

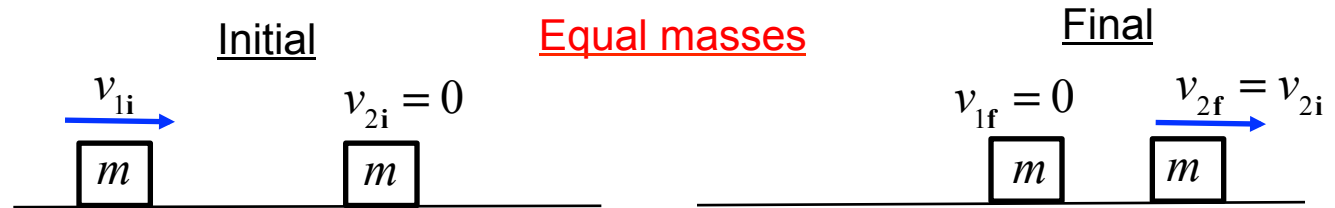
Chapter 6

Impulse and Momentum

Continued

6.3 Collisions in One Dimension

Elastic collisions (x – components of the velocities)



($m_1 = m_2 = m$) masses cancel in both equations

Momentum conservation: $v_{1i} = v_{1f} + v_{2f}$

Energy conservation: $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$

$$v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f} = v_{1f}^2 + v_{2f}^2$$

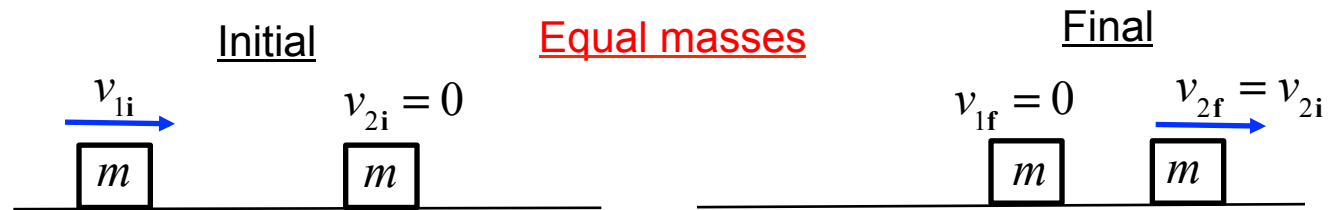
$$2v_{1f}v_{2f} = 0$$

$$v_{1f} = 0; \quad v_{2f} = v_{1i}$$

Incoming mass stops, target mass gets initial momentum

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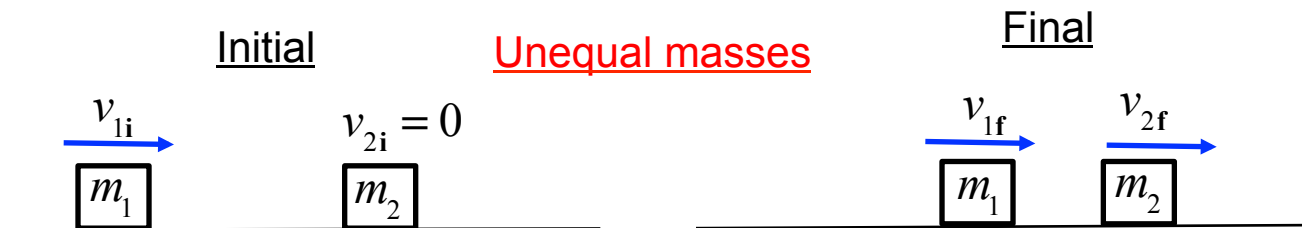
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$$v_{1f} = 0; \quad v_{2f} = v_{1i}$$

Incoming mass stops, target mass gets initial momentum



Momentum conservation:

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \Rightarrow \quad v_{1i} - v_{1f} = \frac{m_2}{m_1} v_{2f}$$

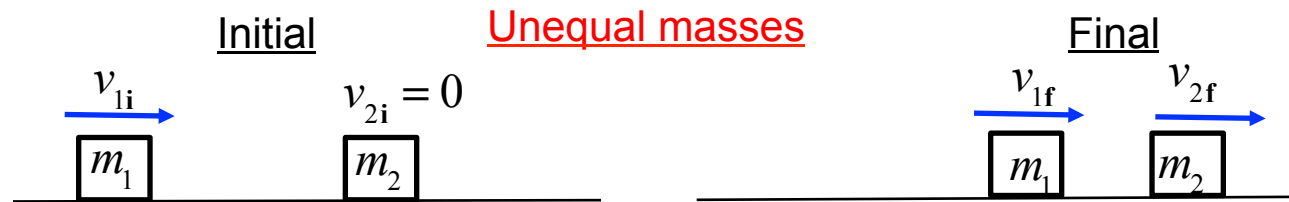
Energy conservation:

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \Rightarrow \quad v_{1i}^2 - v_{1f}^2 = \frac{m_2}{m_1} v_{2f}^2$$

Solve for v_{1f}, v_{2f}

6.3 Collisions in One Dimension

Elastic collisions with arbitrary masses (solve for final velocities)



Momentum conservation: $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \Rightarrow v_{1i} - v_{1f} = \frac{m_2}{m_1} v_{2f} \quad (1)$

Energy conservation: $\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \Rightarrow v_{1i}^2 - v_{1f}^2 = \frac{m_2}{m_1} v_{2f}^2 \quad (2)$

rewrite (2) as $(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = \frac{m_2}{m_1} v_{2f}^2$

insert (1) $\frac{m_2}{m_1} v_{2f} (v_{1i} + v_{1f}) = \frac{m_2}{m_1} v_{2f}^2 \Rightarrow v_{1i} + v_{1f} = v_{2f} \quad (3)$

(1) with (3) $v_{1i} - v_{1f} = \frac{m_2}{m_1} (v_{1i} + v_{1f}) \Rightarrow v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2} \right] v_{1i} \quad (4)$

(1) with (4) $v_{1i} - \left[\frac{m_1 - m_2}{m_1 + m_2} \right] v_{1i} = \frac{m_2}{m_1} v_{2f} \Rightarrow v_{2f} = \left[\frac{2m_1}{m_1 + m_2} \right] v_{1i}$

6.3 Collisions in One Dimension

Elastic collisions with arbitrary masses

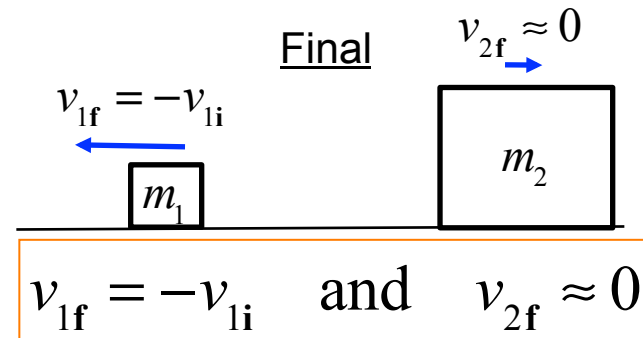
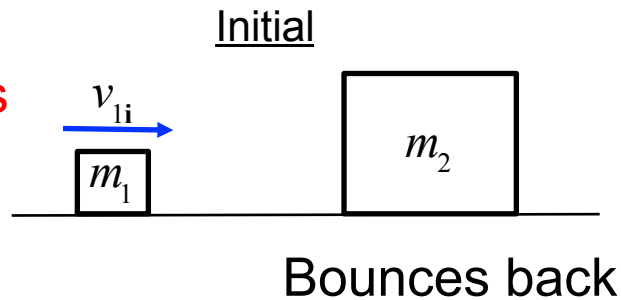
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \quad \text{and} \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Equal mass
solution is here too

Little mass
hits big mass

$$m_2 \gg m_1$$

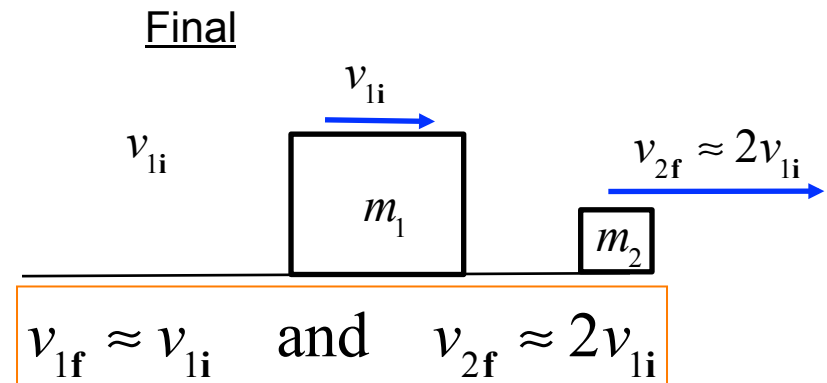
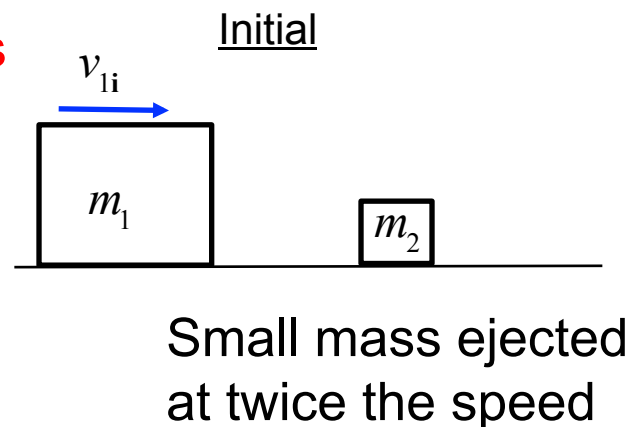
Let $m_1 \rightarrow 0$



Big mass
hits little mass

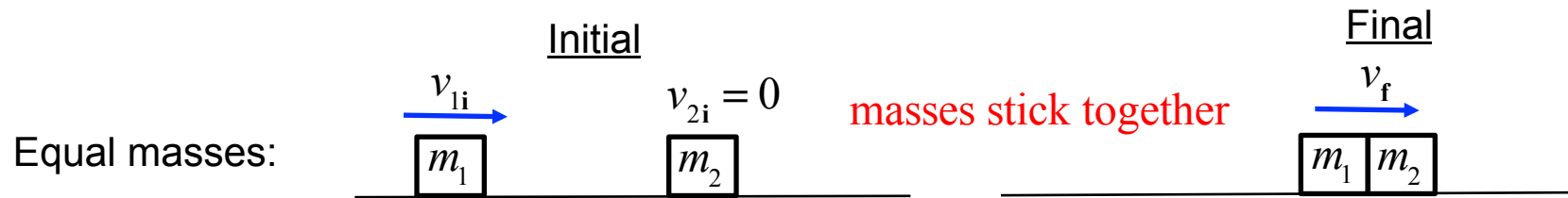
$$m_1 \gg m_2$$

Let $m_1 \rightarrow \infty$



6.3 Collisions in One Dimension

Inelastic collisions (with only internal forces affecting the motion)



(x – components of the velocities)

Momentum conservation: $m_1 v_{1i} = (m_1 + m_2) v_f$

Energy **NOT** conserved:

$$v_f = \frac{m_1}{m_1 + m_2} v_{1i}$$

How much kinetic energy was converted to heat in the collision.

$$K_i = \frac{1}{2} m_1 v_{1i}^2$$

$$K_f = \frac{1}{2} (m_1 + m_2) v_f^2 = \frac{1}{2} (m_1 + m_2) \left[\frac{m_1}{m_1 + m_2} v_{1i} \right]^2$$

$$= \frac{m_1^2}{2(m_1 + m_2)} v_{1i}^2 = \frac{m_1}{m_1 + m_2} \left[\frac{1}{2} m_1 v_{1i}^2 \right]$$

$$= \frac{m_1}{m_1 + m_2} K_i$$

$$\begin{aligned} \Delta K &= K_f - K_i = \frac{m_1}{m_1 + m_2} K_i - K_i \\ &= \left(\frac{m_2}{m_1 + m_2} \right) K_i \quad (\text{converted to heat}) \end{aligned}$$

If $m_2 \gg m_1$, $\Delta K \approx K_i$
(all kinetic energy converted to heat)

6.3 Collisions in One Dimension

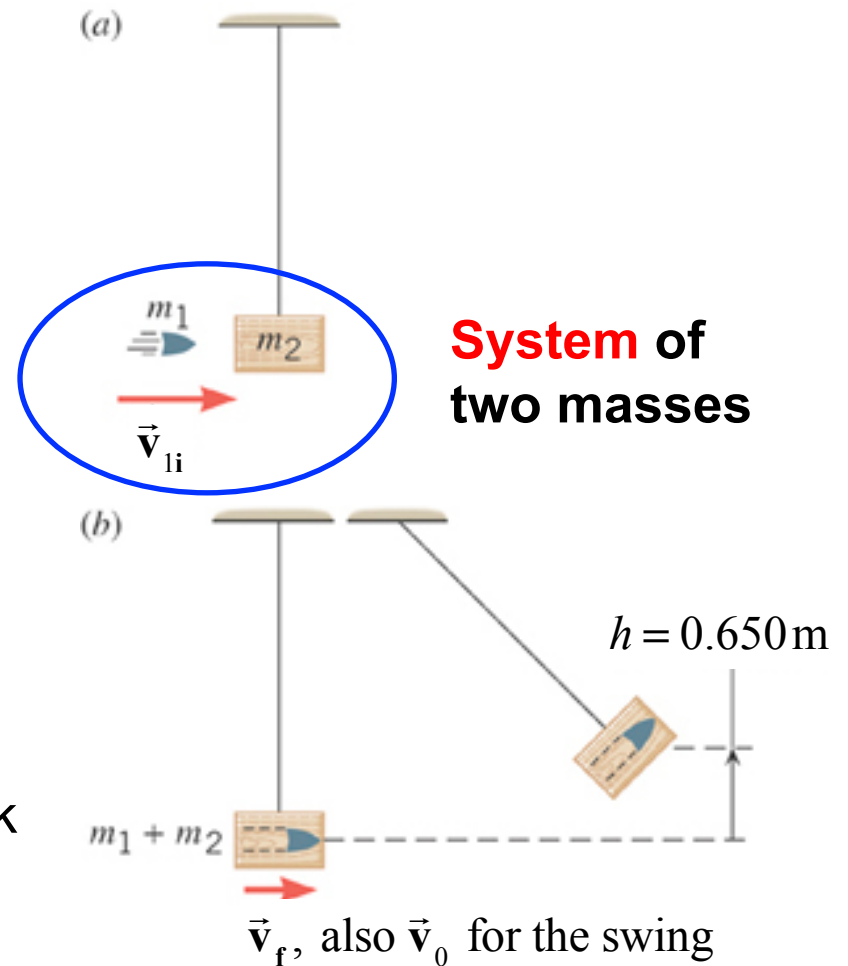
Example: A Ballistic Pendulum

The mass of the block of wood is 2.50-kg and the mass of the bullet is 0.0100-kg. The block swings to a maximum height of 0.650 m above the initial position.

Find the initial speed of the bullet.

Strategy – 1) After the bullet hits the block the swing will conserve energy. From the height determine the starting velocity.

2) Use this velocity as the final velocity of the collision. Then momentum conservation determines the bullet's initial velocity.



6.3 Collisions in One Dimension

Apply conservation of energy to the swinging motion:

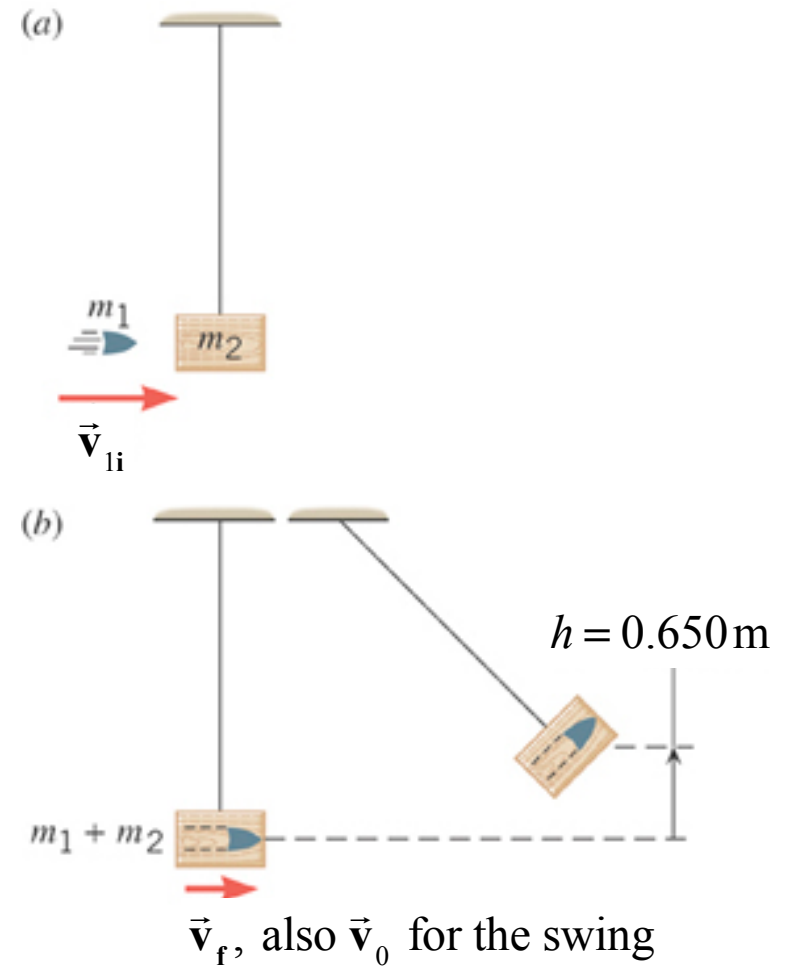
$$\begin{aligned} K + U &= K_0 + U_0 \\ 0 + mgh &= \frac{1}{2}mv_0^2 + 0 \\ v_0 &= \sqrt{2gh} \end{aligned}$$

Apply conservation of momentum in the collision:

$$\begin{aligned} \vec{\mathbf{p}}_{Total,i} &= \vec{\mathbf{p}}_{bullet,i} + \vec{\mathbf{p}}_{block,i} = m_1 v_{1i} + 0 \\ \vec{\mathbf{p}}_{Total,f} &= (m_1 + m_2)v_f \end{aligned}$$

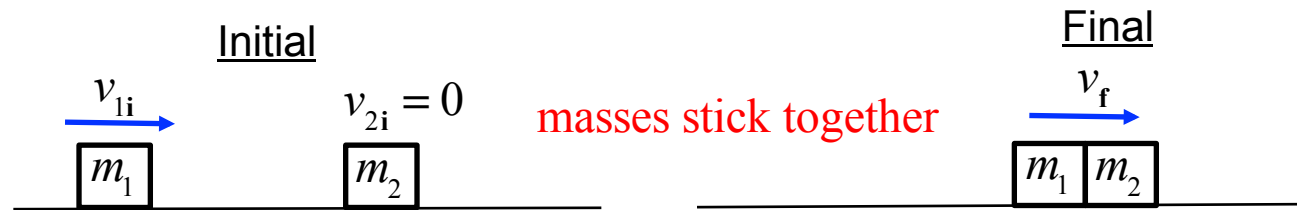
Momentum conservation: $\vec{\mathbf{p}}_{Total,f} = \vec{\mathbf{p}}_{Total,i}$

$$\begin{aligned} v_{1i} &= \frac{(m_1 + m_2)}{m_1} v_f = \frac{(m_1 + m_2)}{m_1} \sqrt{2gh} \\ &= \frac{2.50 + .01}{.01} \sqrt{2(9.81)(0.65)} \text{ m/s} = 896 \text{ m/s} \end{aligned}$$



6.3 Collisions in One Dimension

Inelastic collisions (with only internal forces affecting the motion)



(x – components of the velocities)

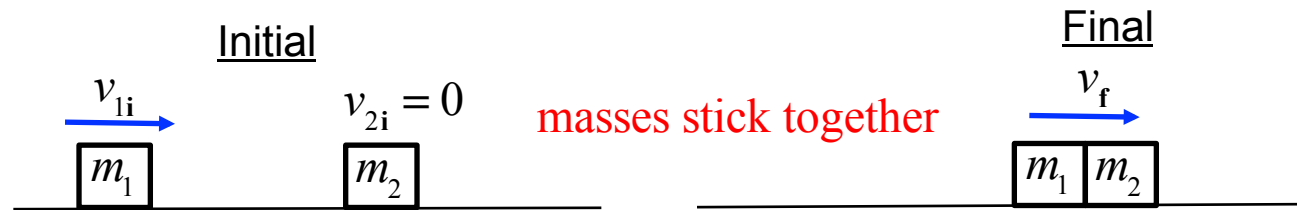
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Momentum conservation: $m_1 v_{1i} = (m_1 + m_2) v_f$

Energy **NOT** conserved:

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Masses initially moving
toward each other



(x – components of the velocities)

Momentum conservation: $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$

Energy **NOT** conserved:

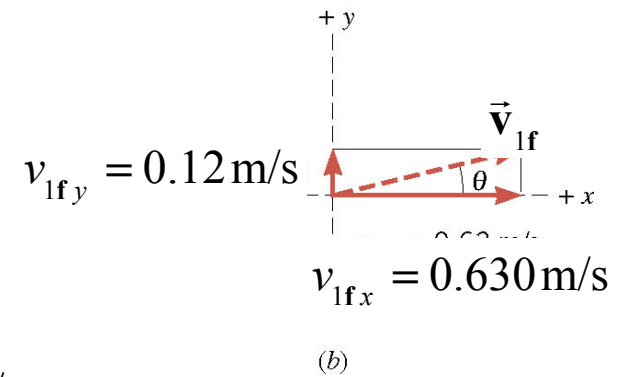
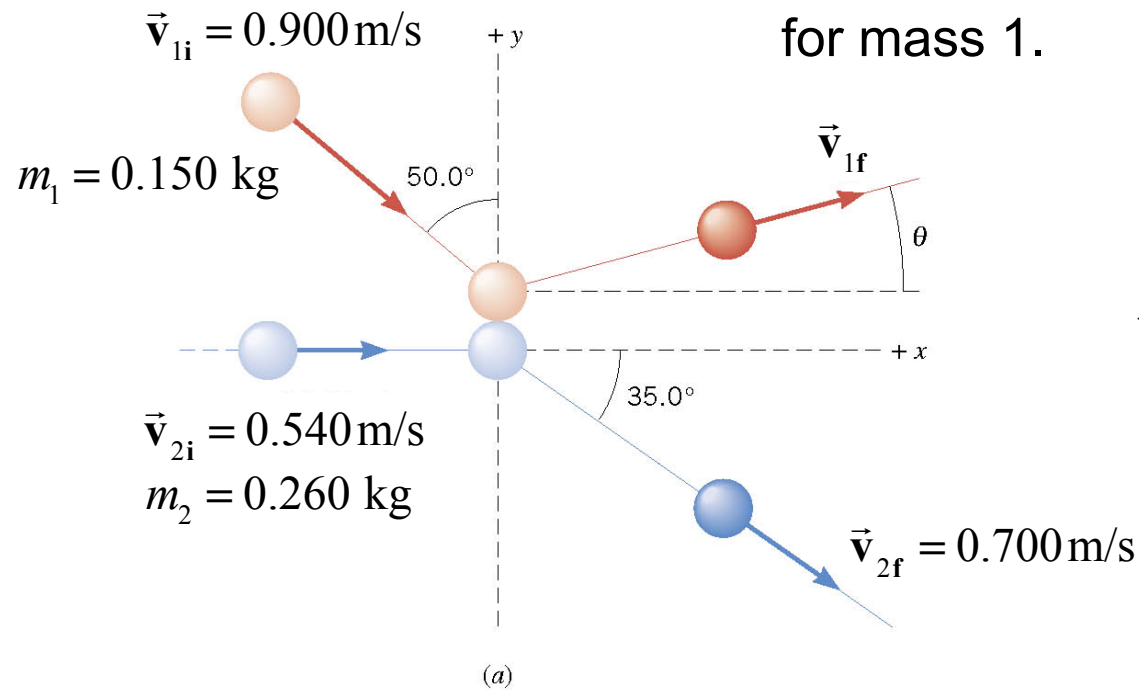
$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

For example: $v_{1i} = +5.0$ m/s, $v_{2i} = -10.0$ m/s, $m_1 = m_2 = m$ (same mass)

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{m}{2m} (v_{1i} + v_{2i}) = 0.5(+5 - 10) \text{ m/s} = -2.5 \text{ m/s}$$

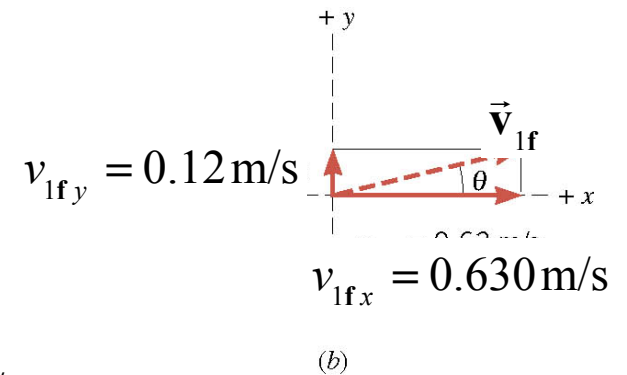
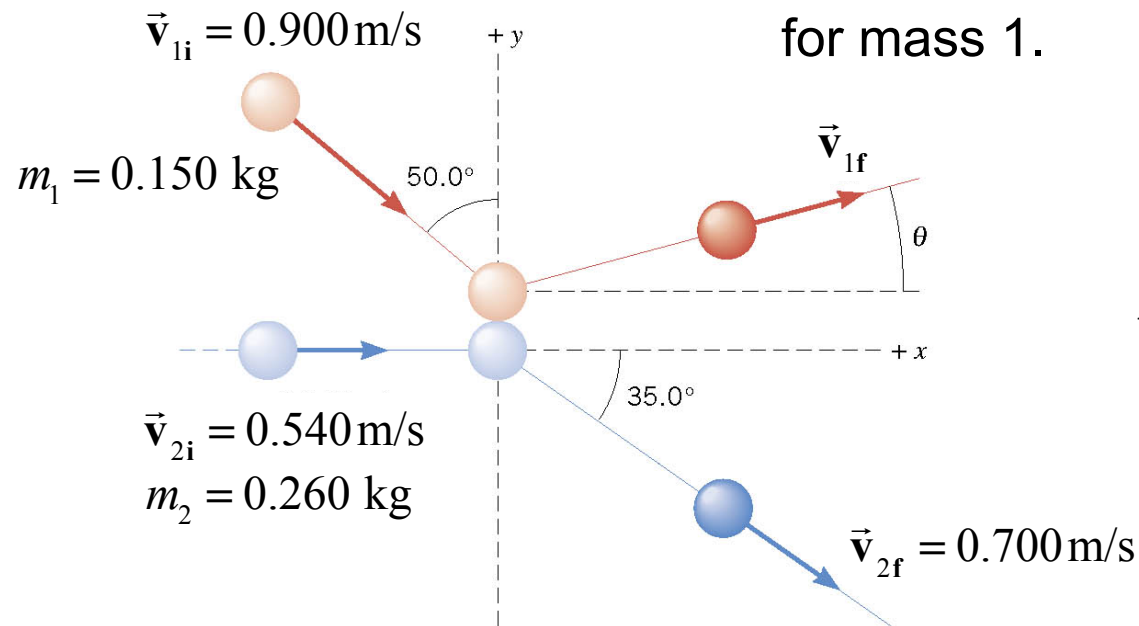
6.4 Collisions in Two Dimensions

Determine the final momentum vector for mass 1.



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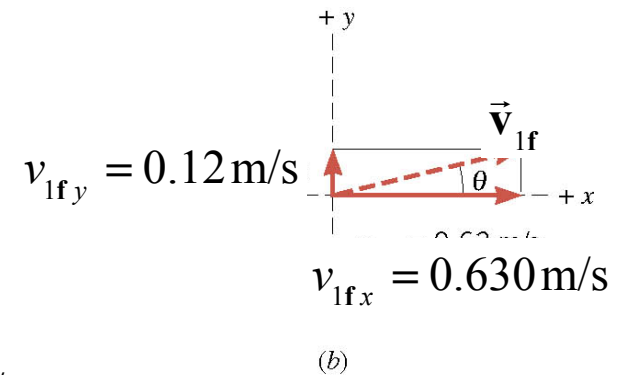
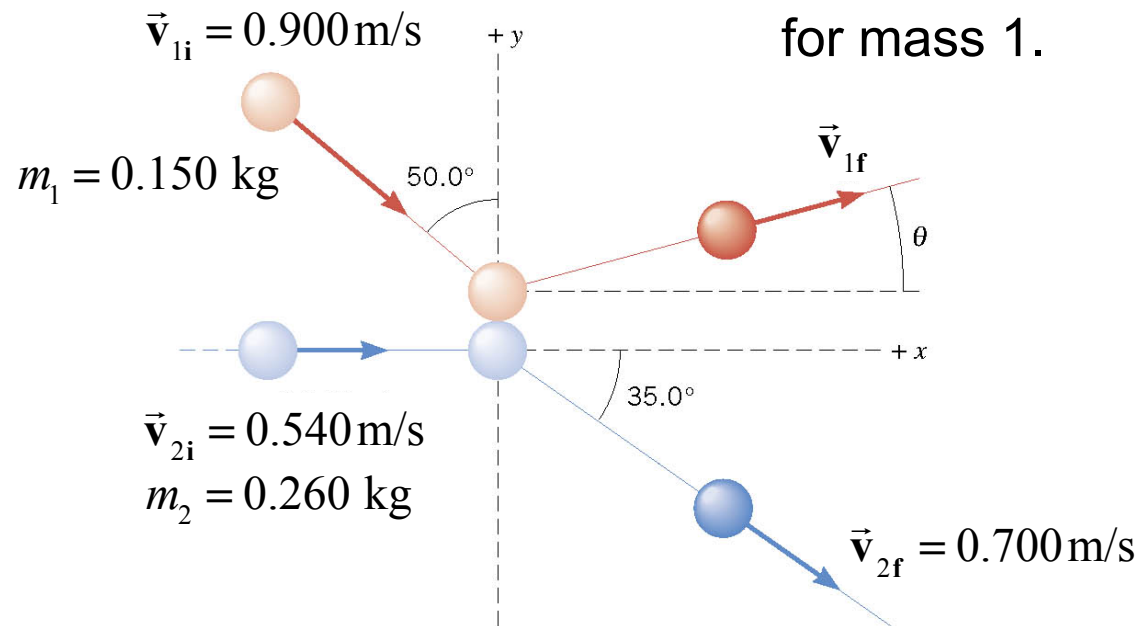


x-components: $m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$

$v_{1i} = +0.900 \sin 50^\circ \text{ m/s}$, $v_{2i} = +0.540 \text{ m/s}$, $v_{2f} = +0.700 \cos 35^\circ \text{ m/s}$

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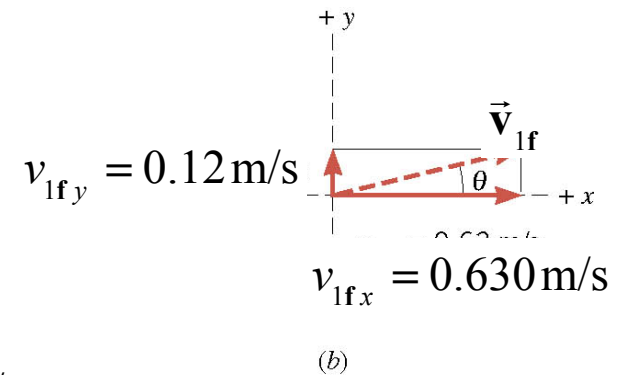
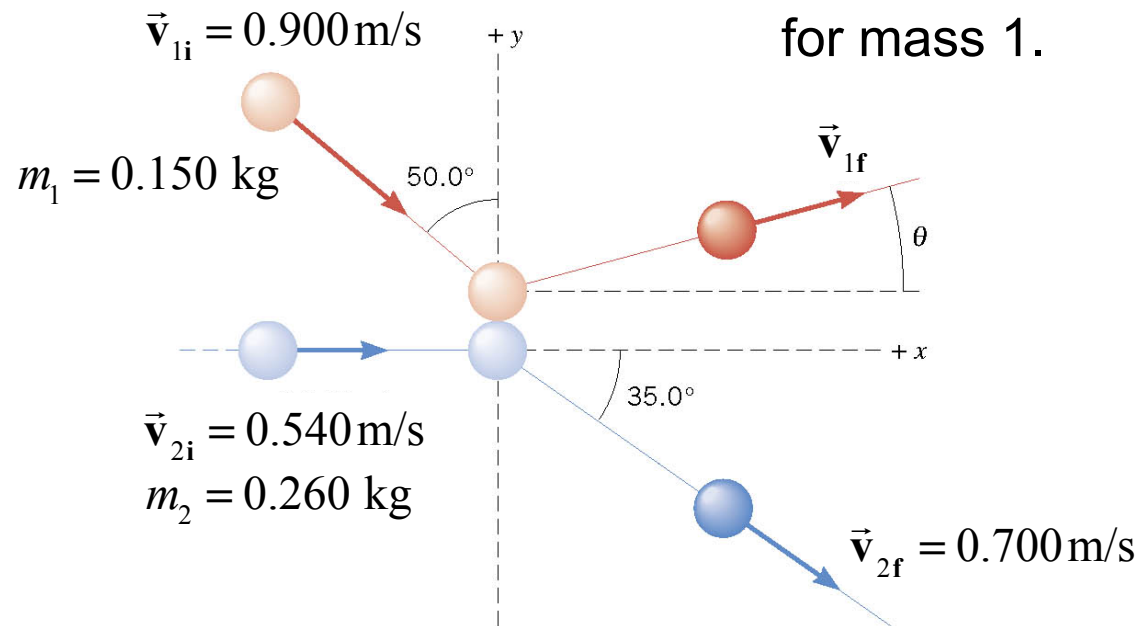
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y-components: $m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$

$v_{1i} = -0.900 \cos 50^\circ \text{ m/s}$, $v_{2i} = 0$, $v_{2f} = -0.700 \sin 35^\circ \text{ m/s}$

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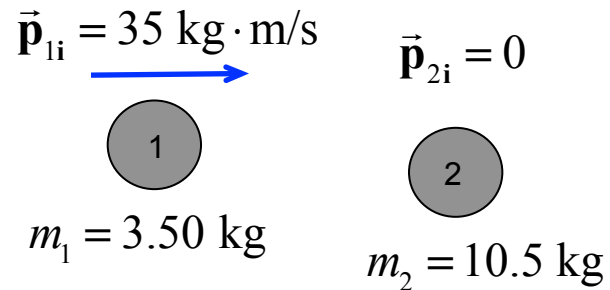
$v_{1i} = -0.900 \cos 50^\circ \text{ m/s}$, $v_{2i} = 0$, $v_{2f} = -0.700 \sin 35^\circ \text{ m/s}$

final x: $v_{1x} = +0.63 \text{ m/s}$ **final y:** $v_{1y} = +0.12 \text{ m/s}$

$v_1 = \sqrt{v_{1x}^2 + v_{1y}^2} = +0.64 \text{ m/s}$; $\theta_1 = \tan^{-1}(v_{1y}/v_{1x}) = 11^\circ$

6.4 Collisions in Two Dimensions

In the elastic collision, m_1 is deflected upward at 90° .

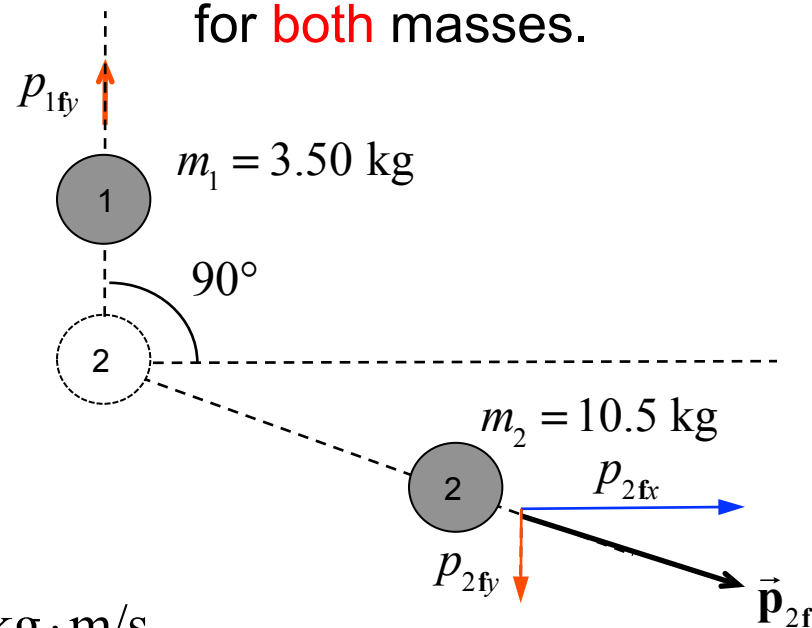


Momentum conservation

x-components: $p_{1i} = p_{2fx} = 35 \text{ kg} \cdot \text{m/s}$

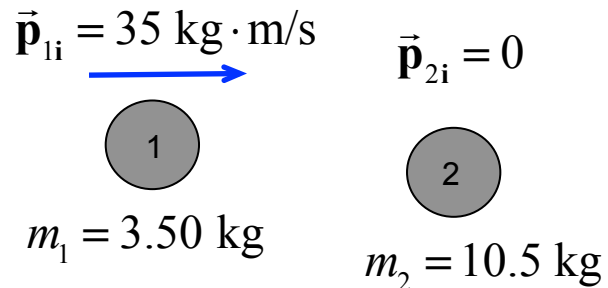
y-components: $p_{1fy} = -p_{2fy}$ (need this)

Determine the final momentum vector for **both** masses.



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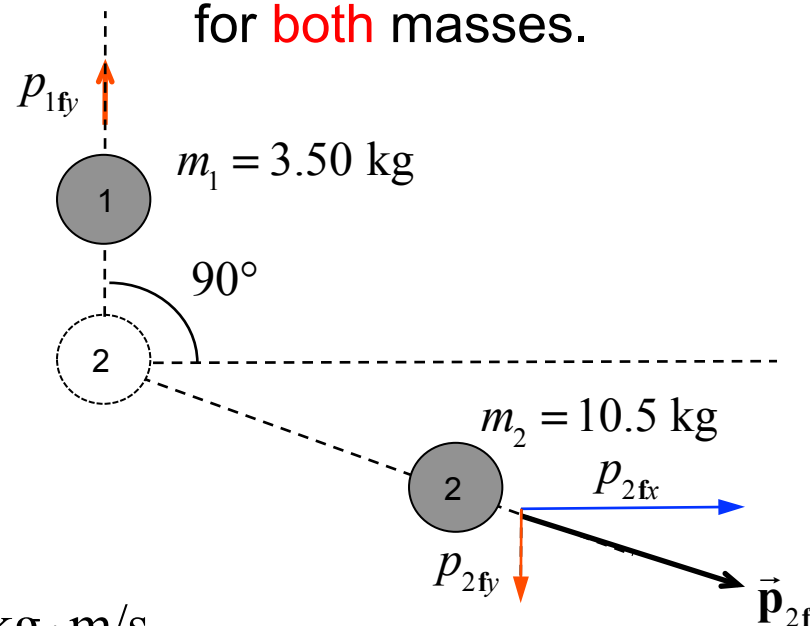
y-components: $p_{1fy} = -p_{2fy}$ (need this)

Kinetic Energies

$$K_i = \frac{p_{1i}^2}{2m_1}$$

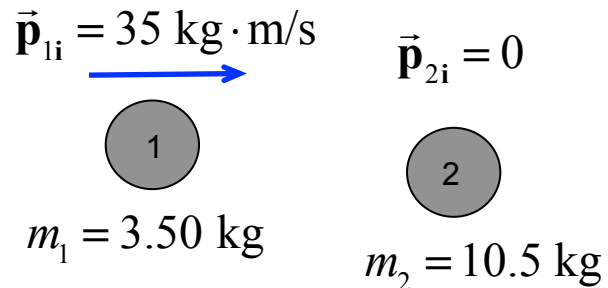
$$K_f = \frac{p_{1fy}^2}{2m_1} + \frac{p_{2fx}^2 + p_{2fy}^2}{2m_2}$$

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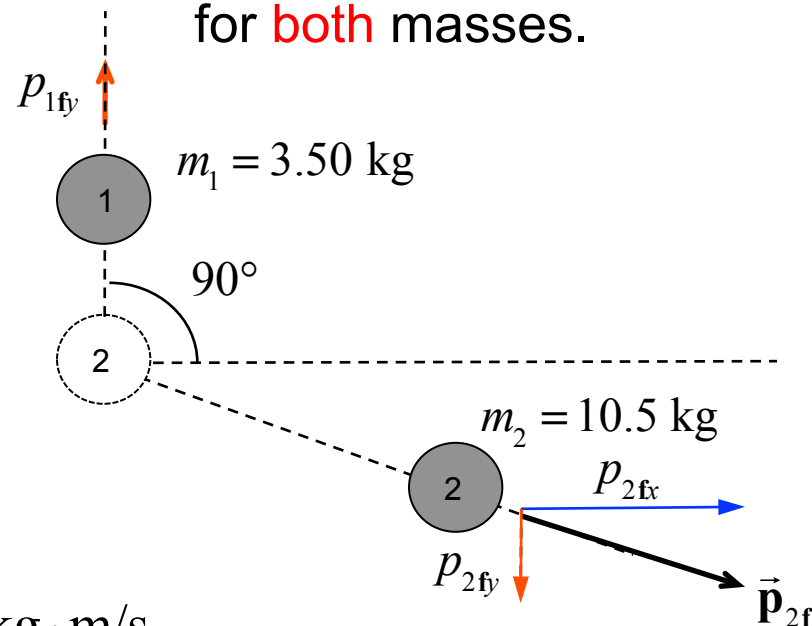


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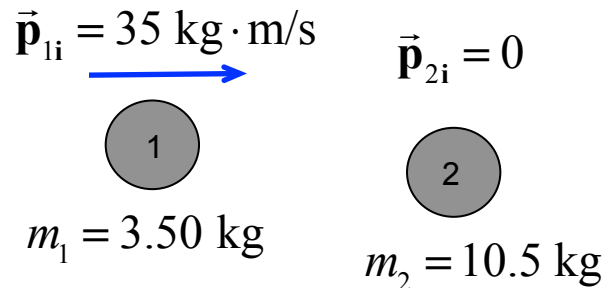
$$K_i = \frac{p_{1i}^2}{2m_1}$$

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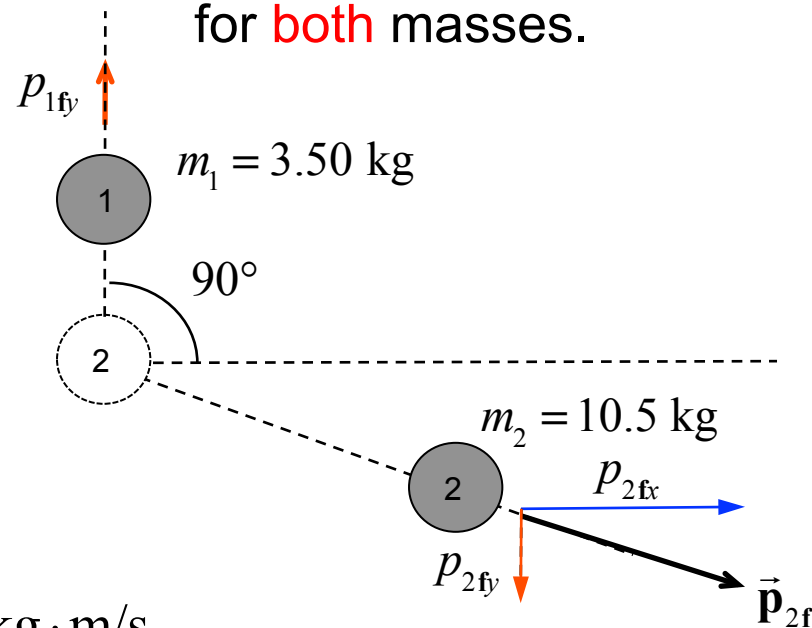
$$\frac{p_{2fy}^2}{2m_1} + \frac{p_{1i}^2 + p_{2fy}^2}{2m_2}$$

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$$K_f = \frac{p_{1fy}^2}{2m_1} + \frac{p_{2fx}^2 + p_{2fy}^2}{2m_2}$$

$$\frac{p_{2fy}^2}{2m_1} + \frac{p_{1i}^2 + p_{2fy}^2}{2m_2}$$

Energy conservation

$$K_f = K_i$$

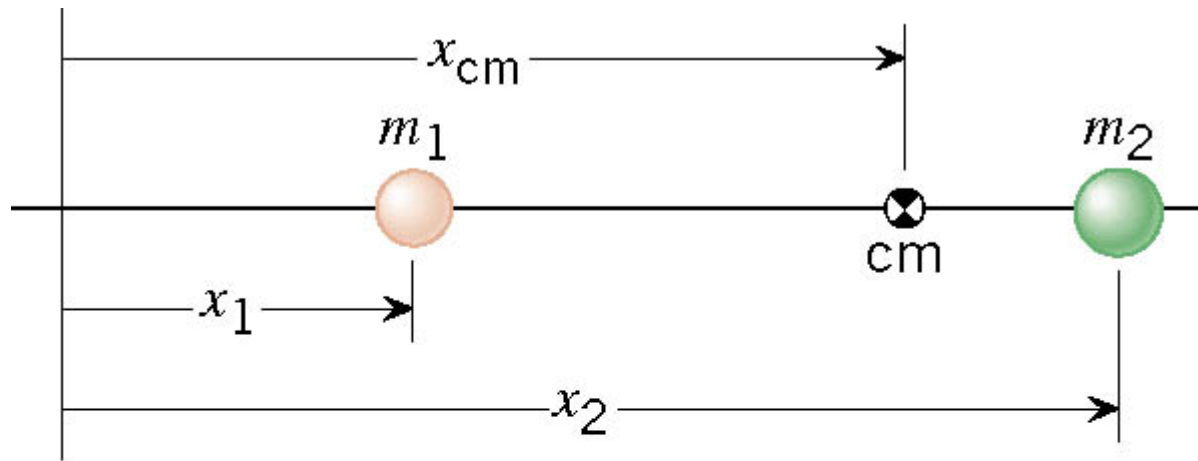
$$p_{2fy}^2 \left(\frac{1}{m_1} + \frac{1}{m_2} \right) = p_{1i}^2 \left(\frac{1}{m_1} - \frac{1}{m_2} \right)$$

$$p_{2fy} = \pm \frac{1}{\sqrt{2}} p_{1i} \Rightarrow \underline{p_{2fy} = -24.7 \text{ kg} \cdot \text{m/s}}$$

$$\underline{\text{therefore, } p_{1fy} = +24.7 \text{ kg} \cdot \text{m/s}}$$

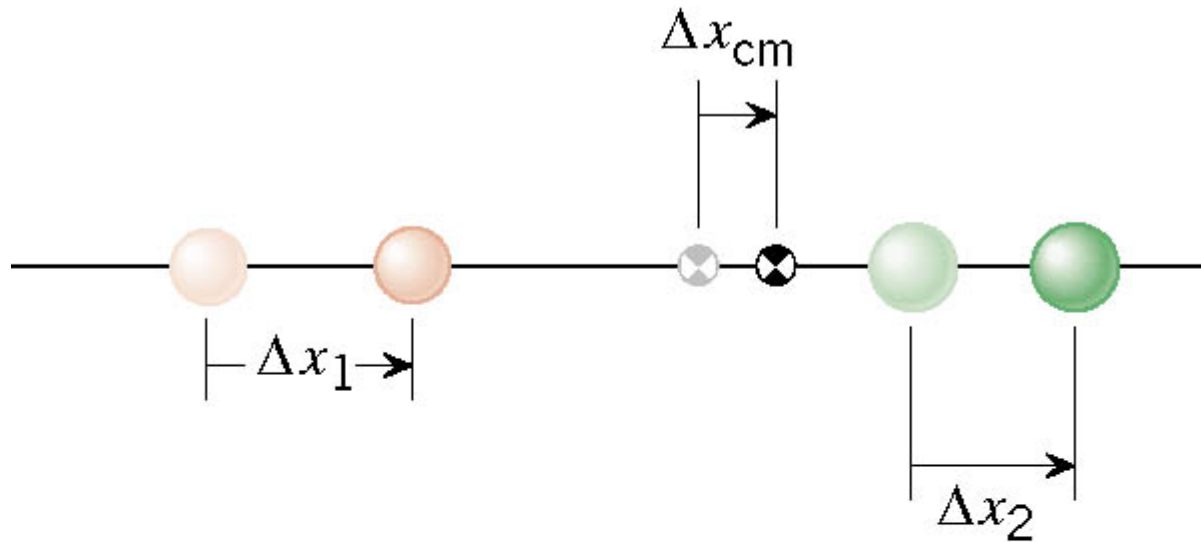
6.5 Center of Mass

The center of mass is a point that represents the average location for the total mass of a system.



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

6.5 Center of Mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} \quad \Rightarrow \quad v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

6.5 Center of Mass

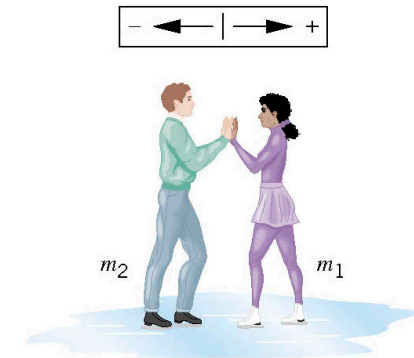
$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

6.5 Center of Mass

BEFORE

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$



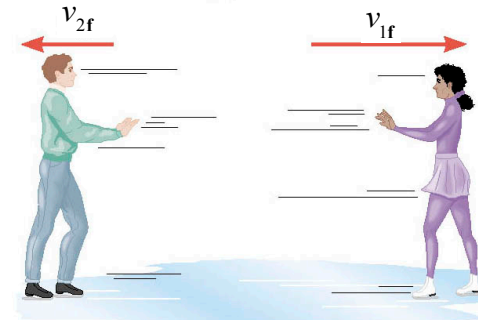
(a) Before

AFTER

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{(88 \text{ kg})(-1.5 \text{ m/s}) + (54 \text{ kg})(+2.5 \text{ m/s})}{88 \text{ kg} + 54 \text{ kg}}$$

$$= 0.00$$



(b) After