Chapter 6

Impulse and Momentum

Continued

6.4 Collisions in Two Dimensions

Momentum conservation can be used to solve collision problems if there are no external forces affecting the motion of the masses.

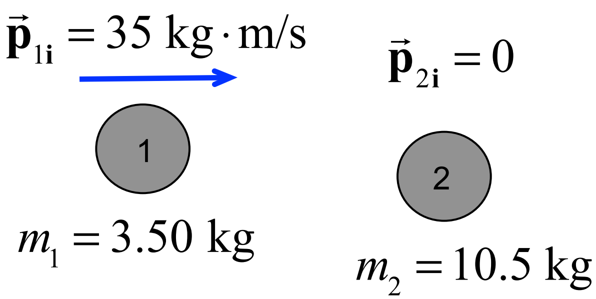
Energy conservation can be used to solve a collision problem if it is stated explicity that the collision is ELASTIC.

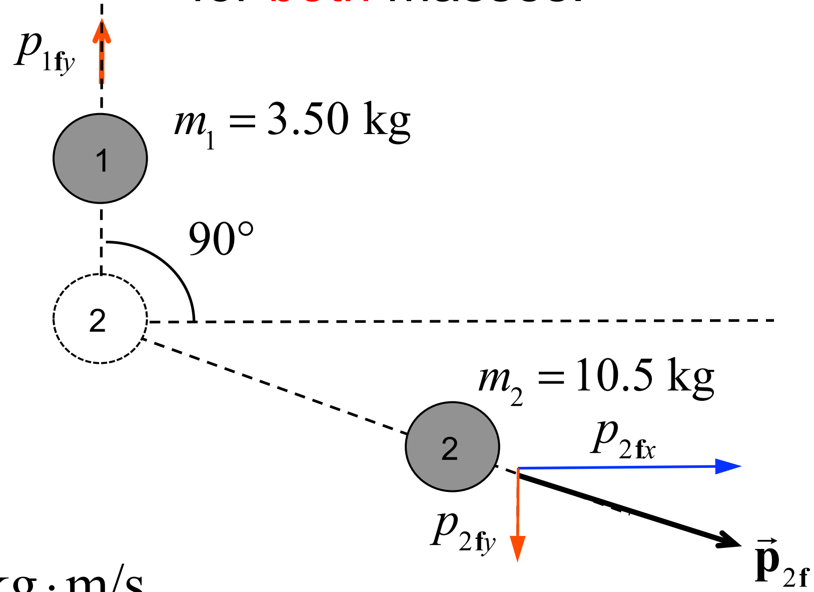
If two masses in a collision stick together, the collision must be INELASTIC. Energy conservation cannot be used unless the energy lost or gained is provided.

6.4 ***Collisions in Two Dimensions***

Determine the final momentum vector for both masses.

In the elastic collision, m_1 is deflected upward at 90°.





<u>Momentum conservation</u>

x-components: $p_{1\mathbf{i}} = \underline{p_{2\mathbf{f}x} = 35\ \text{kg}\cdot\text{m/s}}$

y-components: $p_{1\mathbf{f}y} = \underline{-p_{2\mathbf{f}y}}$ (need this)

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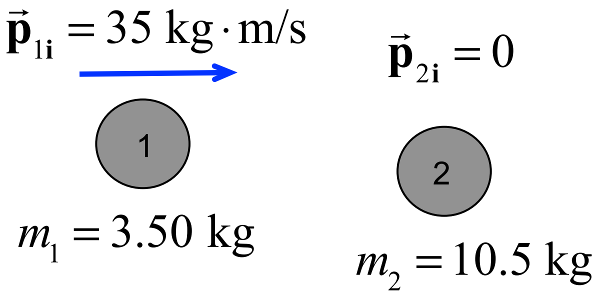

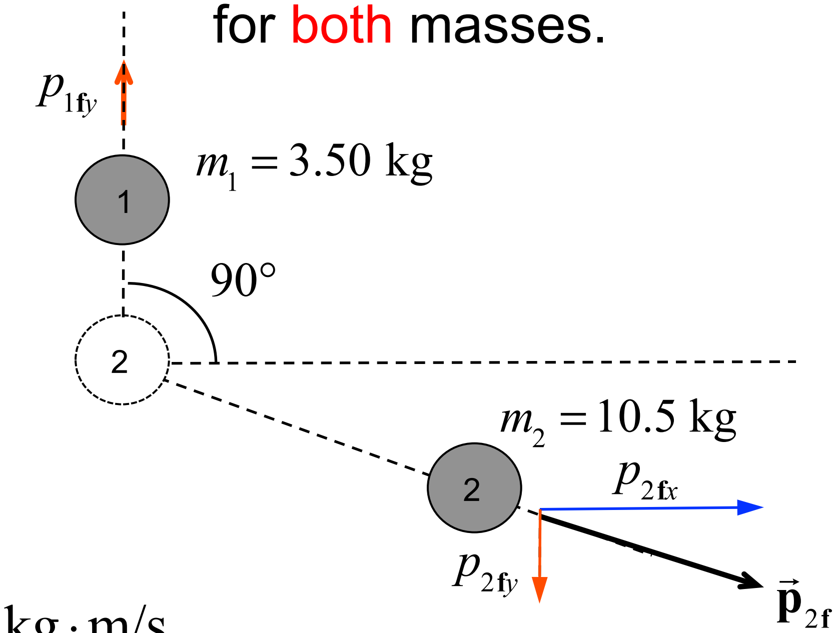

<u>Momentum conservation</u>

x-components: $p_{1i} = \underline{p_{2fx} = 35 \text{ kg}\cdot\text{m/s}}$

y-components: $p_{1fy} = \underline{-p_{2fy}}$ (need this)

<u>Kinetic Energies</u>

$$K_i = \frac{p_{1i}^2}{2m_1}$$

$$K_f = \frac{p_{1fy}^2}{2m_1} + \frac{p_{2fx}^2 + p_{2fy}^2}{2m_2}$$

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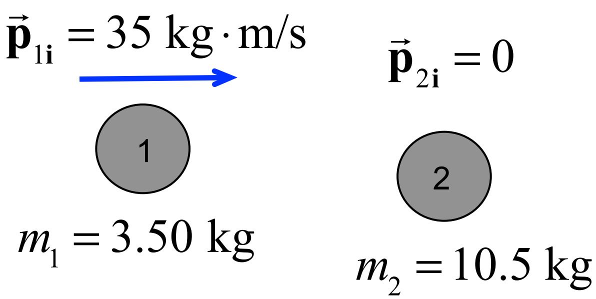

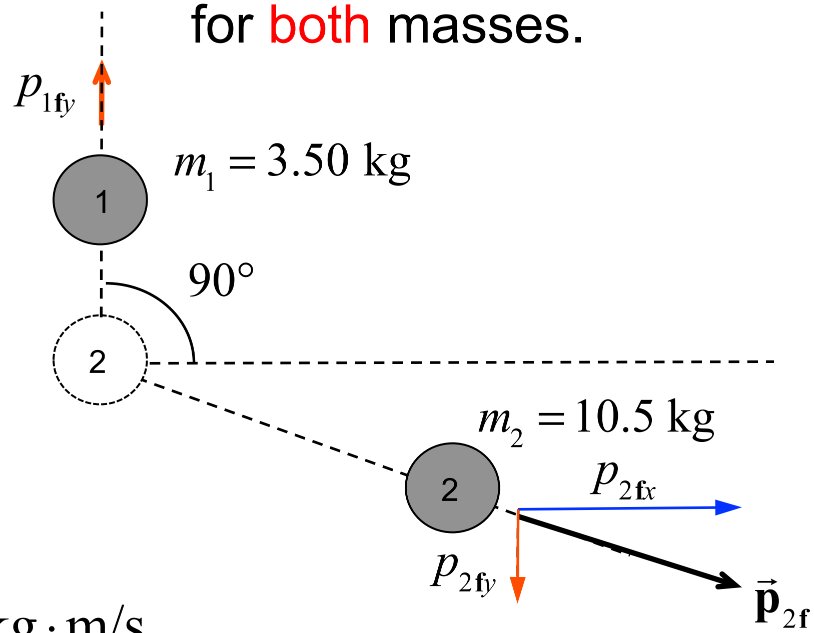

Momentum conservation

x-components: $p_{1\mathrm{i}} = p_{2\mathrm{f}x} = 35\ \mathrm{kg \cdot m/s}$

y-components: $p_{1\mathrm{f}y} = -p_{2\mathrm{f}y}$ (need this)

Kinetic Energies

$$K_\mathrm{i} = \frac{p_{1\mathrm{i}}^2}{2m_1}$$

$$K_\mathrm{f} = \frac{p_{1\mathrm{f}y}^2}{2m_1} + \frac{p_{2\mathrm{f}x}^2 + p_{2\mathrm{f}y}^2}{2m_2}$$

$$\frac{p_{2\mathrm{f}y}^2}{2m_1} + \frac{p_{1\mathrm{i}}^2 + p_{2\mathrm{f}y}^2}{2m_2}$$

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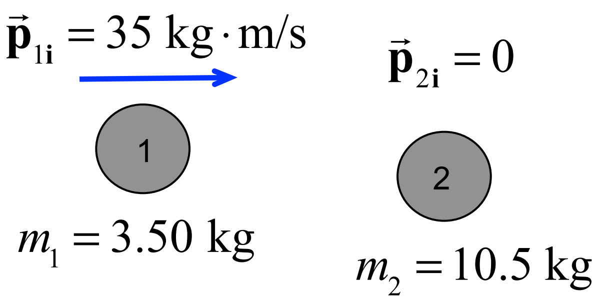

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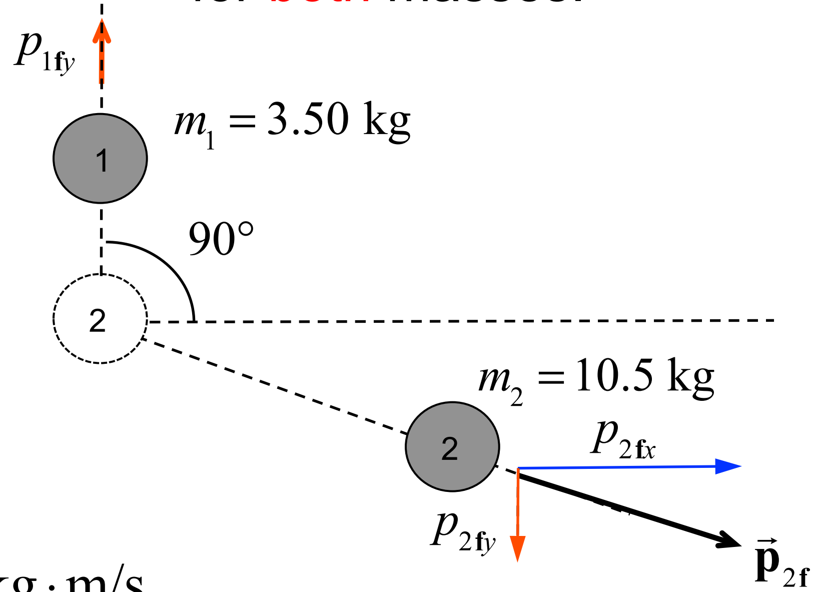

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$$\frac{p_{2\mathrm{f}y}^2}{2m_1} + \frac{p_{1\mathrm{i}}^2 + p_{2\mathrm{f}y}^2}{2m_2}$$

Energy conservation

$$K_\mathrm{f} = K_i$$

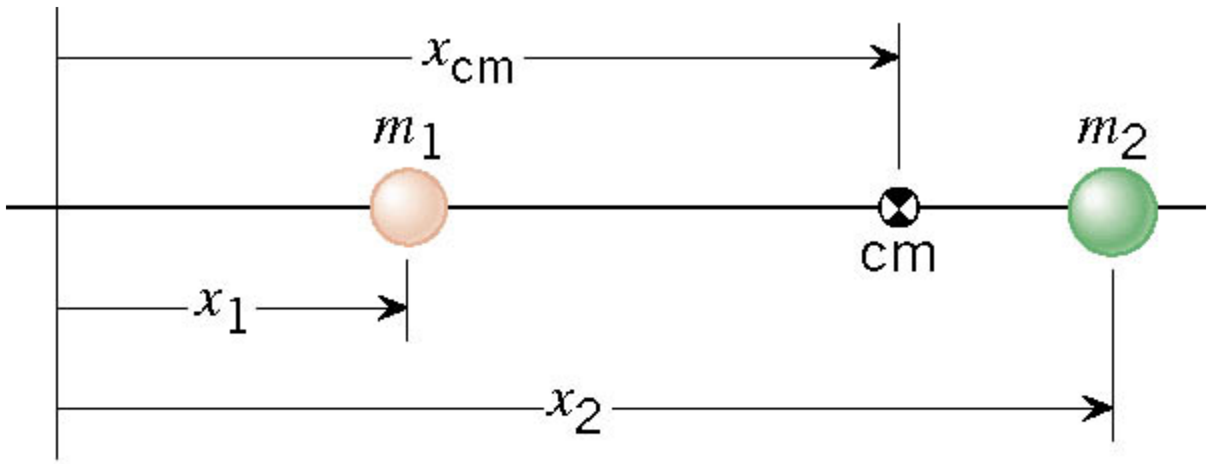
$$p_{2\mathrm{f}y}^2\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = p_{1\mathrm{i}}^2\left(\frac{1}{m_1} - \frac{1}{m_2}\right)$$

$$p_{2\mathrm{f}y} = \pm\frac{1}{\sqrt{2}}\,p_{1\mathrm{i}} \quad \Rightarrow \quad \underline{p_{2\mathrm{f}y} = -24.7\ \mathrm{kg \cdot m/s}}$$

$$\underline{\text{therefore, } p_{1\mathrm{f}y} = +24.7\ \mathrm{kg \cdot m/s}}$$

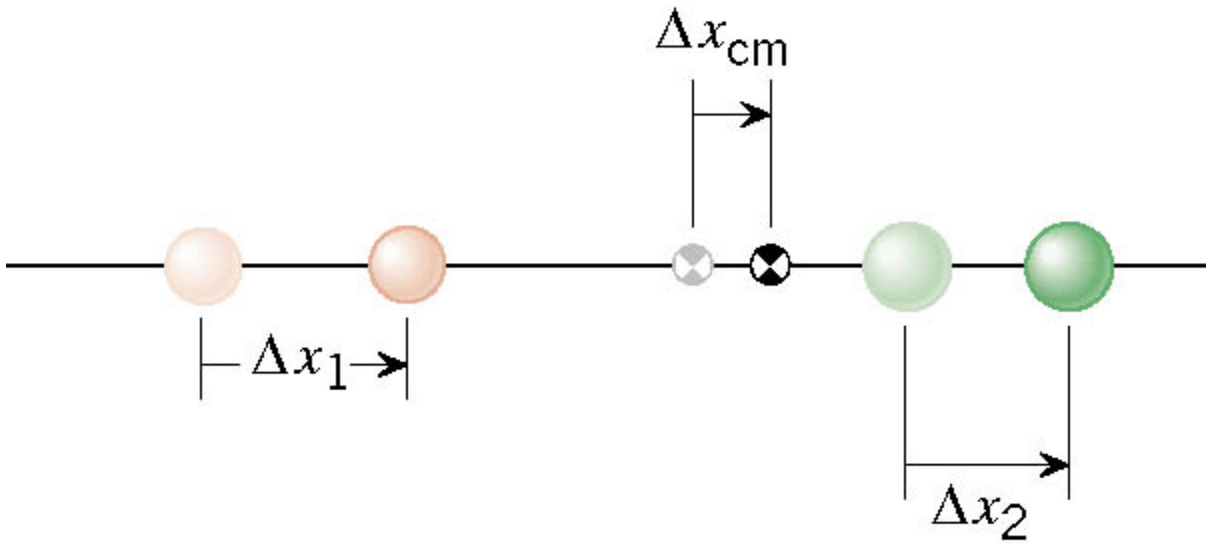
6.5 Center of Mass

The center of mass is a point that represents the average location for the total mass of a system.



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

6.5 Center of Mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} \quad \Rightarrow \quad v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

6.5 Center of Mass

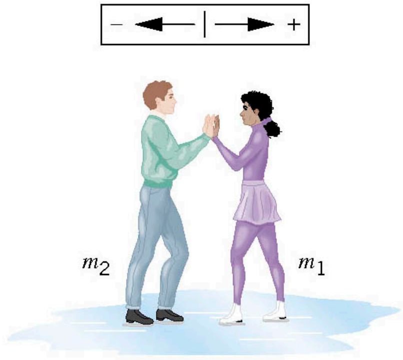
$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

6.5 Center of Mass

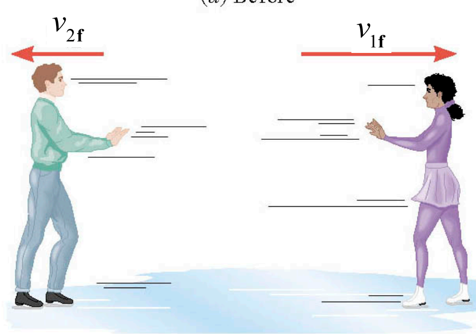
BEFORE

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$



(*a*) Before

AFTER



(*b*) After

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{(54\text{ kg})(+2.5\text{m/s}) + (88\text{ kg})(-1.5\text{m/s})}{88\text{ kg} + 54\text{ kg}}$$

$$= 0.00$$

Forces and Newton's Laws of Motion

Force Magnitudes

Gravitational Force: $F_G = mg$ (magnitude), direction downward, weight

Elastic Forces: (stretched spring) $F_S = kx$ (magnitude), inward at the ends

Compression: (squeezed spring or object) normal forces, n, $\perp$ contact

Frictional: opposes motion, Static: $\mathrm{f}_{Max} = \mu_S F_\perp$, then Kinetic: $\mathrm{f}_K = \mu_K F_\perp$

Net Force: $\vec{\mathbf{F}}_{\text{Net}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \ldots$ on a single object (magnitude and direction)

Newton's 1st Law: speed and direction do not change, if & only if $\vec{\mathbf{F}}_{\text{Net}} = 0$

Newton's 2nd Law: $\vec{\mathbf{F}}_{\text{Net}} = m\vec{\mathbf{a}}$

Newton's 3rd Law for two masses (1,2) in contact: $\vec{\mathbf{F}}_{12} = -\vec{\mathbf{F}}_{21}$

Decomposition to components: $\vec{\mathbf{F}} = F_x \hat{\mathbf{i}} + F_y \hat{\mathbf{j}}$, $F_x = F\cos\theta$, $F_y = F\sin\theta$

Components to magnitude angle: $F = \sqrt{F_x^2 + F_y^2}$, $\theta = \tan^{-1}(F_y / F_x)$

Summaries and Examples
Energy and energy conservation
Momentum and momentum conservation

Energy and energy conservation

Work: $W = F(\cos\theta)\Delta x$

Kinetic Energy: $K = \frac{1}{2}mv^2$

Work changes kinetic energy: $W = K - K_0 = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

Conservative forces $\Rightarrow$ Potential Energy

Gravitational Potential Energy: $U_G = mgy$

Ideal Spring Potential Energy: $U_S = \frac{1}{2}kx^2$

Total Energy: $E = K + U$

Work by non-conservative forces (friction, humans, explosions) changes total energy: $K + U = K_0 + U_0 + W_{\text{NC}}$

If $W_{\text{NC}} = 0$, there is total energy conservation:

$$E = E_0 \quad \Rightarrow \quad K + U = K_0 + U_0$$

Average power = Work/time = (Energy change)/time = $F\,\overline{v}$

Momentum and momentum conservation

Impulse: $\vec{\mathbf{J}} = \vec{\mathbf{F}}t$

Momentum: $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

Net average impulse changes momentum:

$$\sum \bar{\vec{\mathbf{F}}}\Delta t = \vec{\mathbf{p}} - \vec{\mathbf{p}}_0 = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_0$$

Momentum of 2 masses in collision: $\vec{\mathbf{p}}_{\text{total}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$

No net external force active, momentum is conserved

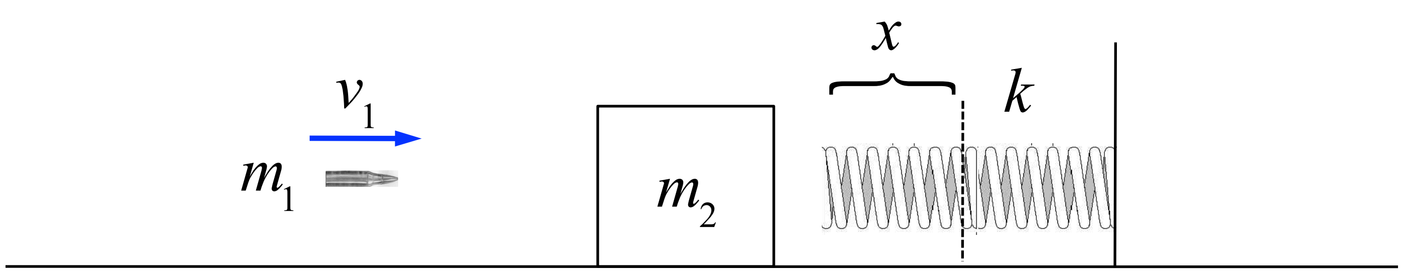
$$\vec{\mathbf{p}}_{Total,\mathbf{f}} = \vec{\mathbf{p}}_{Total.\mathbf{i}} \quad \Rightarrow \quad m_1\vec{\mathbf{v}}_{1\mathbf{f}} + m_2\vec{\mathbf{v}}_{2\mathbf{f}} = m_1\vec{\mathbf{v}}_{1\mathbf{i}} + m_2\vec{\mathbf{v}}_{2\mathbf{i}}$$

Center of mass position: $x_{\text{cm}} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$, and velocity $\vec{\mathbf{v}}_{\text{cm}} = \dfrac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}$

Conservation of momentum $\Rightarrow$ $\vec{\mathbf{v}}_{\text{cm}}$ remains constant

Example:

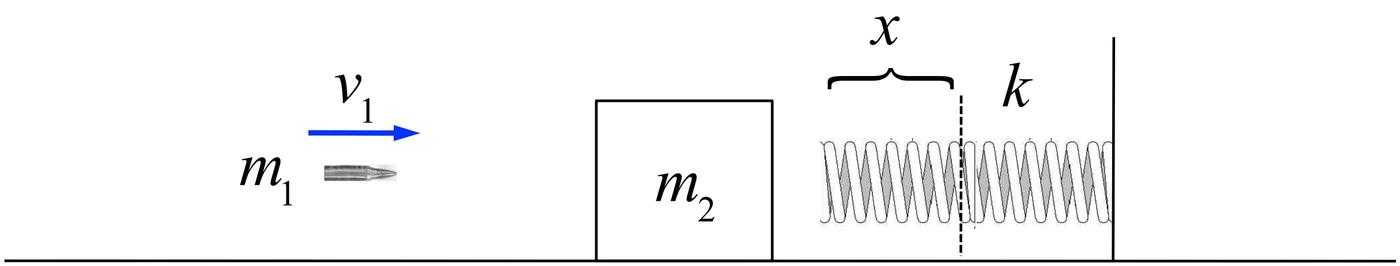
An 11-g bullet is fired horizontally into a 110-g wooden block that is initially at rest on a frictionless horizontal surface and a fixed spring having spring constant 183 N/m. The bullet becomes embedded in the block. The bullet-block system compresses the spring by a maximum amount 85 cm. What was the initial velocity of the bullet?



Strategy:

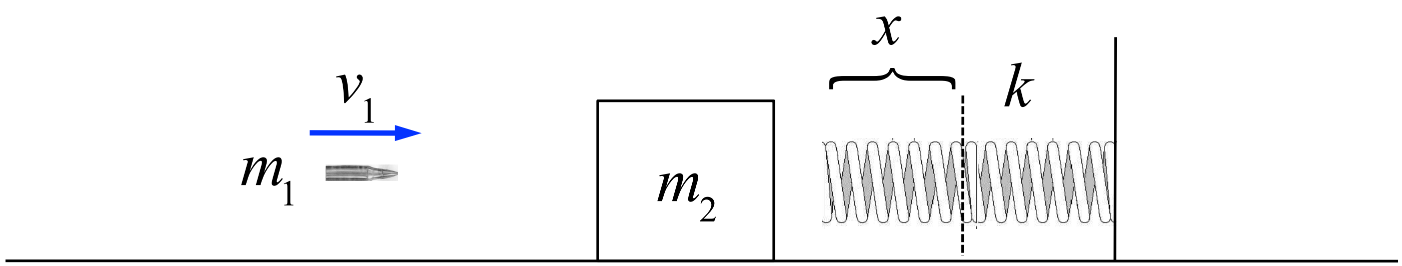
1) Momentum conservation bullet + block

2) Energy conservation in spring compression



1) Momentum conservation bullet + block

$$p_1 = m_1 v_1, \quad p = (m_1 + m_2)v \quad \Rightarrow \quad p_1 = p$$



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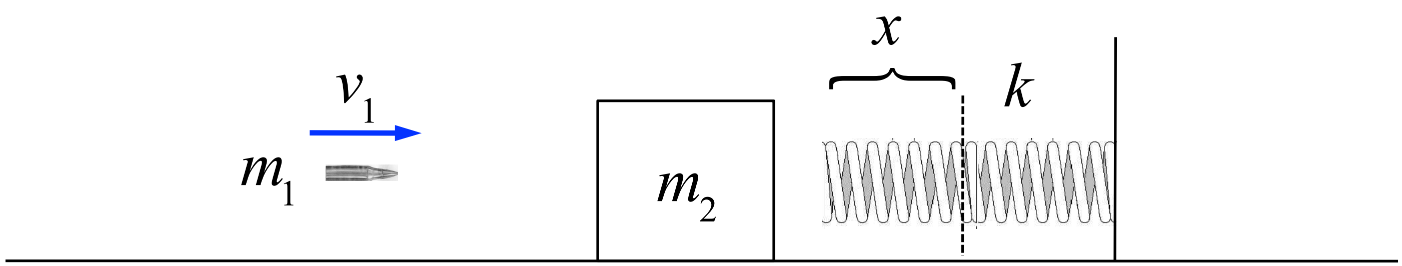
2) Energy conservation in spring compression

Kinetic Energy of block and bullet : $K_i = p_1^2 / 2(m_1 + m_2)$

Potential Energy of compressed spring : $U = \frac{1}{2} k x^2$



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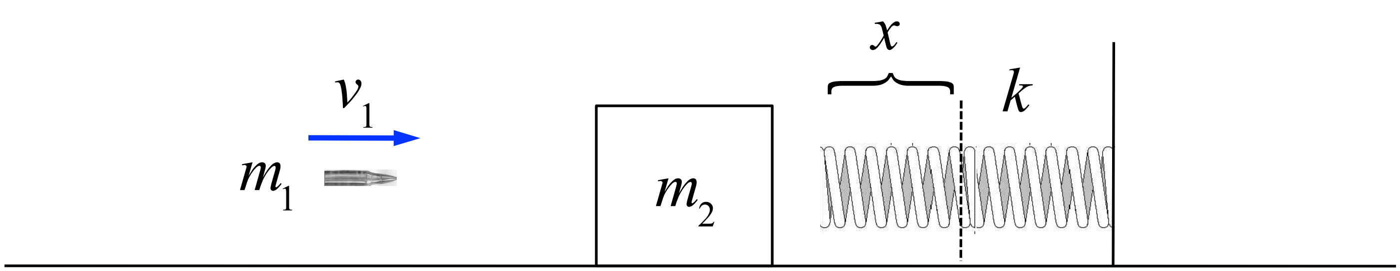
Energy Conservation : $K + U = K_i + U_i \quad ; \quad U_i = 0, K = 0$



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$$\frac{1}{2}kx^2 = p_1^2 / 2(m_1 + m_2)$$

$$p_1 = x\sqrt{k(m_1 + m_2)}$$

$$m_1 v_1 = (0.85\text{ m})\sqrt{(183\text{N/m})(0.121\text{kg})} = 4.00\,\text{N}\cdot\text{s}$$

$$v_1 = 4.00\,\text{N}\cdot\text{s}\big/\left(0.011\text{kg}\right) = 364\text{ m/s}$$