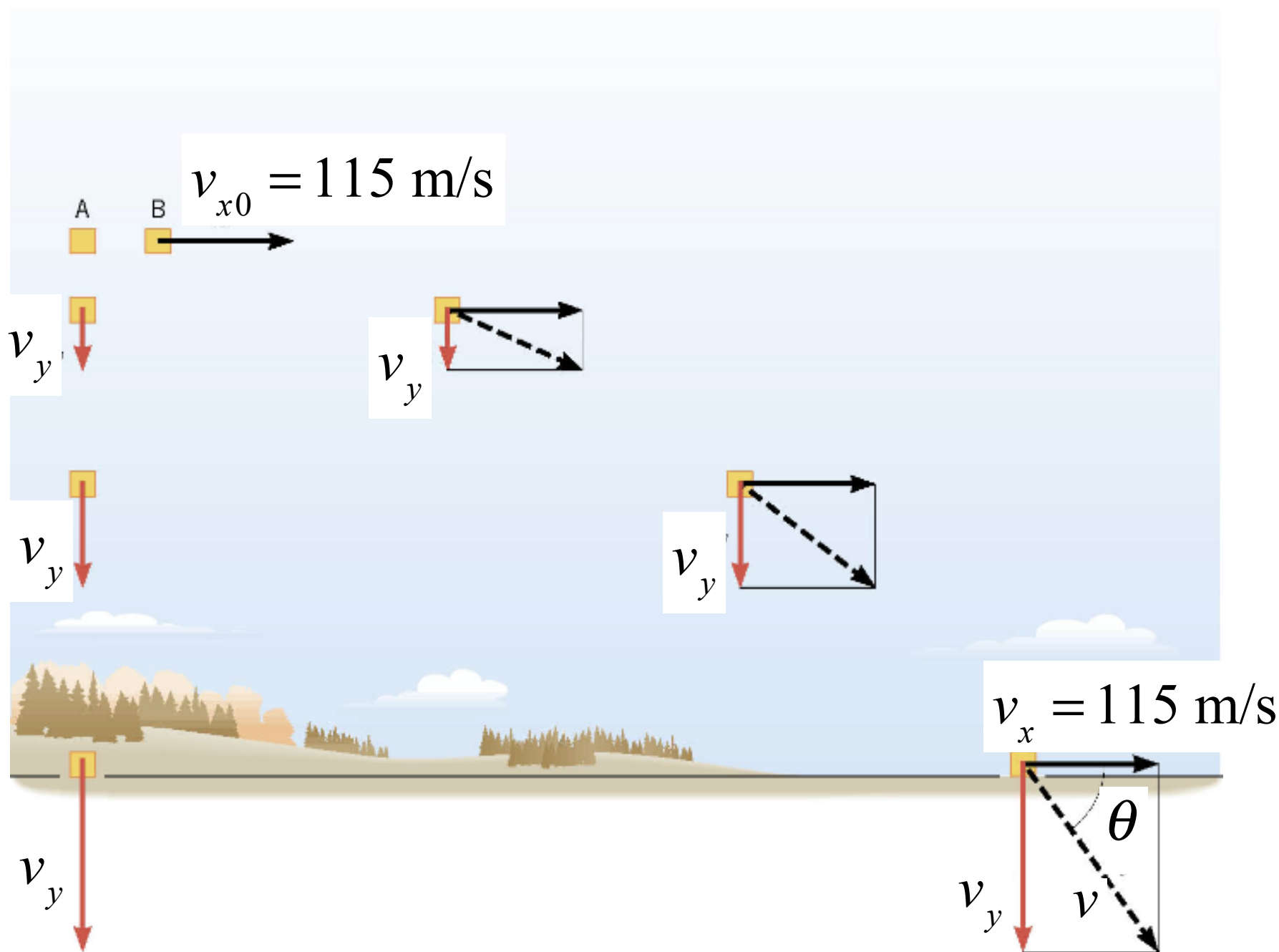
$$a_y = -9.81\text{m/s}^2$$

x direction

$$a_x = 0$$



$$v_x = v_{x0} = \text{constant}$$

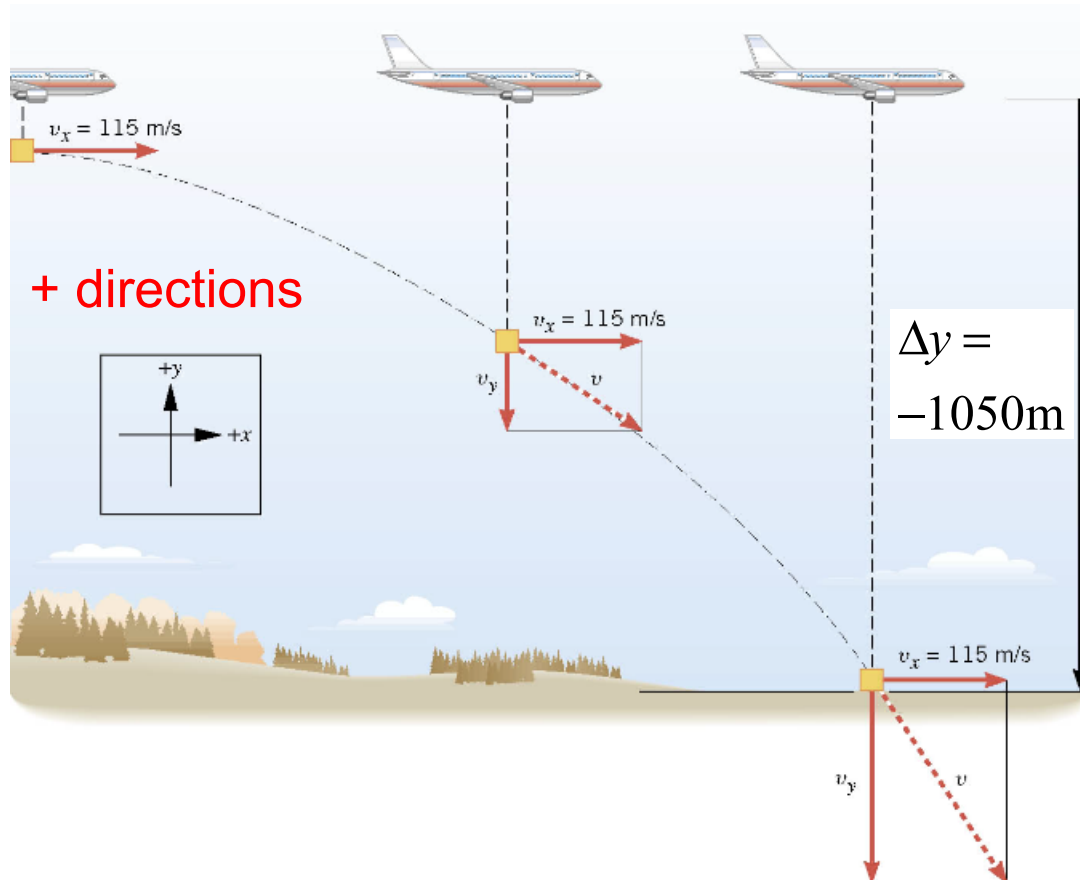


3.3 Projectile Motion

Example: A Falling Care Package

The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.

Time to hit the ground depends
ONLY on vertical (y) motion



$$v_{y0} = 0$$

$$a_y = -9.81 \text{ m/s}^2$$

$$\Delta y = -1050 \text{ m}$$

Displacement in y is in
the negative direction

Why not positive?

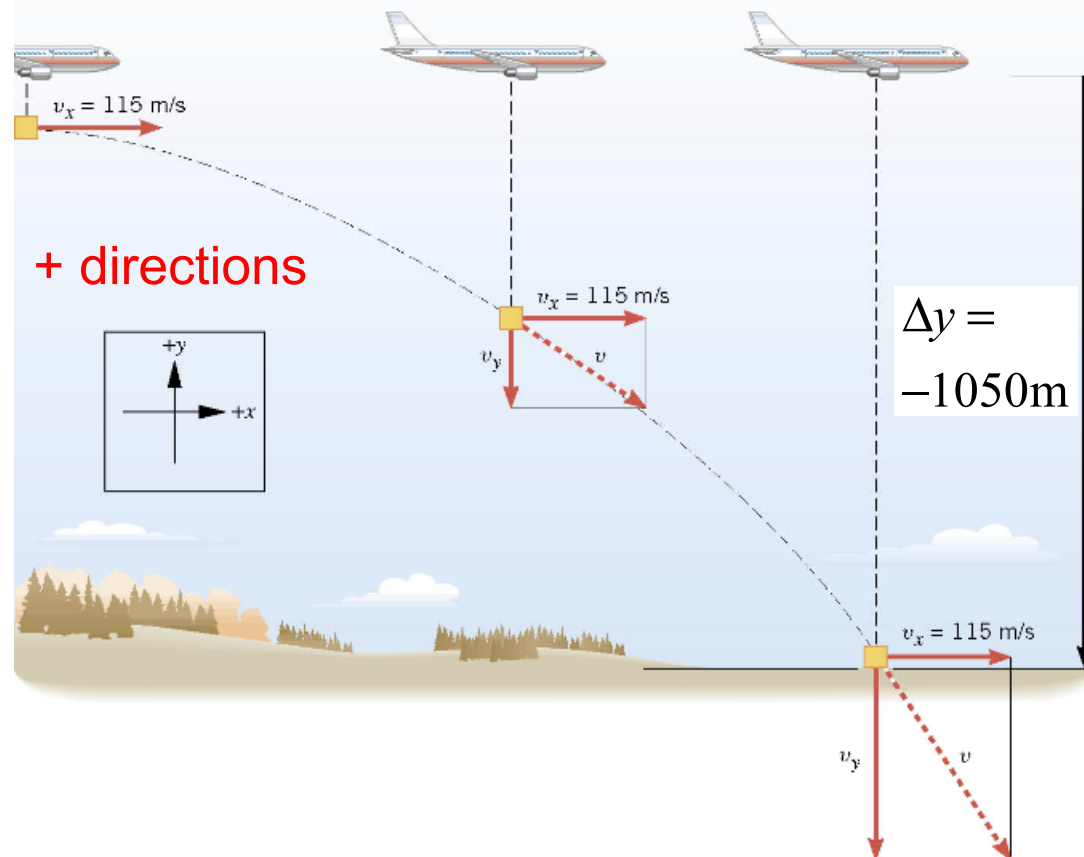
3.3 Projectile Motion

Example: A Falling Care Package

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Time to hit the ground depends
ONLY on vertical (y) motion

Put $y = 0$ at the TOP



$$y_0 = 0 \text{ m}$$

$$\begin{aligned}\Delta y &= y - y_0 \\ &= (-1050 - 0) \\ &= -1050\end{aligned}$$

$$y = -1050 \text{ m}$$

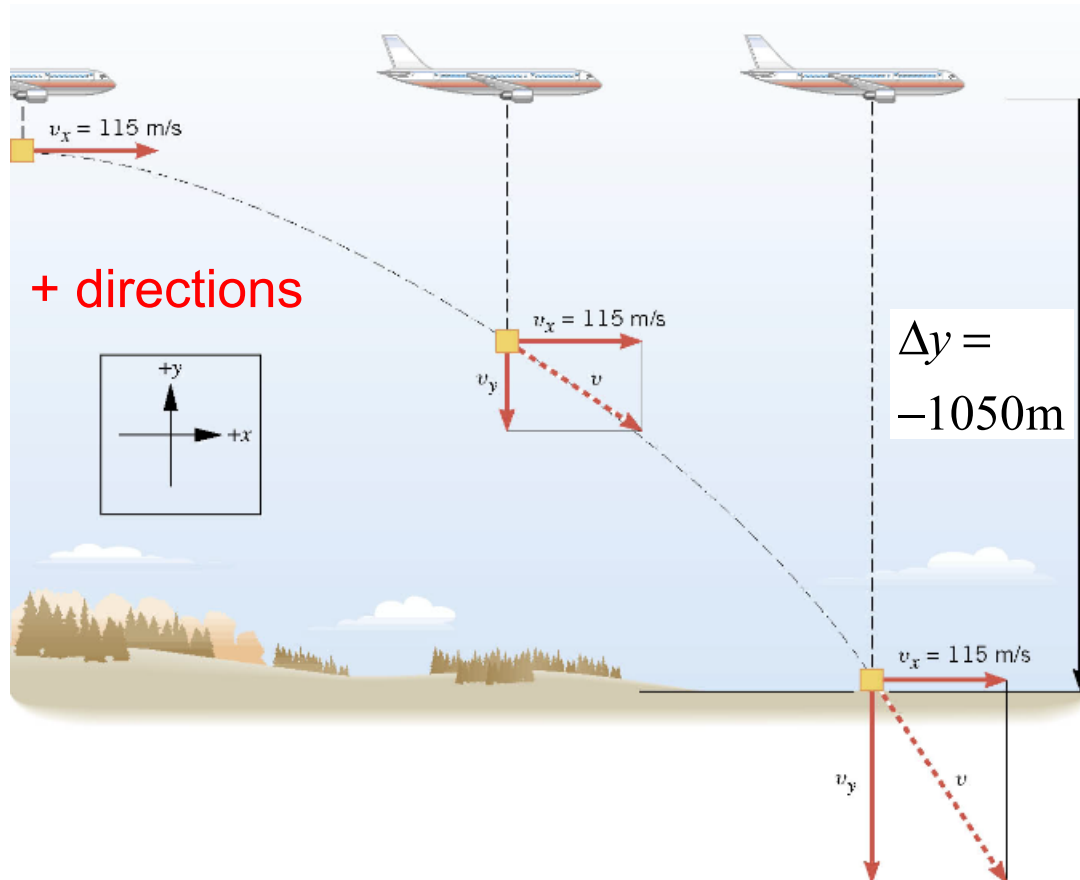
Final y is 1050m BELOW $y = 0$

3.3 Projectile Motion

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The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.

Time to hit the ground depends
ONLY on vertical (y) motion



$$y_0 = +1050 \text{ m}$$

$$\begin{aligned}\Delta y &= y - y_0 \\ &= (0 - 1050) \\ &= -1050\end{aligned}$$

$$y = 0 \text{ m}$$

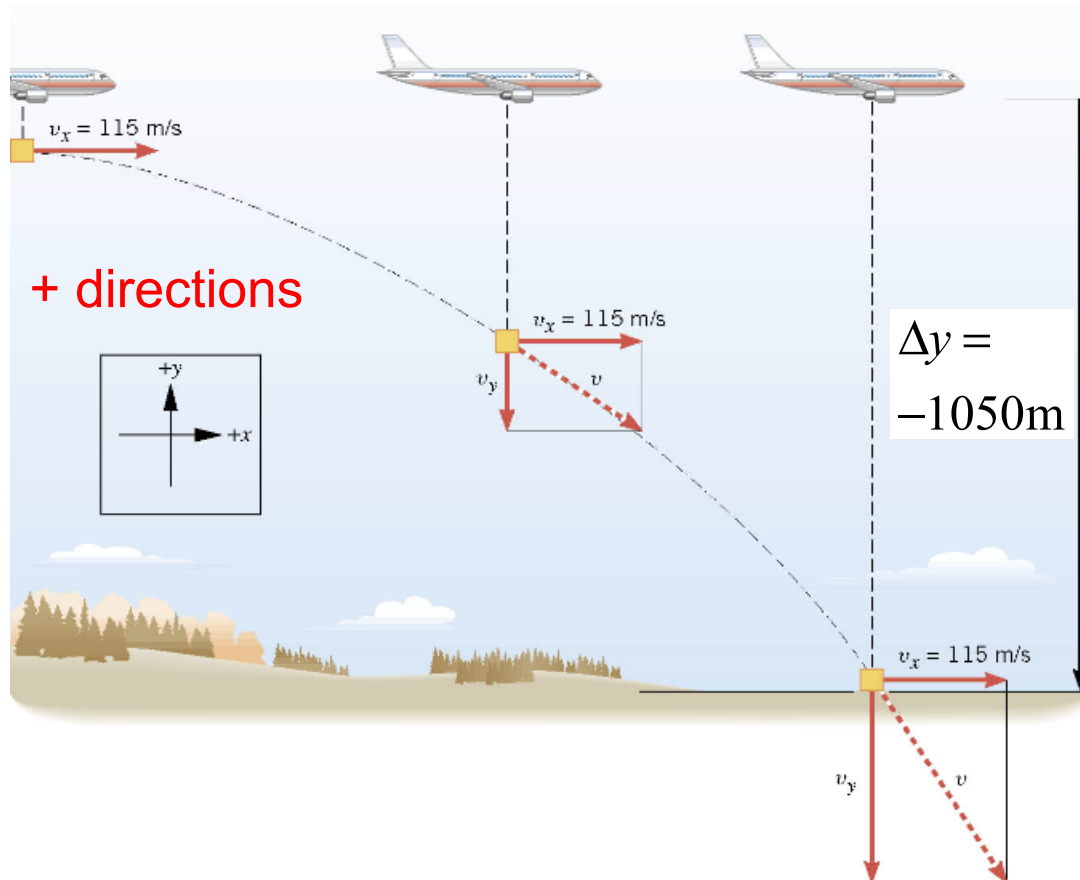
Put $y = 0$ at the BOTTOM

3.3 Projectile Motion

Example: A Falling Care Package

The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.

Time to hit the ground depends
ONLY on vertical (y) motion



$$v_{y0} = 0$$

$$a_y = -9.81 \text{ m/s}^2$$

$$\Delta y = -1050 \text{ m}$$

3.3 *Projectile Motion*

Δy	a_y	v_y	v_{y0}	t
-1050 m	-9.81 m/s ²		0 m/s	?

$$\Delta y = v_{y0}t + \frac{1}{2} a_y t^2 \quad \longrightarrow \quad \Delta y = \frac{1}{2} a_y t^2$$

$$t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-1050 \text{ m})}{-9.81 \text{ m/s}^2}} \\ = 14.6 \text{ s}$$

3.3 Projectile Motion

Example: The Velocity of the Care Package

What are the magnitude and direction of the final velocity of the care package?

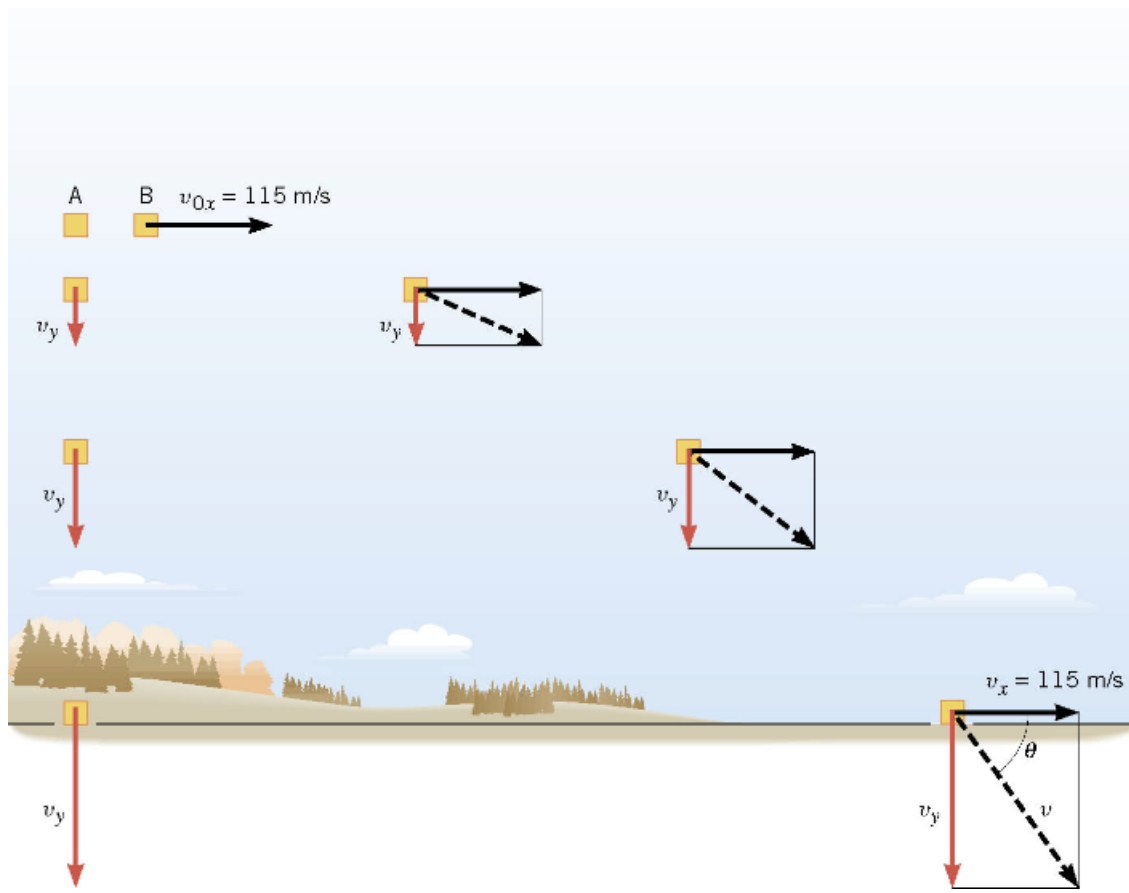
$$t = 14.6 \text{ s}$$

BECAUSE x-component of acceleration is zero

$$a_x = 0; \quad v_{x0} = +115 \text{ m/s}$$

$$\begin{aligned} v_x &= v_{x0} + a_x t \\ &= +115 \text{ m/s} \end{aligned}$$

x-component of velocity does not change



3.3 Projectile Motion

Δy	a_y	v_y	v_{y0}	t
-1050 m	-9.81 m/s ²	?	0 m/s	14.6 s

$$\begin{aligned}v_y &= v_{y0} + a_y t = 0 + (-9.81 \text{ m/s}^2)(14.6 \text{ s}) \\&= -143 \text{ m/s} \quad \text{y-component of final velocity.}\end{aligned}$$

Now ready to get final speed and direction

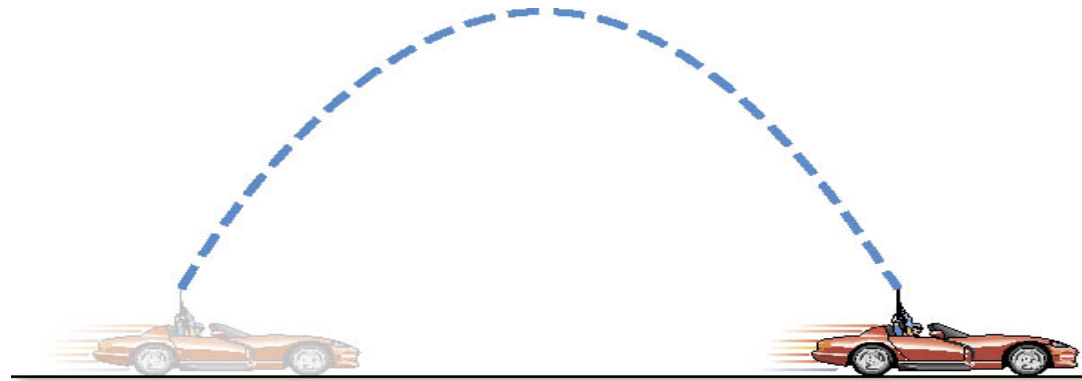
$$v_x = v_{x0} = +115 \text{ m/s} \qquad v = \sqrt{v_x^2 + v_y^2} = 184 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-143}{+115}\right) = -51^\circ$$

3.3 *Projectile Motion*

Conceptual Example: I Shot a Bullet into the Air...

Suppose you are driving a convertible with the top down. The car is moving to the right at constant velocity. You point a rifle straight up into the air and fire it. In the absence of air resistance, where would the bullet land – behind you, ahead of you, or in the barrel of the rifle?



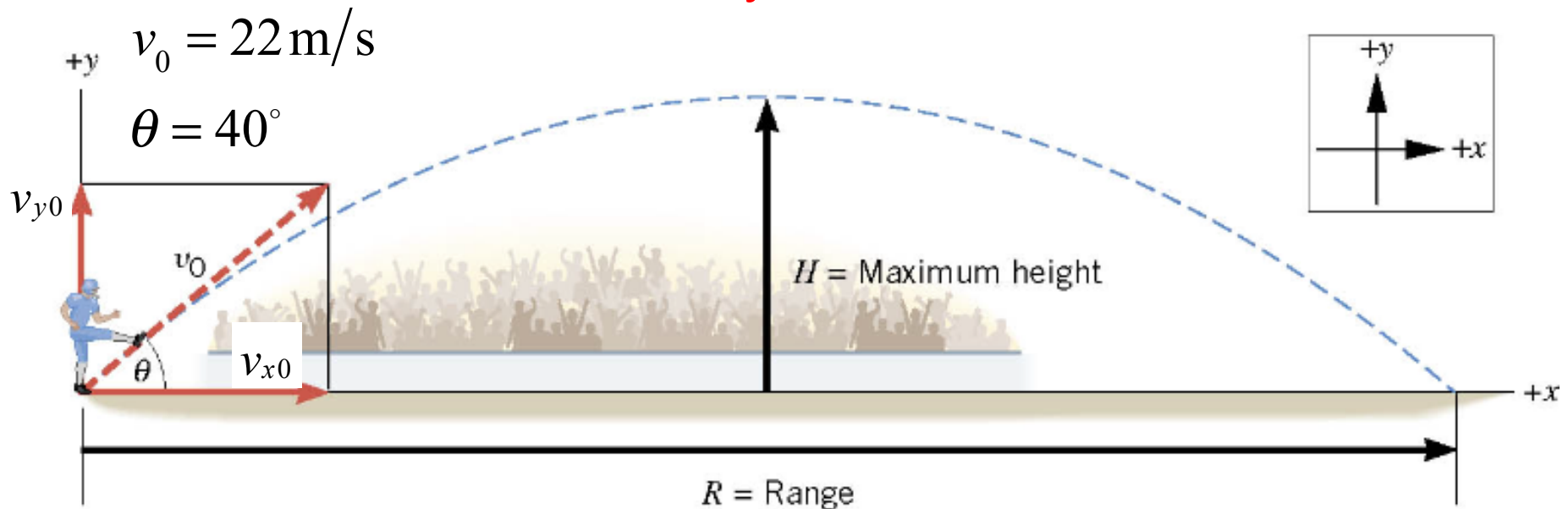
Ballistic Cart Demonstration

3.3 Projectile Motion

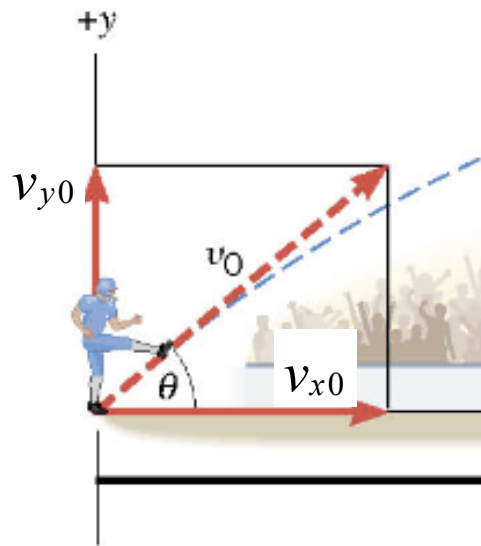
Example: The Height of a Kickoff

A placekicker kicks a football at an angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.

maximum height and “hang time”
depend only on the y-component of
initial velocity

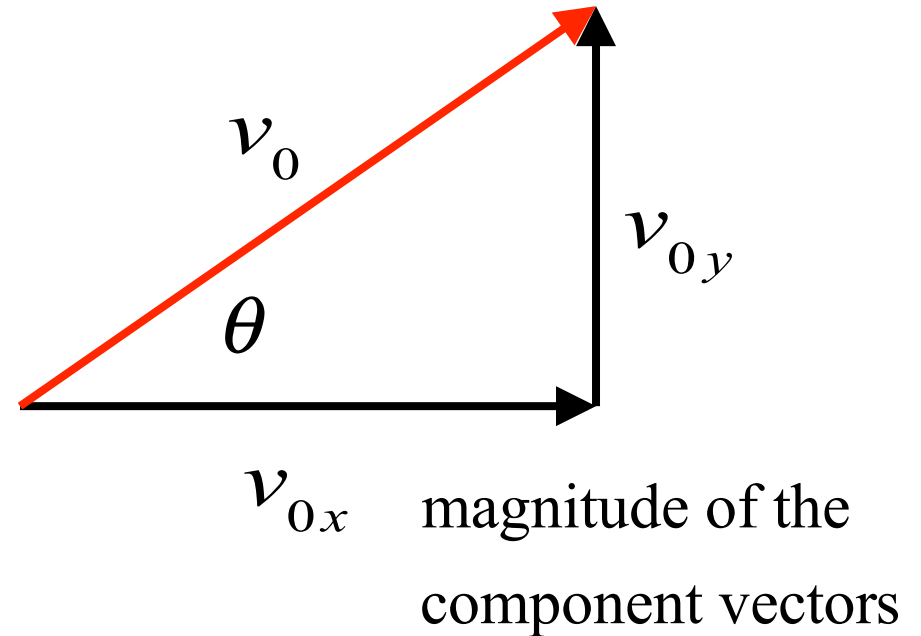


3.3 Projectile Motion



$$v_0 = 22 \text{ m/s}$$

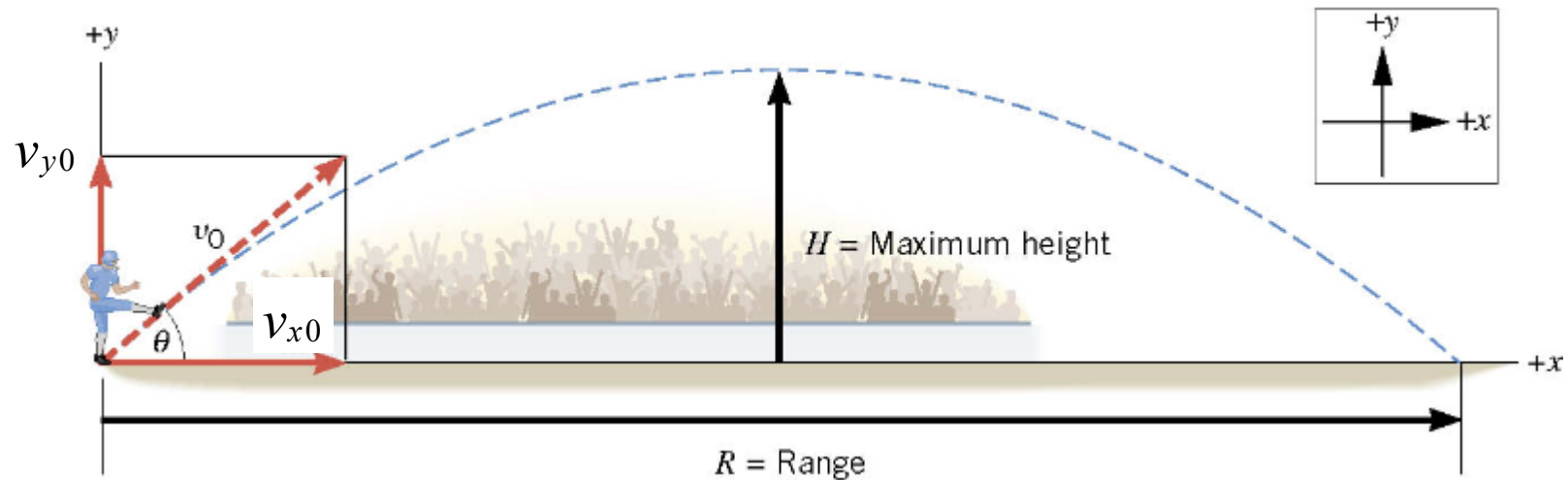
$$\theta = 40^\circ$$



$$v_{y0} = v_0 \sin \theta = (22 \text{ m/s}) \sin 40^\circ = 14 \text{ m/s}$$

$$v_{x0} = v_0 \cos \theta = (22 \text{ m/s}) \cos 40^\circ = 17 \text{ m/s}$$

3.3 Projectile Motion



Δy	a_y	v_y	v_{y0}	t
?	-9.80 m/s^2	0	14 m/s	

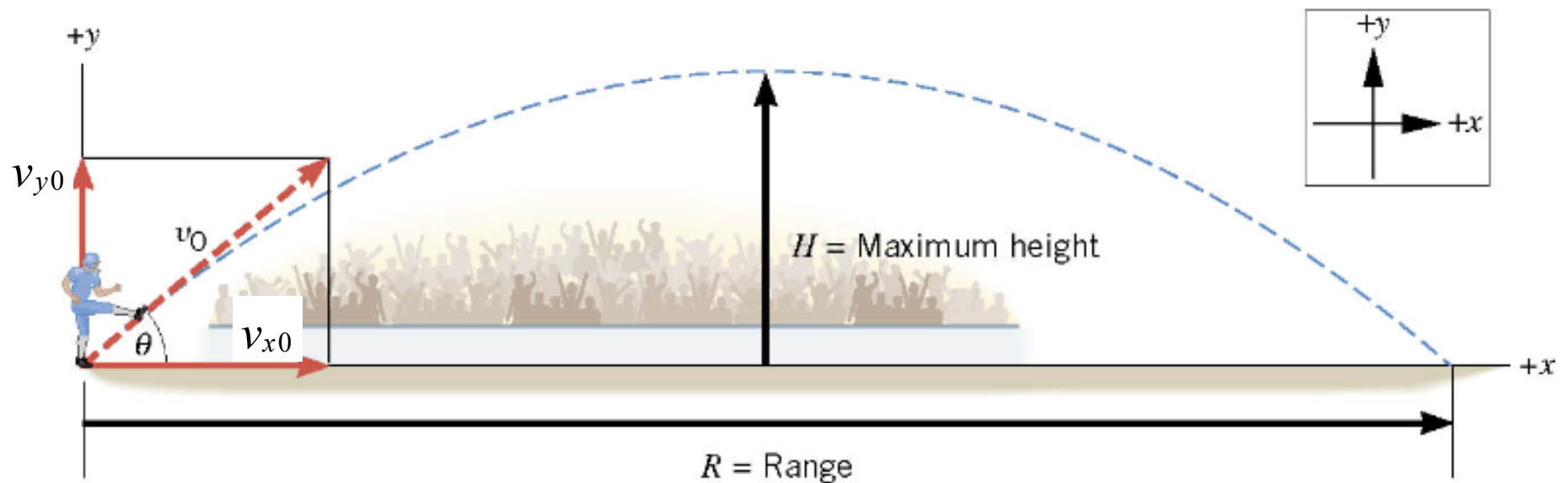
$$v_y^2 = v_{0y}^2 + 2a_y \Delta y \quad \longrightarrow \quad \Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y}$$

maximum height $H = \Delta y = \frac{0 - (14 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = +10 \text{ m}$

3.3 Projectile Motion

Example: The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?



Δy	a_y	v_y	v_{y0}	t
0	-9.80 m/s^2		14 m/s	?

3.3 *Projectile Motion*

Δy	a_y	v_y	v_{y0}	t
0	-9.81 m/s ²		14 m/s	?

$$\Delta y = v_{y0}t + \frac{1}{2}a_y t^2$$

$$0 = (14 \text{ m/s})t + \frac{1}{2}(-9.81 \text{ m/s}^2)t^2$$

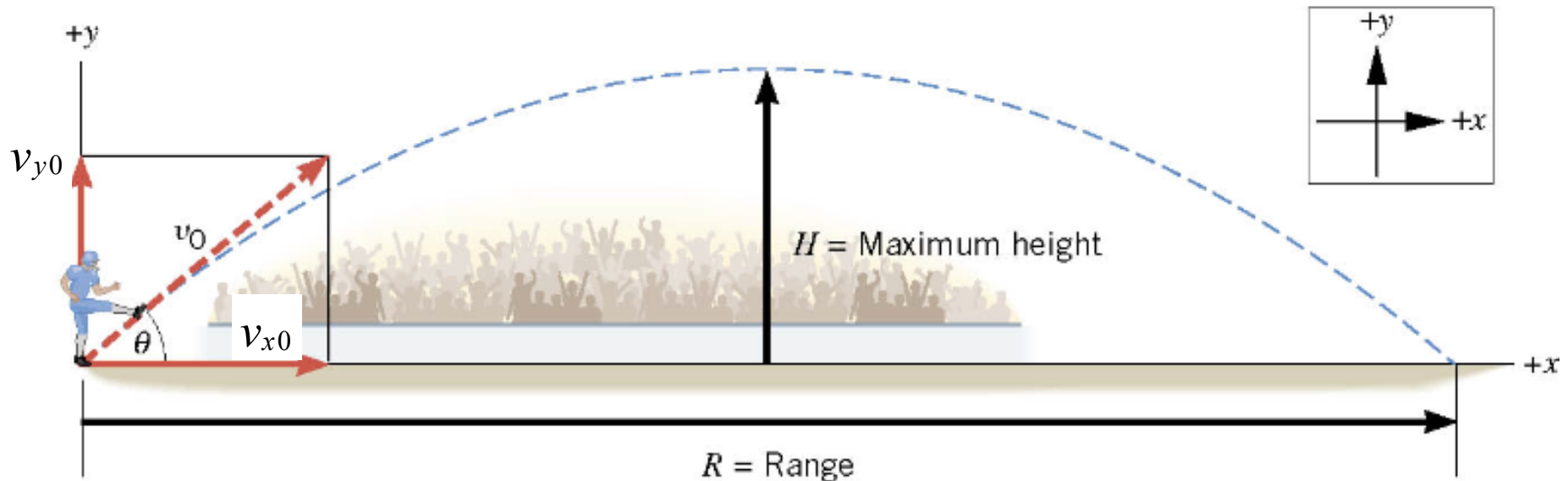
$$0 = 2(14 \text{ m/s}) + (-9.81 \text{ m/s}^2)t$$

$$t = 2.9 \text{ s}$$

3.3 Projectile Motion

Example: The Range of a Kickoff
Calculate the range R of the projectile.

Range depends on the hang time
and x-component of initial velocity



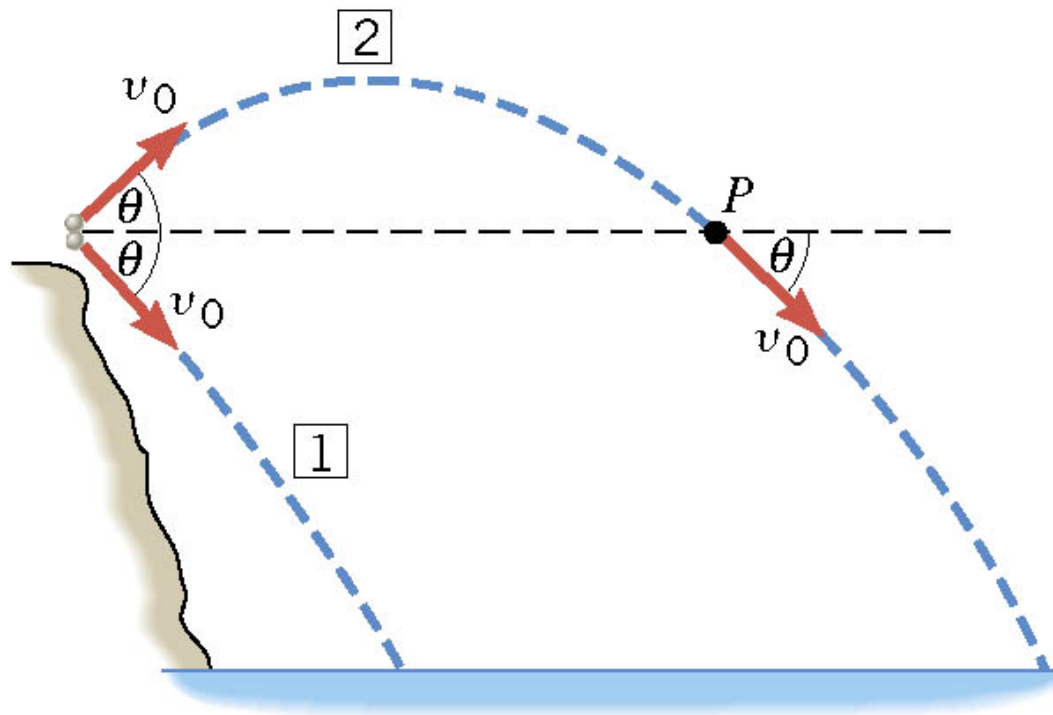
x-direction $\Delta x = v_{x0}t + \frac{1}{2}a_x t^2 = v_{x0}t$ $a_x = 0!$

$$= (17 \text{ m/s})(2.9 \text{ s}) = +49 \text{ m}$$

3.3 Projectile Motion

Conceptual Example: Two Ways to Throw a Stone

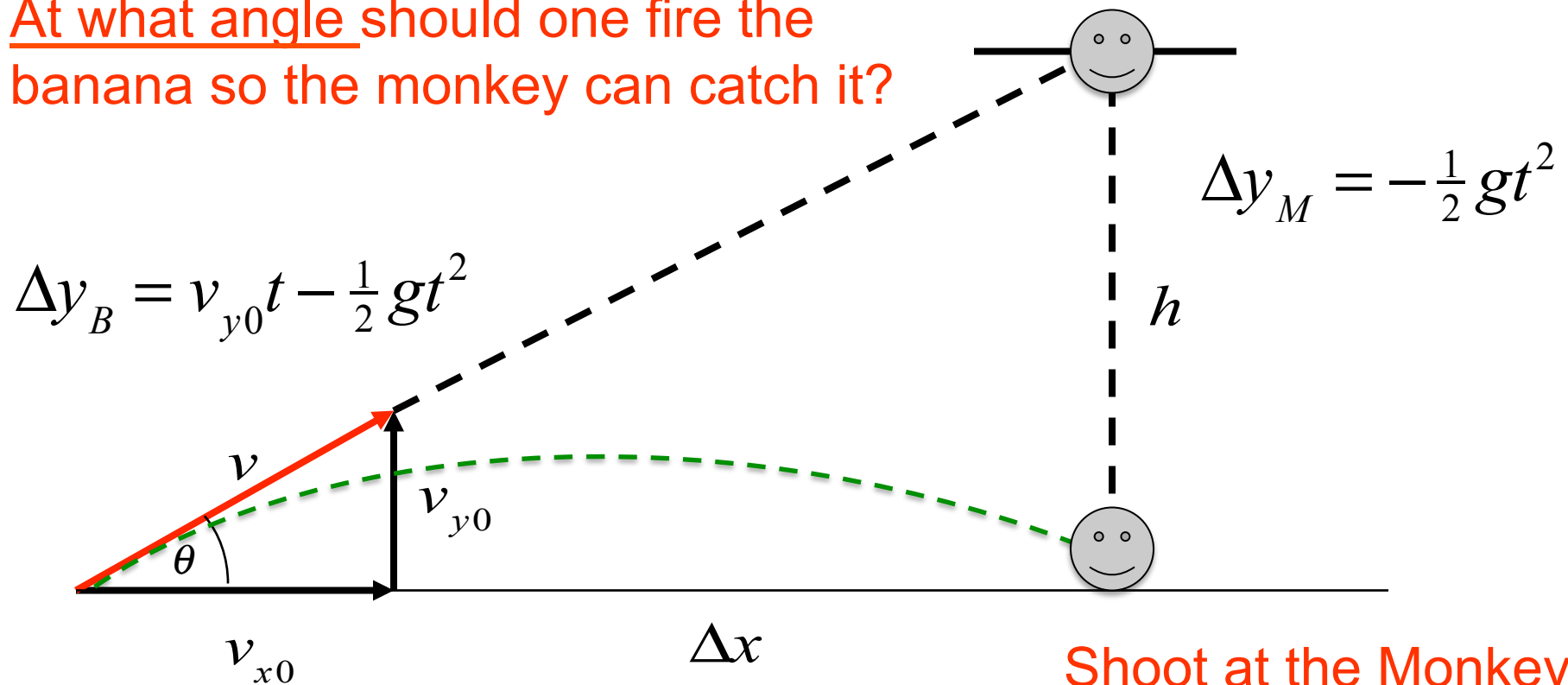
From the top of a cliff, a person throws two stones. The stones have identical initial speeds, but stone 1 is thrown downward at some angle above the horizontal and stone 2 is thrown at the same angle below the horizontal. Neglecting air resistance, which stone, if either, strikes the water with greater velocity?



Shoot the Monkey Demonstration

Banana fired and monkey drops at the same time

At what angle should one fire the banana so the monkey can catch it?



Hit means: $\Delta y_B - \Delta y_M = h \Rightarrow v_{0y}t = h$

Hit time: $t = \frac{\Delta x}{v_{x0}}$

$$\frac{v_{y0}}{v_{x0}} \Delta x = h$$

$$\frac{v_{y0}}{v_{x0}} = \frac{h}{\Delta x} = \tan \theta$$