Chapter 5

Work and Energy

conclusion

5.5 Conservative Versus Nonconservative Forces

In many situations both conservative and non-conservative forces act simultaneously on an object. Work by conservative forces has been incorporated into the potential energies. Only work by non-conservative forces, W_{NC} has to be included.

Work-Energy Theorem becomes:

$$K + U = K_0 + U_0 + W_{\text{NC}}$$
$$E = E_0 + W_{\text{NC}}$$

Work by non-conservative forces add or remove energy from the mass.
If $W_{NC} = 0$, then energy is conserved.

Another (equivalent) way to think about it:

$$\left(K - K_0\right) + \left(U - U_0\right) = W_{\text{NC}}$$
$$\Delta K + \Delta U = W_{\text{NC}}$$

If non-conservative forces do work on the mass, energy changes will not sum to zero

NC work by kinetic friction, $W_{\text{NC}} = -f\,\Delta x = -\mu_k F_{\perp} \Delta x$

Chaper 5 Review: Work and Energy – Forces and Displacements

Effect of forces acting over a displacement

Work

$$W = (F\cos\theta)\Delta x$$

Kinetic Energy

$$K = \tfrac{1}{2}mv^2$$

Work - Energy Theorem (true always)

$$W = K - K_0$$

Work by F_{Net} changes the Kinetic Energy of a mass

Conservative Force	Potential Energy
Gravity	$U_G = mgy$
Ideal Spring	$U_S = \tfrac{1}{2}kx^2$

Non-Conservative Forces doing work

W_{NC} Humans, Friction, Explosion, Rocket

Work - Energy Theorem (w/potential energy U)

$$K + U = K_0 + U_0 + W_{\text{NC}}$$

Note: all of these quantities are scalars.

5.6 Power

DEFINITION OF (AVERAGE) POWER

Average power is the rate at which work is done, and it is obtained by dividing the work by the time required to perform the work, usually work by a non-conservative force.

Average Power

$$\bar{P} = \frac{\text{Work}}{\text{Time}} = \frac{W_{\text{NC}}}{t}$$

Power units: joule/s $= \text{watt (W)}$

Note: 1 horsepower $=$ 745.7 watts

$$\text{Work - Energy Theorem: } W_{\text{NC}} = \Delta E$$

$$\bar{P} = \frac{\Delta E}{t} \quad \Rightarrow \quad \Delta E = \bar{P}\Delta t$$

$$\bar{P} = \frac{W}{t} = \frac{F_x \Delta x}{t} = F_x\left(\frac{\Delta x}{t}\right) = F_x \bar{v}_x$$

$$\bar{P} = F_x \bar{v}_x$$

units: $1 \text{ watt} = 1 \text{ N}\cdot\text{m/s} = 1 \text{ J/s}$

Table of **Human Metabolic Rates**[a]

Activity	Rate (watts)
Running (15 km/h)	1340 W
Skiing	1050 W
Biking	530 W
Walking (5 km/h)	280 W
Sleeping	77 W

[a]For a young 70-kg male.

5.8 *Other Forms of Energy and the Conservation of Energy*

THE PRINCIPLE OF CONSERVATION OF ENERGY

Energy can neither be created not destroyed, but can only be converted from one form to another.

The result of a non-conservative force is often to remove mechanical energy and transform it into heat.
Heat energy is the kinetic or vibrational energy of molecules.

Examples of heat generation: sliding friction, muscle forces.

Chapter 6

Impulse and Momentum

6.1 The Impulse-Momentum Theorem

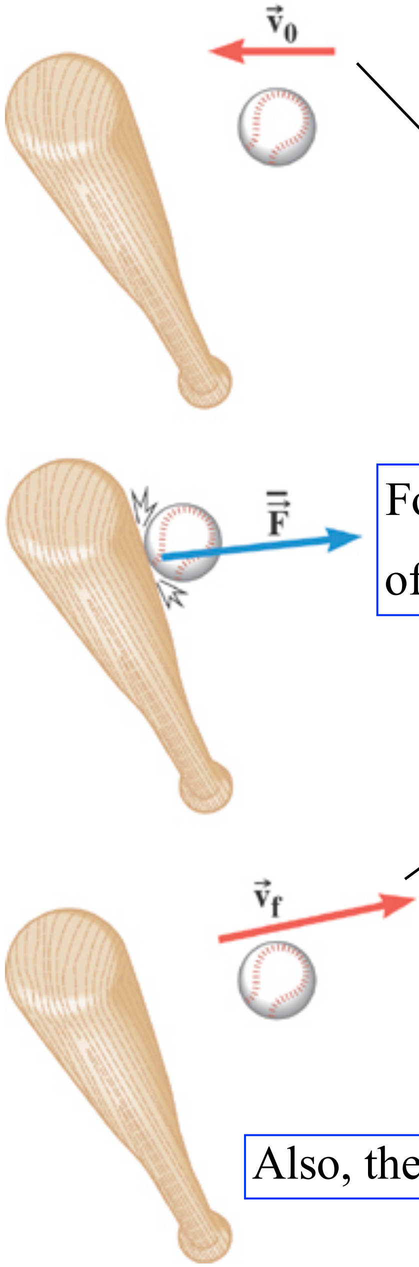
Chapter 6 is about the COLLISION of TWO masses.
To understand the interaction, both masses must be considered.
Newton's 3rd Law plays a very important part.

Collisions involve two new concepts: Impulse and Momentum.
Impulse concept leads to the Momentum definition.

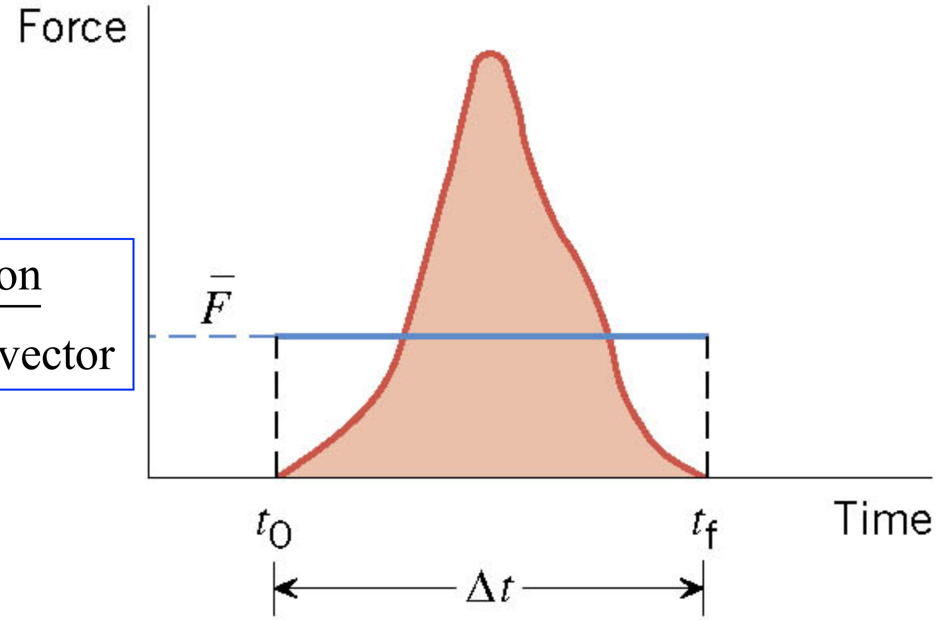
Also applied to two (or more) masses blown apart by an explosion.

6.1 The Impulse-Momentum Theorem

What is the effect of a force acting over a short time?



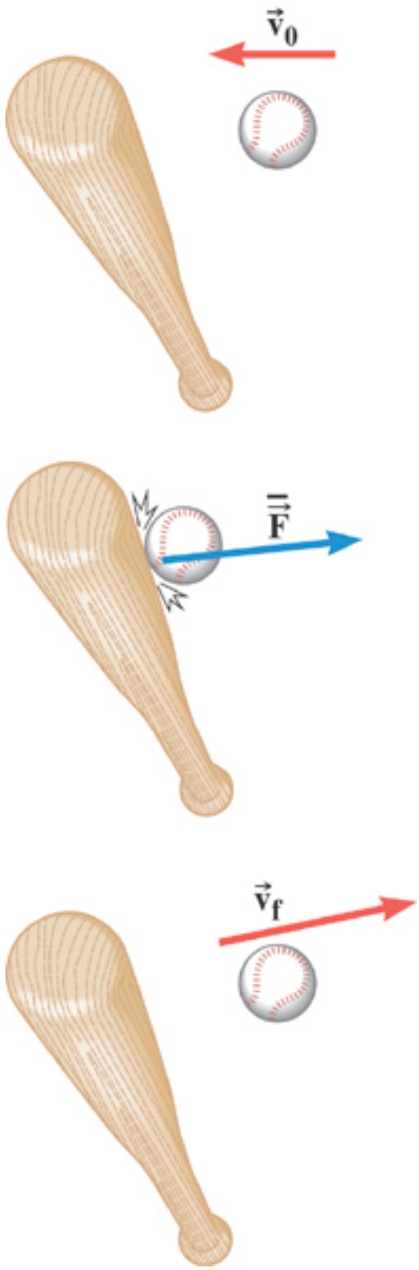
Force changes the direction of the baseball's velocity vector



The bat/ball force is not constant and the mass makes a very short displacement while it acts.

Also, the reaction force changes the magnitude of the bat's velocity vector

6.1 The Impulse-Momentum Theorem



$\bar{\vec{\mathbf{F}}}_{\text{Net}}$ acts on the Baseball

$m, \vec{\mathbf{v}}$, and $\vec{\mathbf{a}}$ are of the Baseball

f $\equiv$ final

i $\equiv$ initial

$$\bar{\vec{\mathbf{F}}}_{\text{Net}} = m\bar{\vec{\mathbf{a}}}$$

$$\bar{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}}_{\text{f}} - \vec{\mathbf{v}}_{\text{i}}}{\Delta t}$$

$$\bar{\vec{\mathbf{F}}}_{\text{Net}} = \frac{m\vec{\mathbf{v}}_{\text{f}} - m\vec{\mathbf{v}}_{\text{i}}}{\Delta t} = \frac{\vec{\mathbf{p}}_{\text{f}} - \vec{\mathbf{p}}_{\text{i}}}{\Delta t}$$

Momentum

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

on the BALL — of the BALL

$$\bar{\vec{\mathbf{F}}}_{\text{Net}}\,\Delta t = \Delta\vec{\mathbf{p}}_{\text{f}}$$

Impulse

$$\bar{\vec{\mathbf{F}}}_{\text{Net}}\,\Delta t$$

Impulse $\Rightarrow$ changes BALL's momentum

reaction — Newton's 3rd Law — action

$$(\bar{\vec{\mathbf{F}}}_{\text{Net}})_{\text{on the BAT}} = -(\bar{\vec{\mathbf{F}}}_{\text{Net}})_{\text{on the BALL}}$$

DEFINITION OF IMPULSE

The impulse of a force is the product of the average force and the time interval during which the force acts:

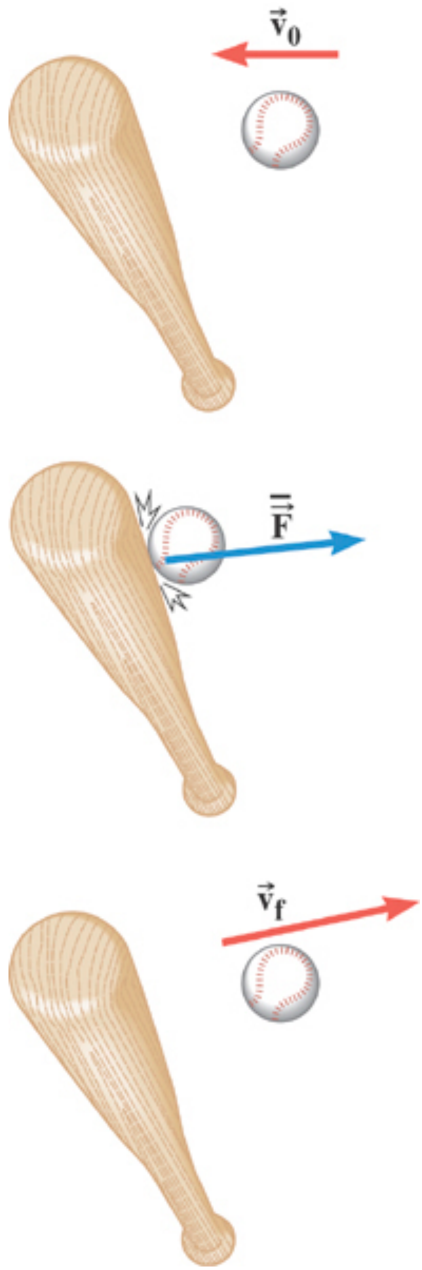
$$\vec{\mathbf{J}} = \overline{\vec{\mathbf{F}}}_{\text{Net}}\,\Delta t$$

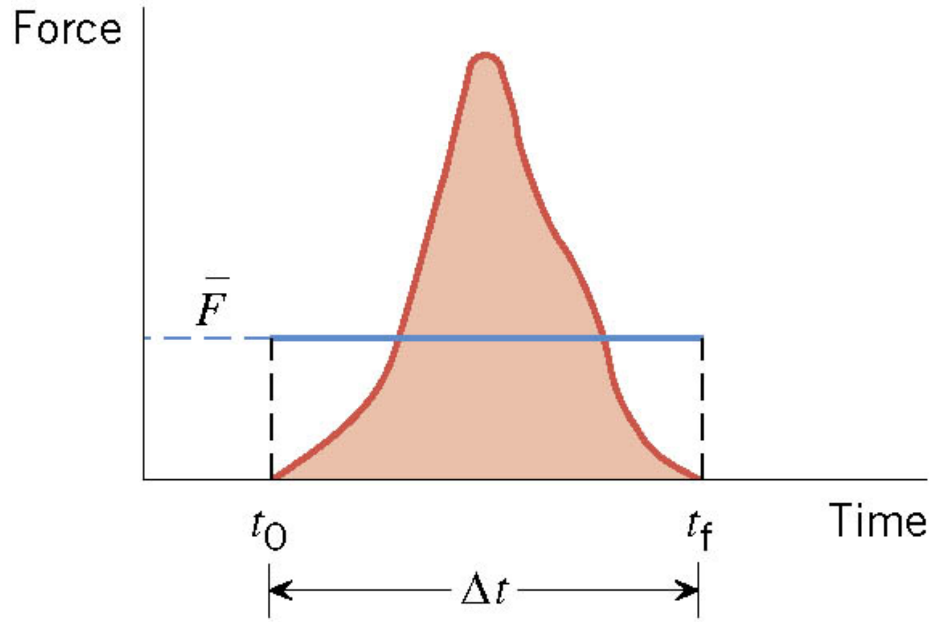
$$\overline{\vec{\mathbf{F}}}_{\text{Net}} = \text{average net force vector}$$

Impulse is a vector quantity and has the same direction as the average force.

$$\text{newton}\cdot\text{seconds}\;(\text{N}\cdot\text{s})$$

6.1 The Impulse-Momentum Theorem





(b)

$$\vec{J} = \overline{\vec{F}}\Delta t$$

Impulse

DEFINITION OF LINEAR MOMENTUM

The linear momentum of an object is the product of the object's mass times its velocity:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Linear momentum is a vector quantity and has the same direction as the velocity.

$$\text{kilogram} \cdot \text{meter/second (kg} \cdot \text{m/s)}$$

IMPULSE-MOMENTUM THEOREM

When a net force acts on an object, the impulse of this force is equal to the change in the momentum of the object

impulse — final momentum — minus — initial momentum

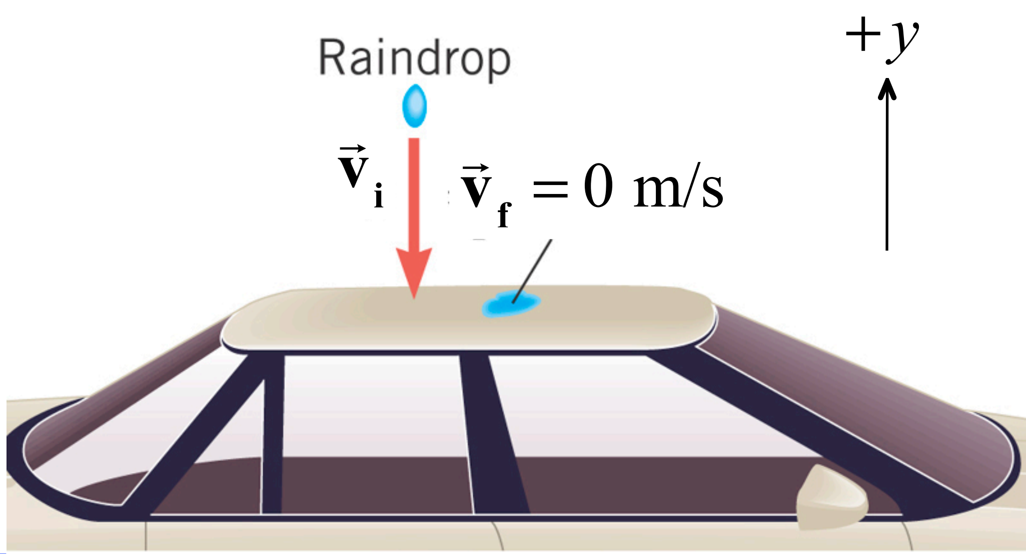
$$\bar{\vec{\mathbf{F}}}_{\text{Net}}\,\Delta t = m\vec{\mathbf{v}}_{\text{f}} - m\vec{\mathbf{v}}_{\text{i}}$$

Time averaged force acting on a mass.	Changes the momentum of the mass.

Example: A Rain Storm

Rain comes down with a velocity of -15 m/s and hits the roof of a car. The mass of rain per second that strikes the roof of the car is 0.060 kg/s. Assuming that rain comes to rest upon striking the car, find the average force exerted by the rain on the roof.

$$\bar{\vec{\mathbf{F}}}_{\text{Net}}\,\Delta t = m\vec{\mathbf{v}}_{\mathbf{f}} - m\vec{\mathbf{v}}_{\mathbf{i}}$$

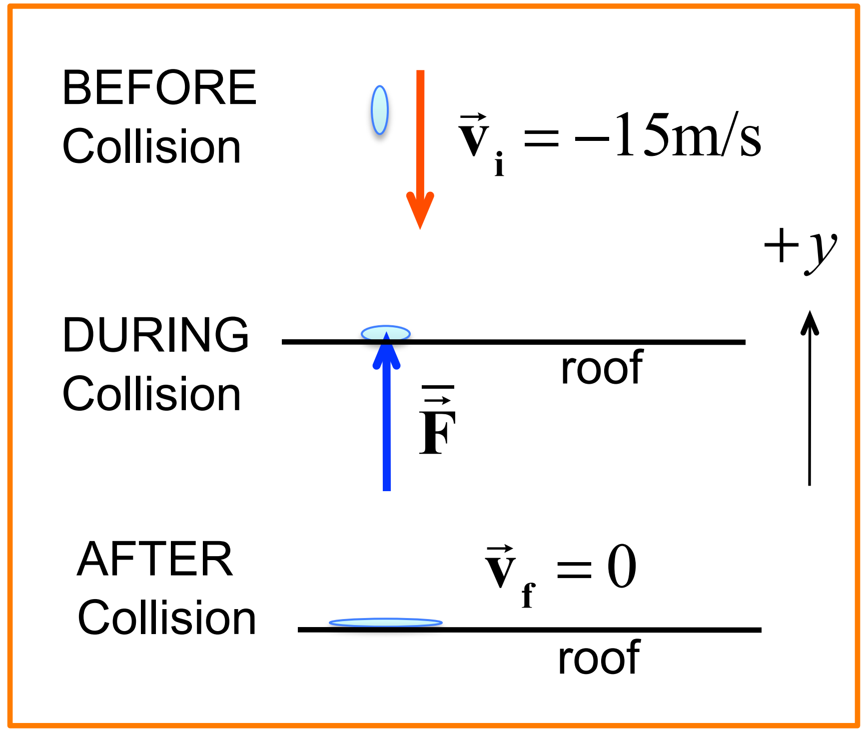


Using this, you will determine the average force on the raindrops.

But, using Newton's 3rd law you can get the average force on the roof .

6.1 The Impulse-Momentum Theorem

Neglecting the raindrop's weight, the average net force on the raindrops caused by the collisions with the roof is obtained.



Impulse of roof on raindrops	Changes momentum of the raindrops

$$\bar{\vec{\mathbf{F}}}\Delta t = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i$$

$\vec{\mathbf{v}}_f = 0$

$$\bar{\vec{\mathbf{F}}} = -\left(\frac{m}{\Delta t}\right)\vec{\mathbf{v}}_i$$

mass of rain per second $\left(\frac{m}{\Delta t}\right) = 0.060$ kg/s

$$\bar{\vec{\mathbf{F}}} = -(0.060\,\text{kg/s})(-15\,\text{m/s})$$
$$= +0.90\text{ N}$$

By Newton's 3rd Law average force of raindrops on the roof is

$$\bar{\vec{\mathbf{F}}} = -0.90\,\text{N}$$

Clicker Question 6.1: Hailstones versus raindrops

Instead of rain, suppose hail has velocity of –15 m/s and one hailstone with a mass 0.060 kg of hits the roof and bounces off with a velocity of +10 m/s. In the collision, what is the change in the momentum vector of the hailstone?

a) $+0.3\ \mathrm{N \cdot s}$

b) $-0.3\ \mathrm{N \cdot s}$

c) $0.0\ \mathrm{N \cdot s}$

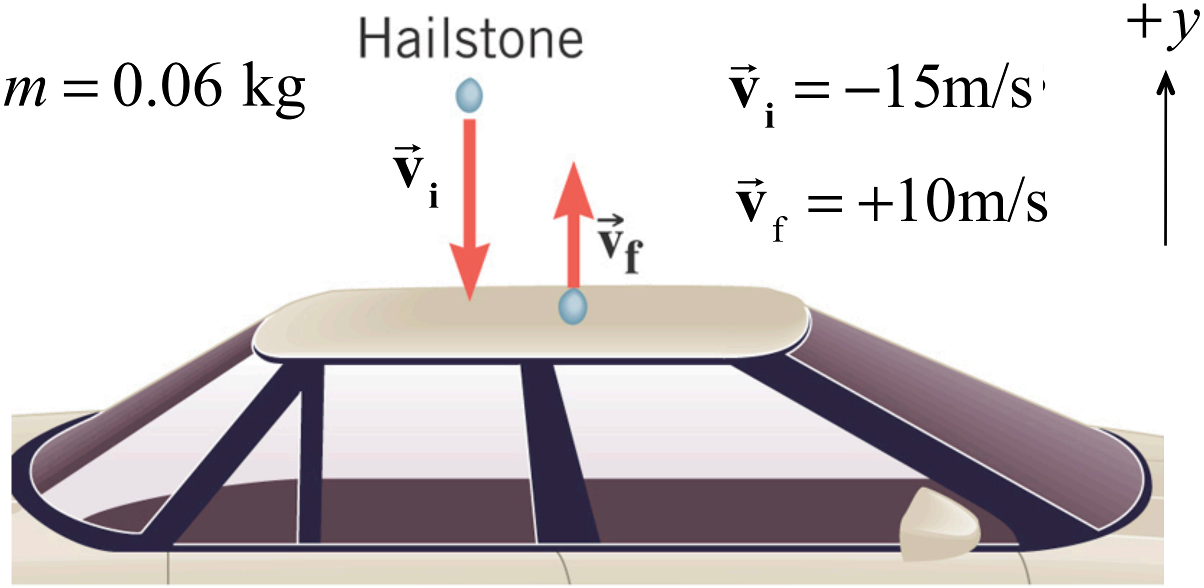
d) $+1.5\ \mathrm{N \cdot s}$

e) $-1.5\ \mathrm{N \cdot s}$

$m = 0.06\ \mathrm{kg}$

$\vec{\mathbf{v}}_i = -15\mathrm{m/s}$

$\vec{\mathbf{v}}_f = +10\mathrm{m/s}$



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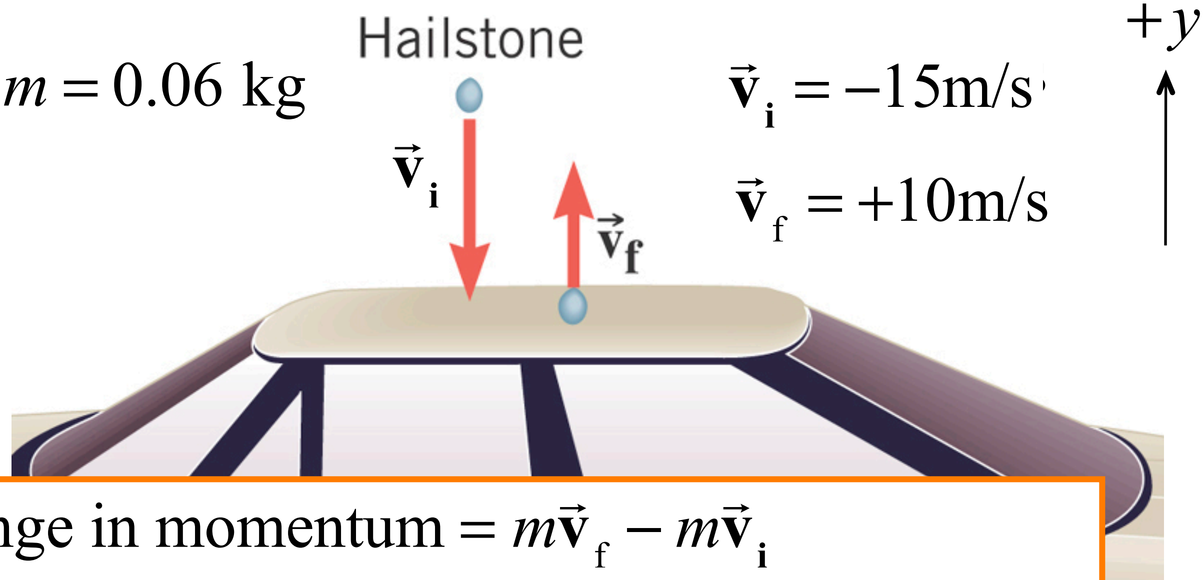


$$\mathbf{F}\Delta t = \text{change in momentum} = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i$$

$$F_y \Delta t = m(v_{yf} - v_{yi}) = (0.060\ \mathrm{kg})\left[+10\ \mathrm{m/s} - (-15\ \mathrm{m/s})\right]$$

$$= +1.5\ \mathrm{kg \cdot m/s}$$

6.2 The Principle of Conservation of Linear Momentum

WORK-ENERGY THEOREM ⇔CONSERVATION OF ENERGY

IMPULSE-MOMENTUM THEOREM ⇔???

Apply the impulse-momentum theorem to the midair collision between two objects while falling due to gravity.

Distinguish the EXTERNAL forces and INTERNAL forces

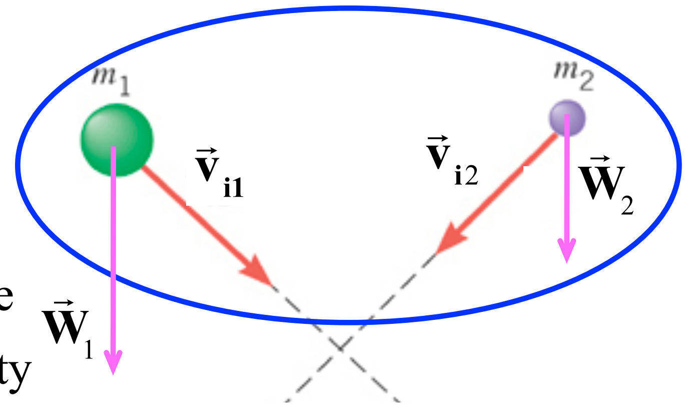
6.2 The Principle of Conservation of Linear Momentum

External forces – Forces exerted on the objects by agents external to the system. Net force changes the velocity (& momentum) of the masses.

Newton's 2nd Law

$\vec{W}$(weight vectors), the external force of gravity

System of two masses



Before the collision

6.2 *The Principle of Conservation of Linear Momentum*

System of two masses

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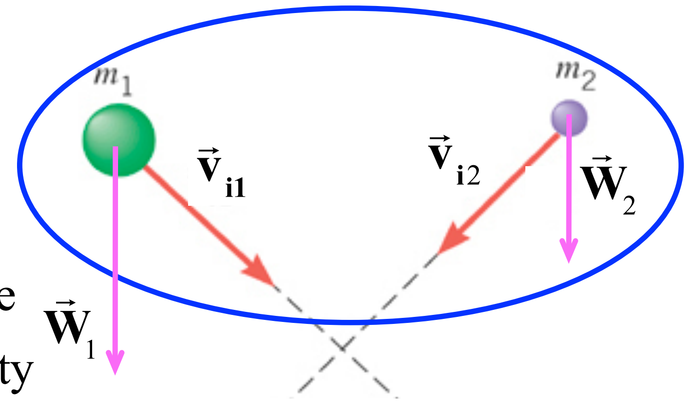
Newton's 2^{nd} Law

$\vec{\mathbf{W}}$(weight vectors), the external force of gravity



Before the collision

Internal forces – Forces **within the system** that objects exert on each other. These forces have equal magnitudes and opposite directions.

Newton's 3^{rd} Law

forces at contact point

also external $\vec{\mathbf{W}}_1$ and $\vec{\mathbf{W}}_2$

During the collision

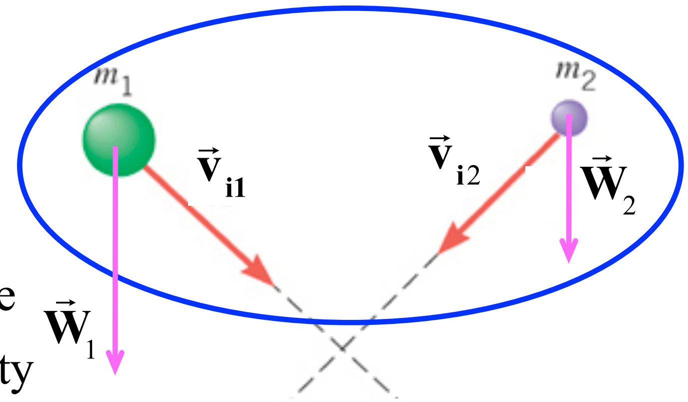
6.2 The Principle of Conservation of Linear Momentum

System of two masses

External forces – Forces exerted on the objects by agents external to the system. Net force changes the velocity (& momentum) of the masses.

Newton's 2nd Law

$\vec{\mathbf{W}}$(weight vectors), the external force of gravity



Before the collision

Internal forces – Forces **within the system** that objects exert on each other. These forces have equal magnitudes and opposite directions.

Newton's 3rd Law

forces at contact point

$$\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$$

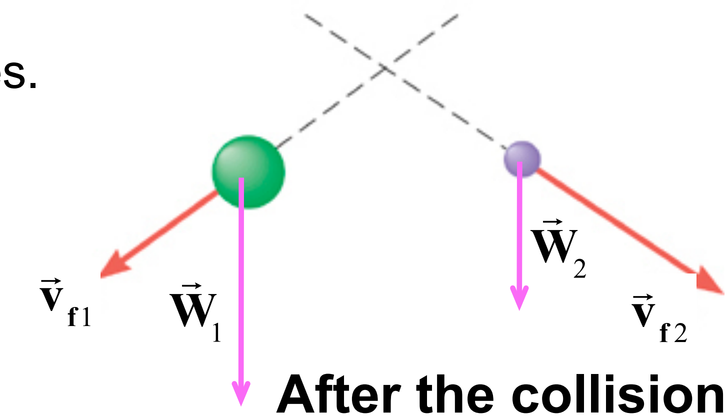
also external $\vec{\mathbf{W}}_1$ and $\vec{\mathbf{W}}_2$

During the collision

External forces – Forces exerted on the objects by agents external to the system. Net force changes the velocity (and momentum) of the masses.

Newton's 2nd Law

$\vec{\mathbf{W}}$(weight vectors), the external force of gravity



After the collision

6.2 The Principle of Conservation of Linear Momentum

During the collision (Δt)

Impulse-Momentum Theorem

Mass 1

Weight of mass 1.

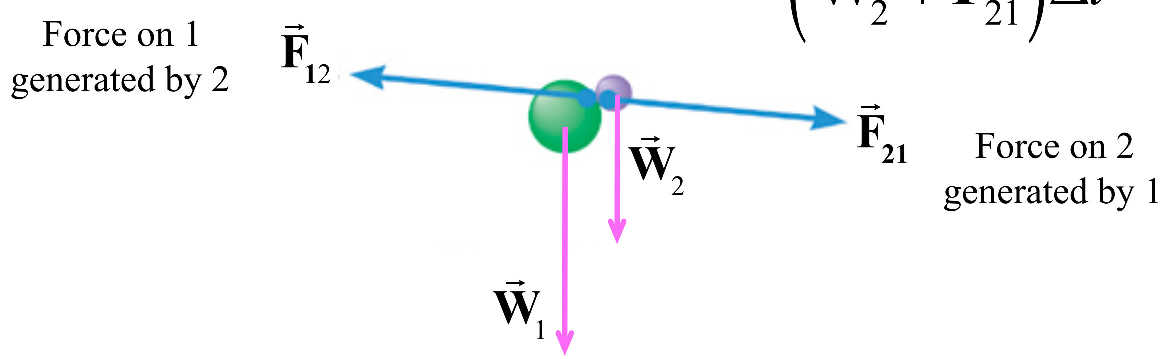
$$\left(\vec{\mathbf{W}}_1 + \vec{\bar{\mathbf{F}}}_{12}\right)\Delta t = m_1\vec{\mathbf{v}}_{\mathbf{f}1} - m_1\vec{\mathbf{v}}_{\mathbf{i}1}$$

Impulse-Momentum Theorem

Mass 2

Weight of mass 2.

$$\left(\vec{\mathbf{W}}_2 + \vec{\bar{\mathbf{F}}}_{21}\right)\Delta t = m_2\vec{\mathbf{v}}_{\mathbf{f}2} - m_2\vec{\mathbf{v}}_{\mathbf{i}2}$$



Net effect on the system of two masses $\Rightarrow$ add the equations together

6.2 The Principle of Conservation of Linear Momentum

During the collision (Δt)

Impulse-Momentum Theorem

Mass 1

Weight of mass 1.

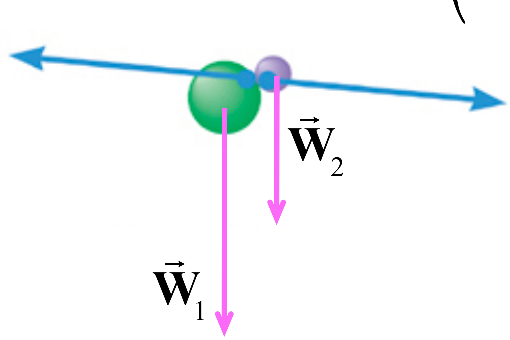
$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{F}}_{12}\right)\Delta t = m_1\vec{\mathbf{v}}_{\mathbf{f}1} - m_1\vec{\mathbf{v}}_{\mathbf{i}1}$$

Impulse-Momentum Theorem

Mass 2

Weight of mass 2.

$$\left(\vec{\mathbf{W}}_2 + \vec{\mathbf{F}}_{21}\right)\Delta t = m_2\vec{\mathbf{v}}_{\mathbf{f}2} - m_2\vec{\mathbf{v}}_{\mathbf{i}2}$$



Net effect on the system of two masses $\Rightarrow$ add the equations together

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{F}}_{12} + \vec{\mathbf{W}}_2 + \vec{\mathbf{F}}_{21}\right)\Delta t = \left(m_1\vec{\mathbf{v}}_{\mathbf{f}1} - m_1\vec{\mathbf{v}}_{\mathbf{i}1}\right) + \left(m_2\vec{\mathbf{v}}_{\mathbf{f}2} - m_2\vec{\mathbf{v}}_{\mathbf{i}2}\right)$$

At contact point: $\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$

put final values together & initial values together

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2\right)\Delta t = \left(m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2\right)_{\mathbf{f}} - \left(m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2\right)_{\mathbf{i}}$$

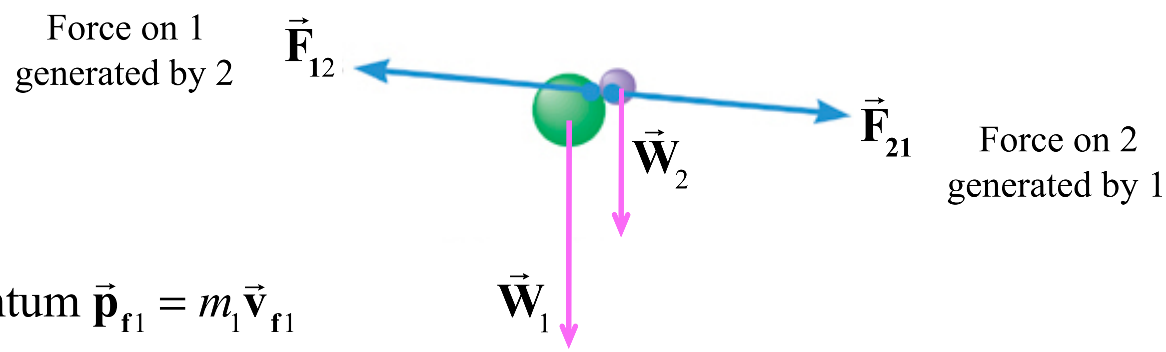
Impulses due only to external forces

Total momentum in the final system

Total momentum in the initial system

6.2 The Principle of Conservation of Linear Momentum

During the collision(Δt)



Using momentum $\vec{\mathbf{p}}_{\mathbf{f}1} = m_1 \vec{\mathbf{v}}_{\mathbf{f}1}$

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2\right)\Delta t \;=\; (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_{\mathbf{f}} - (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_{\mathbf{i}}$$

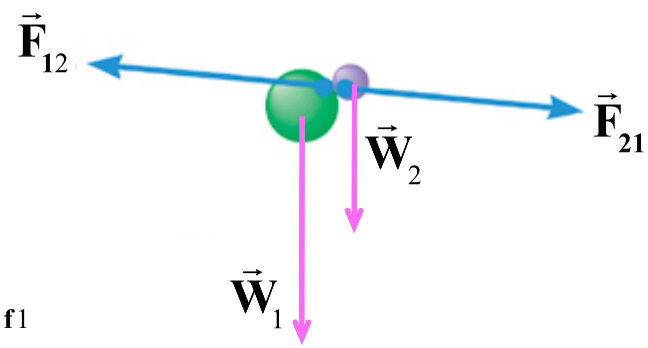
Impulses due only to external forces	Total momentum in the final system	Total momentum in the initial system

Only EXTERNAL forces can change the Total Momentum of a system of masses

With only INTERNAL forces affecting motion (e.g., if external forces are balanced)

6.2 The Principle of Conservation of Linear Momentum

During the collision (Δt)



Using momentum $\vec{\mathbf{p}}_{f1} = m_1 \vec{\mathbf{v}}_{f1}$

$$\left(\vec{\mathbf{W}}_1 + \vec{\mathbf{W}}_2\right)\Delta t = (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_\mathbf{f} - (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_\mathbf{i}$$

Impulses due only to external forces	Total momentum in the final system	Total momentum in the initial system

Only EXTERNAL forces can change the Total Momentum of a system of masses

With only INTERNAL forces affecting motion (e.g., if external forces are balanced)

$$0 = (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_\mathbf{f} - (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_\mathbf{i}$$

$$(\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_\mathbf{f} = (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2)_\mathbf{i}$$

Final value of total momentum	Initial value of total momentum

If only INTERNAL forces affect motion, total momentum VECTOR of a system does not change

6.2 *The Principle of Conservation of Linear Momentum*

If only INTERNAL forces affect the motion,
total momentum VECTOR of a system does not change

$$(\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \ldots)_{\mathbf{f}} = (\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \ldots)_{\mathbf{i}}$$

PRINCIPLE OF CONSERVATION OF LINEAR MOMENTUM

The total linear momentum of an isolated system of masses is constant (conserved). An isolated system is one for which the sum of the average external forces acting on the system is zero.

Most Important example

If the only external forces are gravitational forces that are balanced by normal forces, the total momentum VECTOR of a system is conserved in a collision