

Chapter 6

Impulse and Momentum

Continued

6.4 Collisions in Two Dimensions

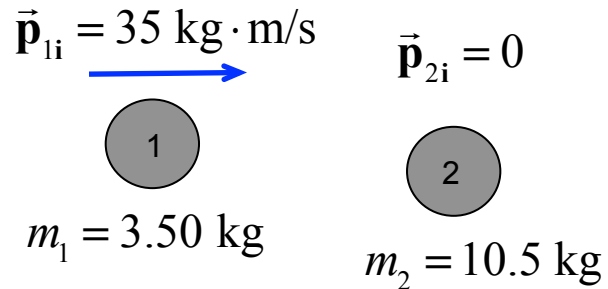
Momentum conservation can be used to solve collision problems if there are no external forces affecting the motion of the masses.

Energy conservation can be used to solve a collision problem if it is stated explicitly that the collision is ELASTIC.

If two masses in a collision stick together, the collision must be INELASTIC. **Energy conservation** cannot be used unless the energy lost or gained is provided.

6.4 Collisions in Two Dimensions

In the elastic collision, m_1 is deflected upward at 90° .

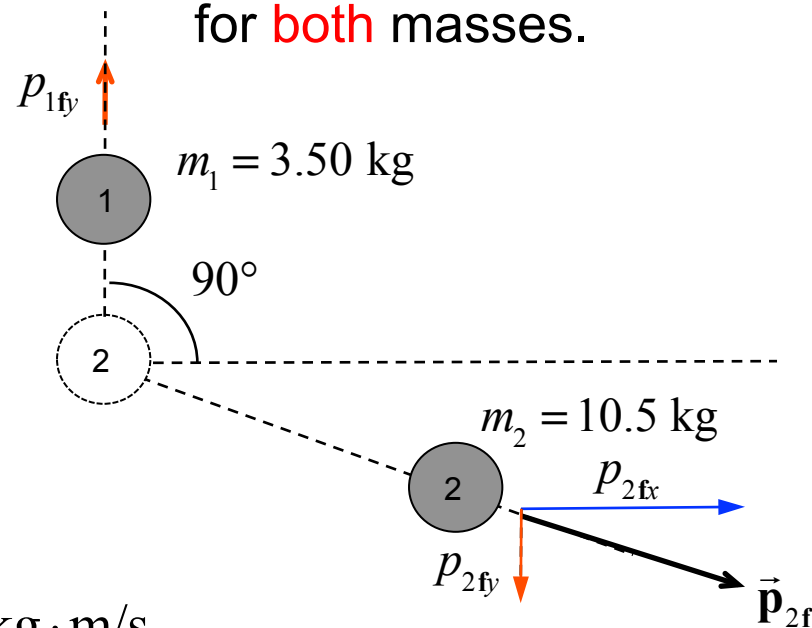


Momentum conservation

x-components: $p_{1i} = p_{2fx} = 35 \text{ kg} \cdot \text{m/s}$

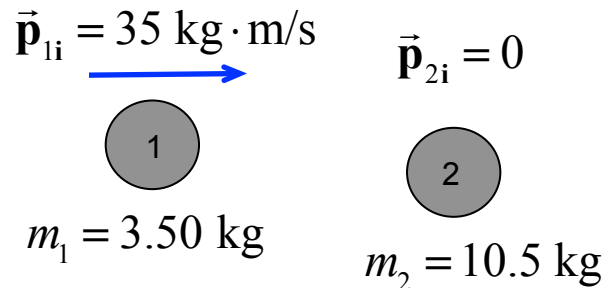
y-components: $p_{1fy} = -p_{2fy}$ (need this)

Determine the final momentum vector for **both** masses.



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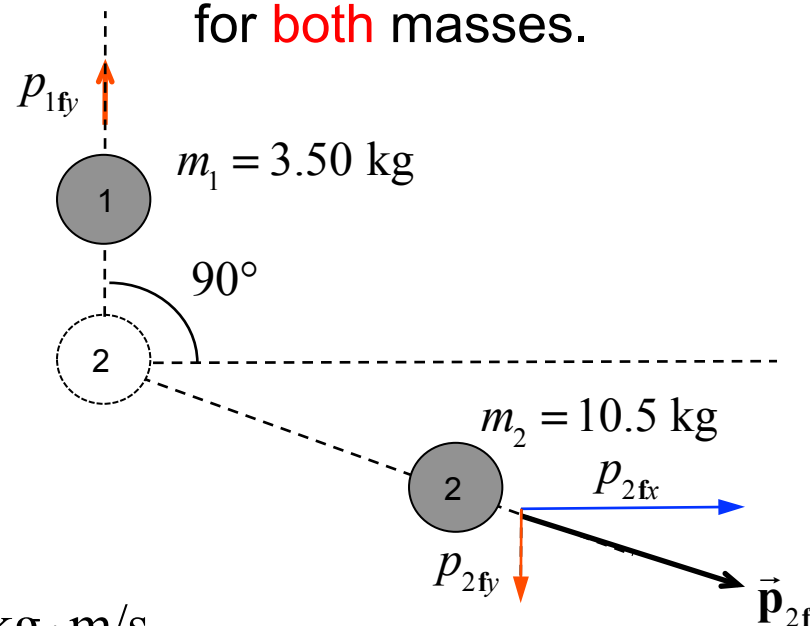
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Kinetic Energies

$$K_i = \frac{p_{1i}^2}{2m_1}$$

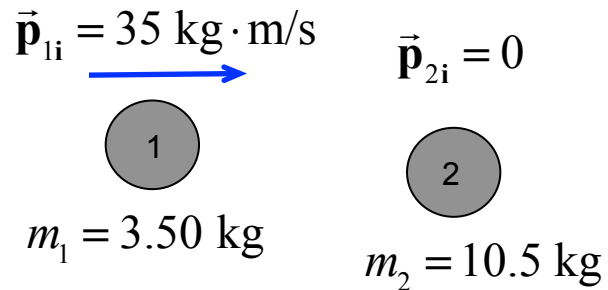
$$K_f = \frac{p_{1fy}^2}{2m_1} + \frac{p_{2fx}^2 + p_{2fy}^2}{2m_2}$$

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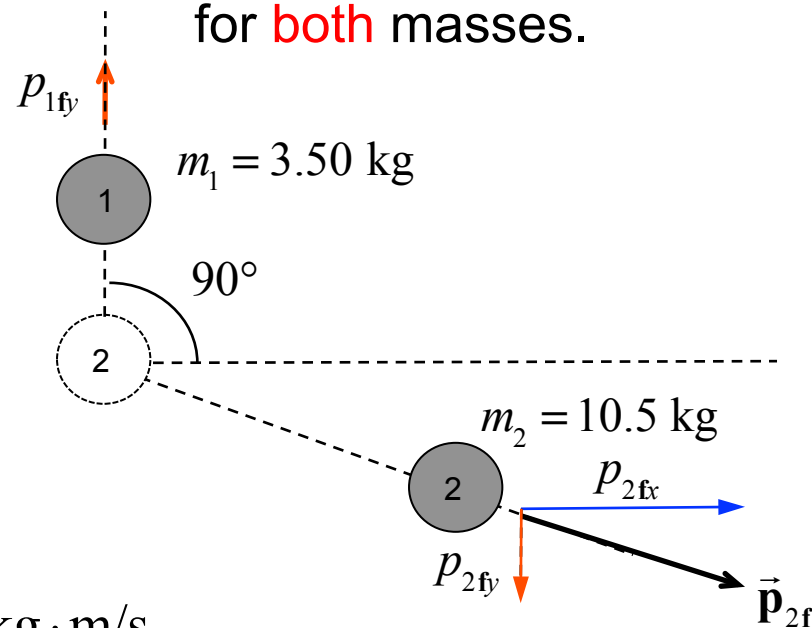


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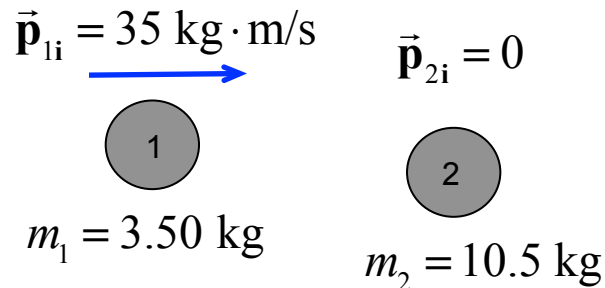
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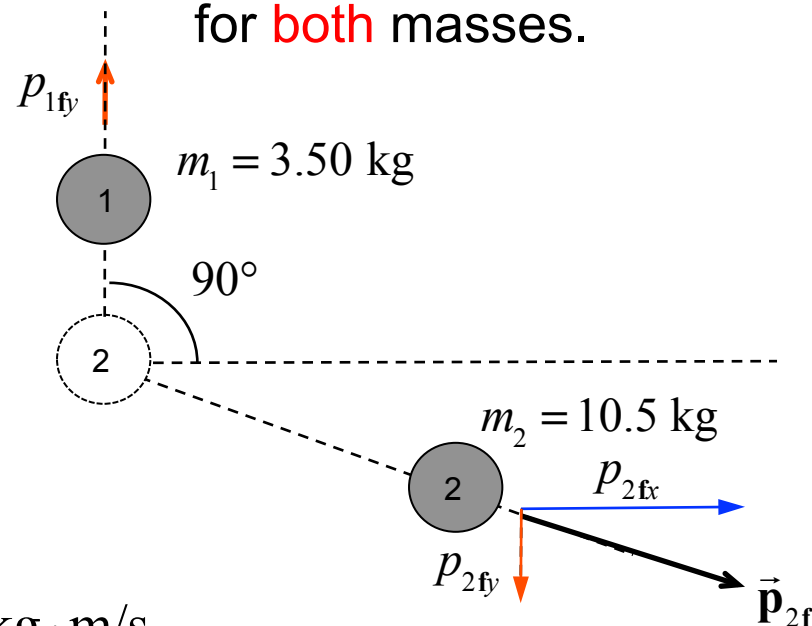
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$$\frac{p_{2fy}^2}{2m_1} + \frac{p_{1i}^2 + p_{2fy}^2}{2m_2}$$

Energy conservation

$$K_f = K_i$$

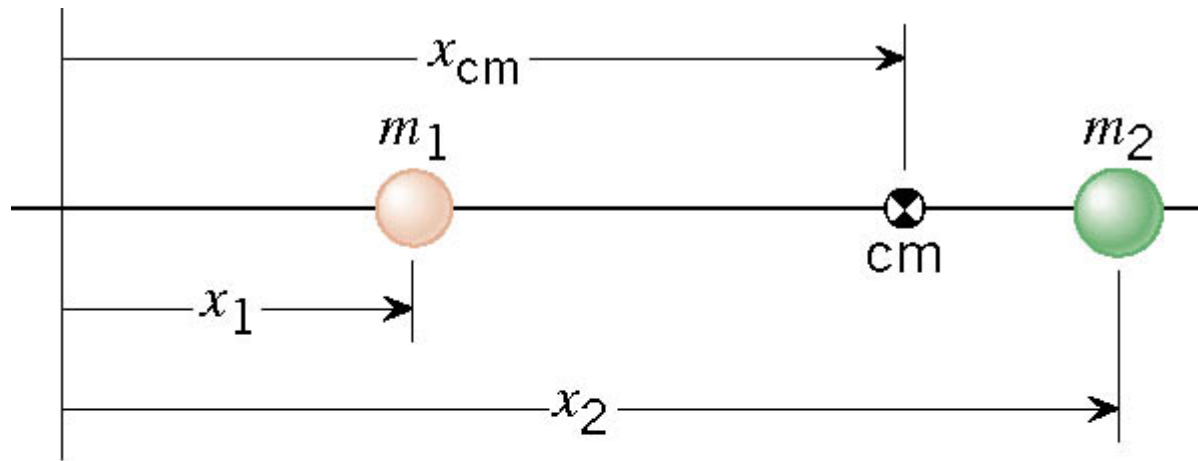
$$p_{2fy}^2 \left(\frac{1}{m_1} + \frac{1}{m_2} \right) = p_{1i}^2 \left(\frac{1}{m_1} - \frac{1}{m_2} \right)$$

$$p_{2fy} = \pm \frac{1}{\sqrt{2}} p_{1i} \Rightarrow \underline{p_{2fy} = -24.7 \text{ kg} \cdot \text{m/s}}$$

$$\underline{\text{therefore, } p_{1fy} = +24.7 \text{ kg} \cdot \text{m/s}}$$

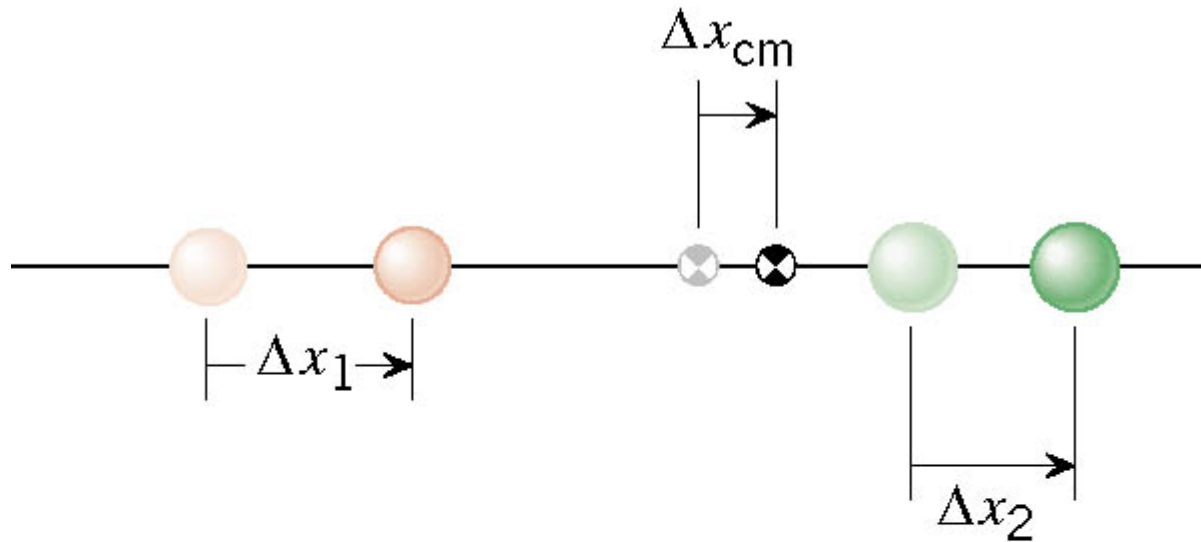
6.5 Center of Mass

The center of mass is a point that represents the average location for the total mass of a system.



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

6.5 Center of Mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} \quad \Rightarrow \quad v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

6.5 Center of Mass

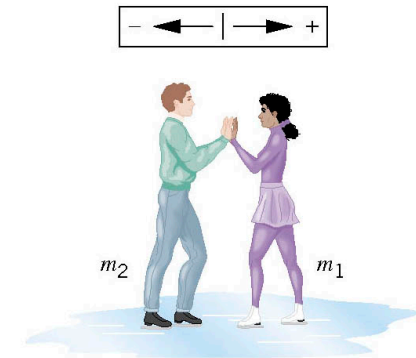
$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

6.5 Center of Mass

BEFORE

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$



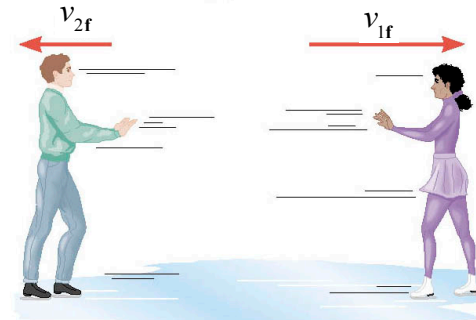
(a) Before

AFTER

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{(54 \text{ kg})(+2.5 \text{ m/s}) + (88 \text{ kg})(-1.5 \text{ m/s})}{88 \text{ kg} + 54 \text{ kg}}$$

$$= 0.00$$



(b) After

Forces and Newton's Laws of Motion

Force Magnitudes

Gravitational Force: $F_G = mg$ (magnitude), direction downward, weight

Elastic Forces: (stretched spring) $F_s = kx$ (magnitude), inward at the ends

Compression: (squeezed spring or object) normal forces, n , \perp contact

Frictional: opposes motion, Static: $f_{Max} = \mu_S F_{\perp}$, then Kinetic: $f_K = \mu_K F_{\perp}$

Net Force: $\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \dots$ on a single object (magnitude and direction)

Newton's 1st Law: speed and direction do not change, if & only if $\vec{F}_{Net} = 0$

Newton's 2nd Law: $\vec{F}_{Net} = m\vec{a}$

Newton's 3rd Law for two masses (1,2) in contact: $\vec{F}_{12} = -\vec{F}_{21}$

Decomposition to components: $\vec{F} = F_x \hat{i} + F_y \hat{j}$, $F_x = F \cos \theta$, $F_y = F \sin \theta$

Components to magnitude angle: $F = \sqrt{F_x^2 + F_y^2}$, $\theta = \tan^{-1}(F_y / F_x)$

Summaries and Examples

Energy and energy conservation

Momentum and momentum conservation

Energy and energy conservation

$$\text{Work: } W = F(\cos \theta) \Delta x$$

$$\text{Kinetic Energy: } K = \frac{1}{2} m v^2$$

$$\text{Work changes kinetic energy: } W = K - K_0 = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$$

$$\text{Conservative forces} \Rightarrow \text{Potential Energy}$$

$$\text{Gravitational Potential Energy: } U_G = mgy$$

$$\text{Ideal Spring Potential Energy: } U_s = \frac{1}{2} kx^2$$

$$\text{Total Energy: } E = K + U$$

Work by non-conservative forces (friction, humans, explosions)

$$\text{changes total energy: } K + U = K_0 + U_0 + W_{\text{NC}}$$

If $W_{\text{NC}} = 0$, there is total energy conservation:

$$E = E_0 \Rightarrow K + U = K_0 + U_0$$

$$\text{Average power} = \text{Work/time} = (\text{Energy change})/\text{time} = F \bar{v}$$

Momentum and momentum conservation

$$\text{Impulse: } \vec{\mathbf{J}} = \vec{\mathbf{F}}t$$

$$\text{Momentum: } \vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Net average impulse changes momentum:

$$\sum \vec{\mathbf{F}}\Delta t = \vec{\mathbf{p}} - \vec{\mathbf{p}}_0 = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_0$$

$$\text{Momentum of 2 masses in collision: } \vec{\mathbf{p}}_{\text{total}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$$

No net external force active, momentum is conserved

$$\vec{\mathbf{p}}_{\text{Total},f} = \vec{\mathbf{p}}_{\text{Total},i} \Rightarrow m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} = m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i}$$

$$\text{Center of mass position: } x_{\text{cm}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \text{ and velocity } \vec{\mathbf{v}}_{\text{cm}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}$$

$$\text{Conservation of momentum} \Rightarrow \vec{\mathbf{v}}_{\text{cm}} \text{ remains constant}$$

Clicker Question 6.7

**A space probe travels with a momentum of $+6.0 \times 10^7 \text{ kg} \cdot \text{m} / \text{s}$.
A retrorocket fires for 10 s, with a force magnitude of $2.0 \times 10^6 \text{ N}$, that slows the speed of the probe. What is the momentum of the probe after the rocket ceases to fire?**

- a) $2.0 \times 10^7 \text{ kg} \cdot \text{m/s}$**
- b) $4.0 \times 10^7 \text{ kg} \cdot \text{m/s}$**
- c) $6.0 \times 10^7 \text{ kg} \cdot \text{m/s}$**
- d) $8.0 \times 10^7 \text{ kg} \cdot \text{m/s}$**
- e) $10.0 \times 10^7 \text{ kg} \cdot \text{m/s}$**

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Impulse changes momentum

$$\mathbf{F}\Delta t = \mathbf{p}_f - \mathbf{p}_i$$

$$\mathbf{p}_f = \mathbf{p}_i + \mathbf{F}\Delta t$$

$$\begin{aligned}\mathbf{p}_f &= \left(+6.0 \times 10^7 \text{ N} \cdot \text{s}\right) + (-2.0 \times 10^6 \text{ N})(10 \text{ s}) \\ &= +4.0 \times 10^7 \text{ N} \cdot \text{s}\end{aligned}$$

Clicker Question 6.8

Two identical 2.0-kg objects are involved in a collision. The objects are on a frictionless track. The initial velocity of object A is +12 m/s and the initial velocity of object B is – 6.0 m/s. What is the final velocity of the two objects?

- a) Object A moves at – 6.0 m/s and B moves at +12 m/s.
- b) Object A moves at +9.0 m/s and B moves at +9.0 m/s.
- c) Object A moves at – 6.0 m/s and B moves at +6.0 m/s.
- d) Object A moves at –12 m/s and B moves at +6.0 m/s.
- e) This cannot be determined without energy information.

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must say if elastic or inelastic

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Momentum is conserved

$$\vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \vec{p}_{2i}$$

(all are x components) $mv_{1f} + mv_{2f} = mv_{1i} + mv_{2i}$ (m cancels in each term)

$$v_{1f} + v_{2f} = (+12\text{m/s}) + (-6\text{m/s}) = +6\text{m/s}$$

Only know the sum: $v_{1f} + v_{2f} = +6\text{m/s}$, Need energy equation

Is this an **ELASTIC** or **INELASTIC** collision?

Clicker Question 6.8


an elastic

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Energy is also conserved

$$v_{1f}^2 + v_{2f}^2 = v_{1i}^2 + v_{2i}^2 = 144 \text{ m}^2/\text{s}^2 + 36 \text{ m}^2/\text{s}^2 \quad (\text{energy conservation, } \frac{1}{2}m \text{ cancels})$$

$$v_{1f}^2 + v_{2f}^2 = 180 \text{ m}^2/\text{s}^2 \quad v_{2f} = +6 \text{ m/s} - v_{1f} \quad (\text{previous slide})$$


$$v_{1f}^2 + (6 \text{ m/s} - v_{1f})^2 = 2v_{1f}^2 - (12 \text{ m/s})v_{1f} + 36 \text{ m}^2/\text{s}^2$$

$$\text{Quadratic Equation: } v_{1f}^2 - (6 \text{ m/s})v_{1f} - 72 \text{ m}^2/\text{s}^2 = 0$$

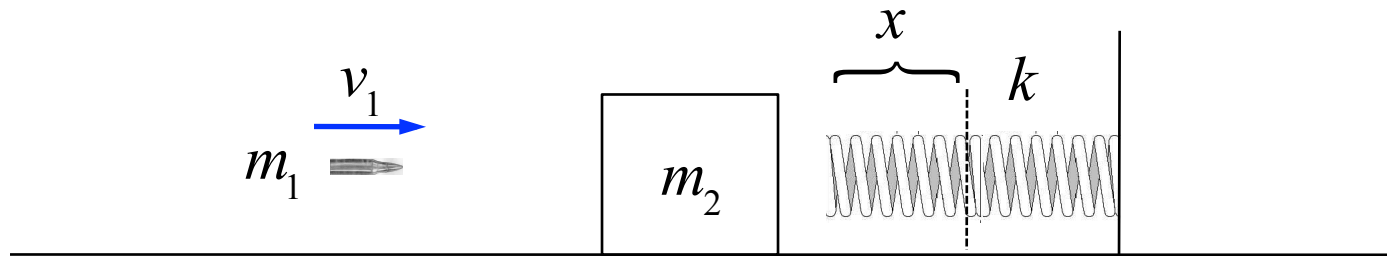
$$v_{1f} = \left(+3 \pm \sqrt{9 + 72} \right) \text{ m/s} = \left(+3 \pm 9 \right) \text{ m/s}$$

$$v_{1f} = -6 \text{ m/s} \quad (\text{other solution - } v_{1f} = +12 \text{ m/s, no collision})$$

- a) Object A moves at – 6.0 m/s and B moves at +12 m/s.**

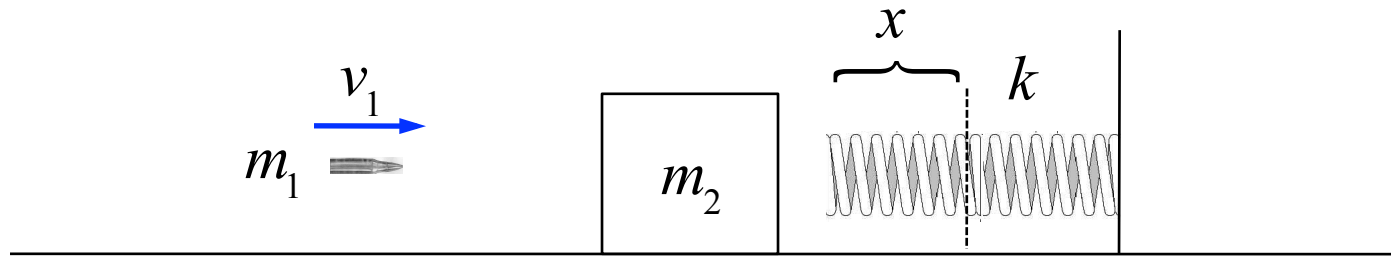
Example:

An 11-g bullet is fired horizontally into a 110-g wooden block that is initially at rest on a frictionless horizontal surface and a fixed spring having spring constant 183 N/m. The bullet becomes embedded in the block. The bullet-block system compresses the spring by a maximum amount 85 cm. What was the initial velocity of the bullet?



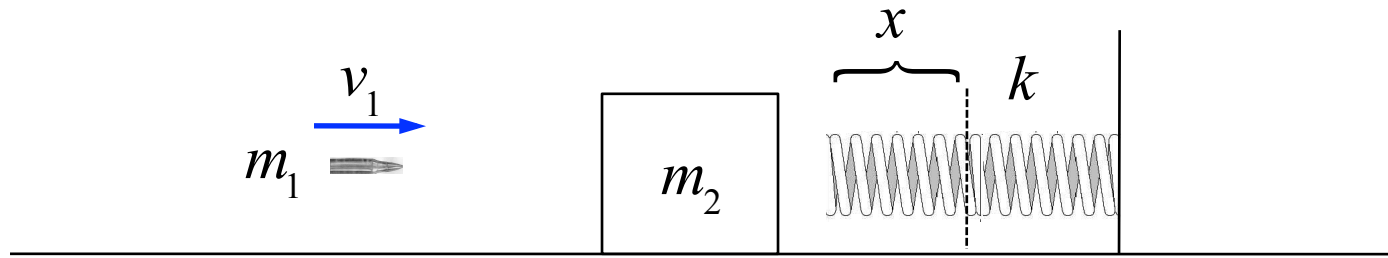
Strategy:

- 1) Momentum conservation bullet + block
- 2) Energy conservation in spring compression



1) Momentum conservation bullet + block

$$p_1 = m_1 v_1, \quad p = (m_1 + m_2) v \quad \Rightarrow \quad p_1 = p$$



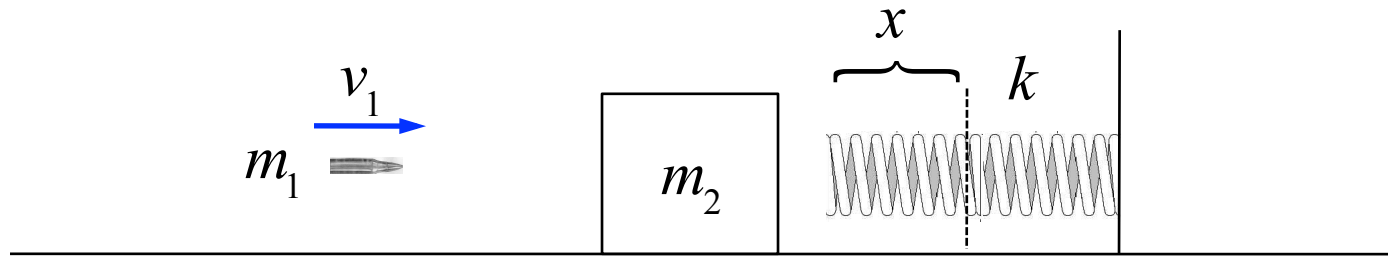
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2) Energy conservation in spring compression

Kinetic Energy of block and bullet : $K_i = p_1^2 / 2(m_1 + m_2)$

Potential Energy of compressed spring : $U = \frac{1}{2} k x^2$



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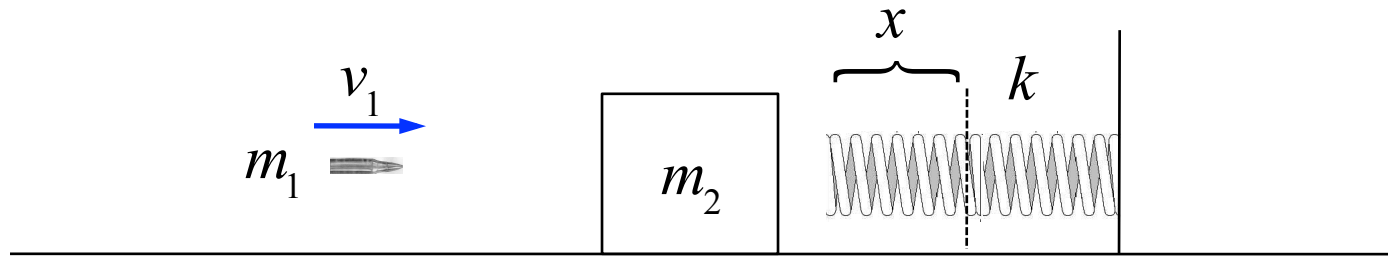
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Energy Conservation : $K + U = K_i + U_i \quad ; \quad U_i = 0, K = 0$



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$$\frac{1}{2} k x^2 = p_1^2 / 2(m_1 + m_2)$$

$$p_1 = x \sqrt{k(m_1 + m_2)}$$

$$m_1 v_1 = (0.85 \text{ m}) \sqrt{(183 \text{ N/m})(0.121 \text{ kg})} = 4.00 \text{ N} \cdot \text{s}$$

$$v_1 = 4.00 \text{ N} \cdot \text{s} / (0.011 \text{ kg}) = 364 \text{ m/s}$$