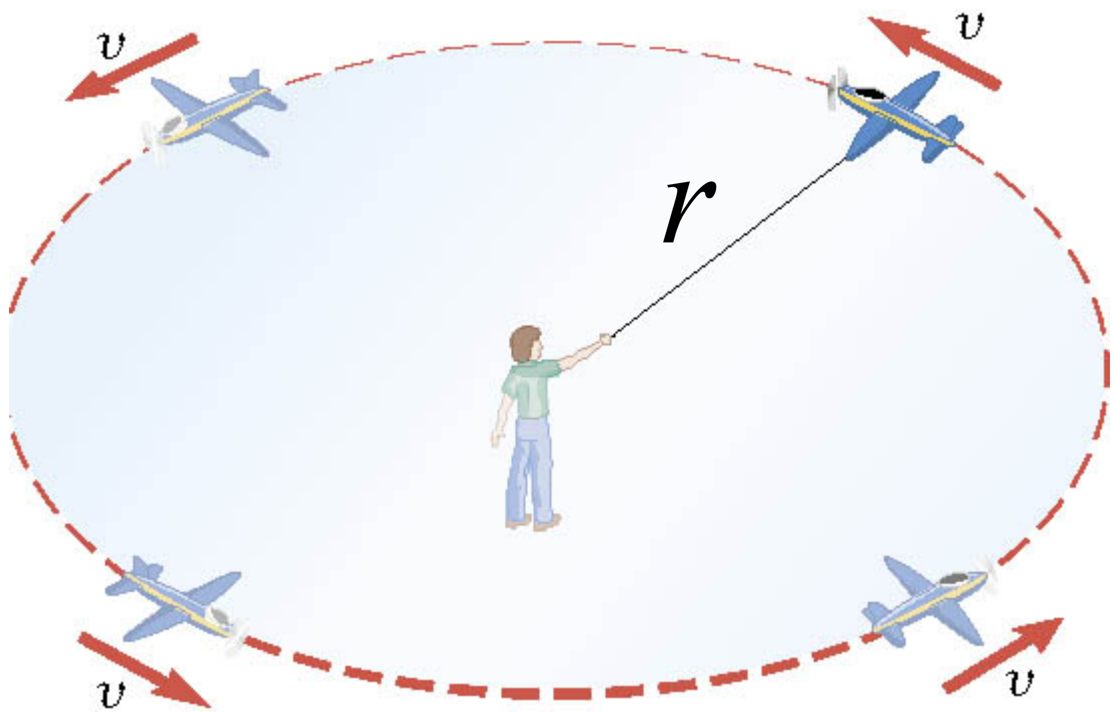
Chapter 3.5

Uniform Circular Motion

DEFINITION OF UNIFORM CIRCULAR MOTION

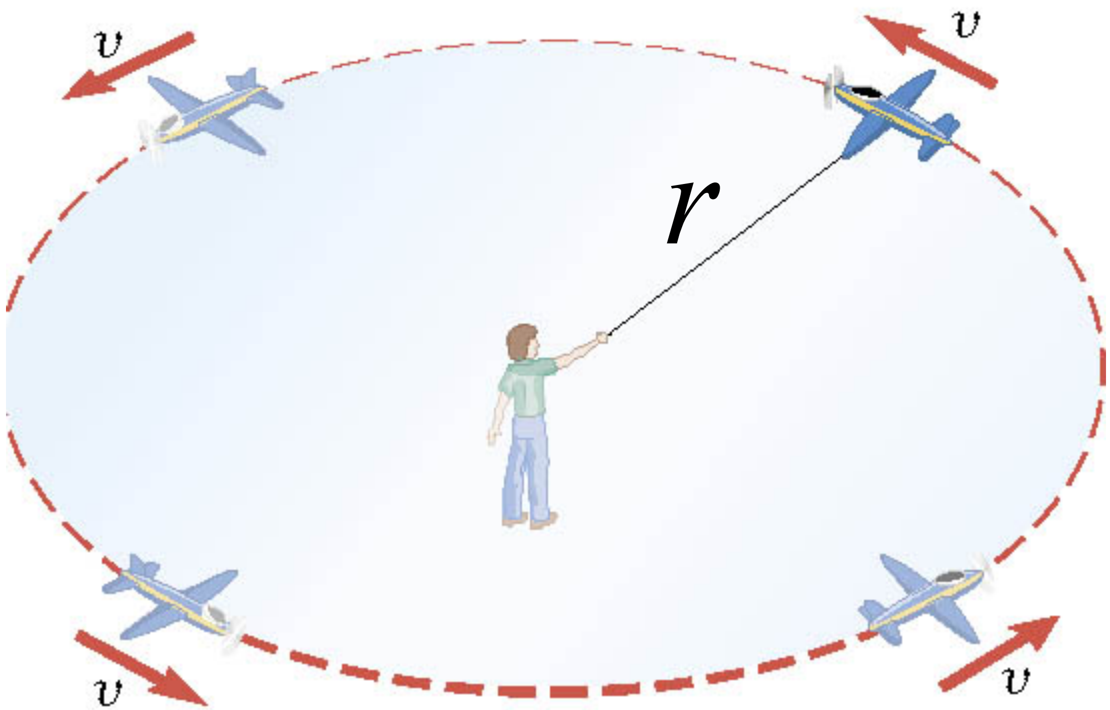
Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.



Circumference of the circle is $2\pi r$.

The time it takes the object to travel once around the circle is *T* (a.k.a. the period)

$$\text{Speed around the circle is, } v = \frac{2\pi r}{T}.$$



Example: A Tire-Balancing Machine

The wheel of a car has a radius of 0.29m and it being rotated at 830 revolutions per minute on a tire-balancing machine. Determine the speed at which the outer edge of the wheel is moving.

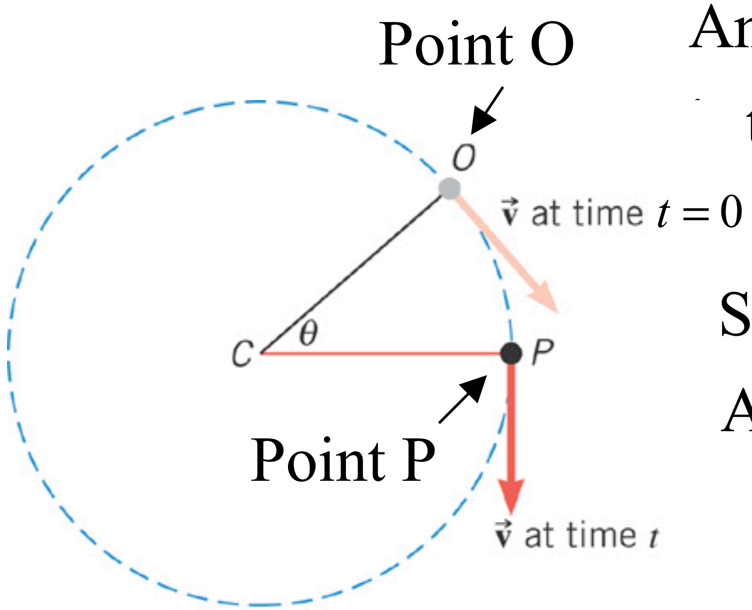
$$\frac{1}{830\,\text{revolutions/min}} = 1.2\times 10^{-3}\ \text{min/revolution}$$

$$T = 1.2\times 10^{-3}\ \text{min} = 0.072\ \text{s}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi\left(0.29\ \text{m}\right)}{0.072\ \text{s}} = 25\ \text{m/s}$$

3.5 *Centripetal Acceleration*

In uniform circular motion, the speed is *constant*, but the direction of the velocity vector is *not constant.*



(*a*)

Angle between point O and point P

the same as between $\vec{\mathbf{v}}_0$ and $\vec{\mathbf{v}}$.

Since velocity vector changes direction

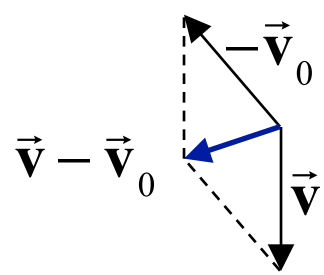
Acceleration vector is NOT ZERO.

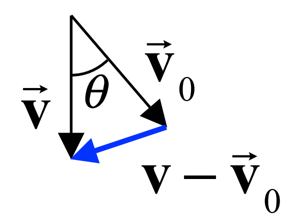
$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_0}{t}$$

Need to understand: $\vec{\mathbf{v}} - \vec{\mathbf{v}}_0$

NOTE: $\vec{\mathbf{v}} - \vec{\mathbf{v}}_0$ and $\vec{\mathbf{a}}$ point

in toward center of circle!





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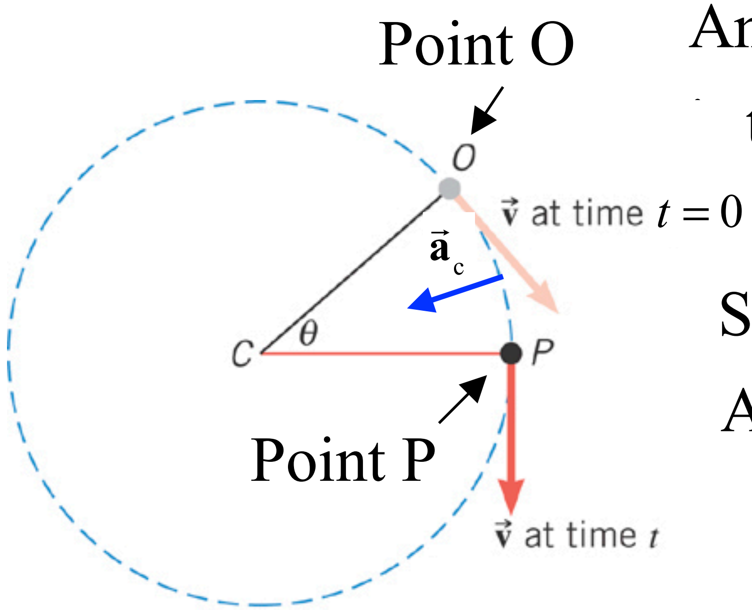

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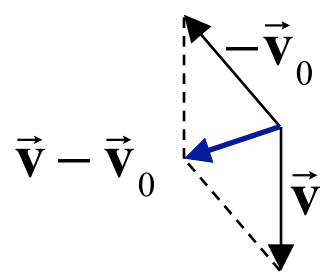
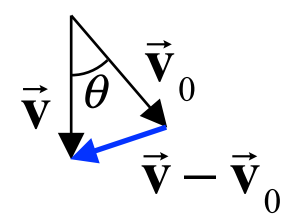
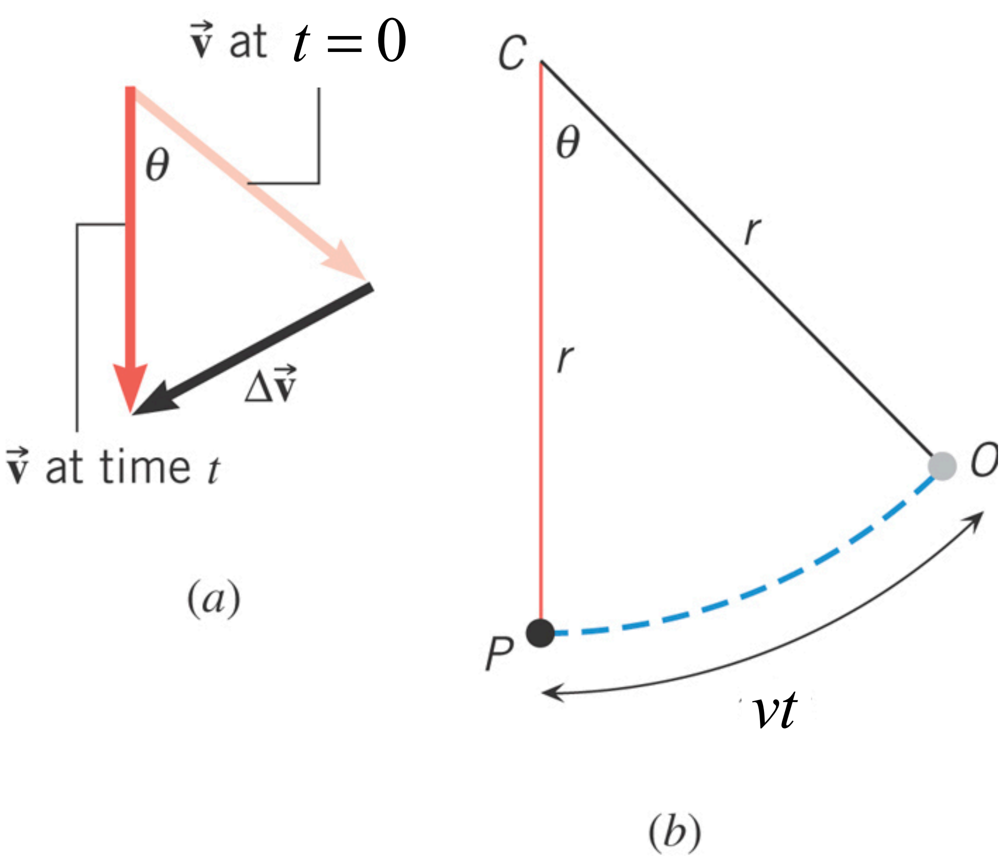




3.5 Centripetal Acceleration

Compare geometry of velocity vectors and the portion of the circle.

$$\theta = \frac{\Delta v}{v} \qquad \theta = \frac{vt}{r}$$



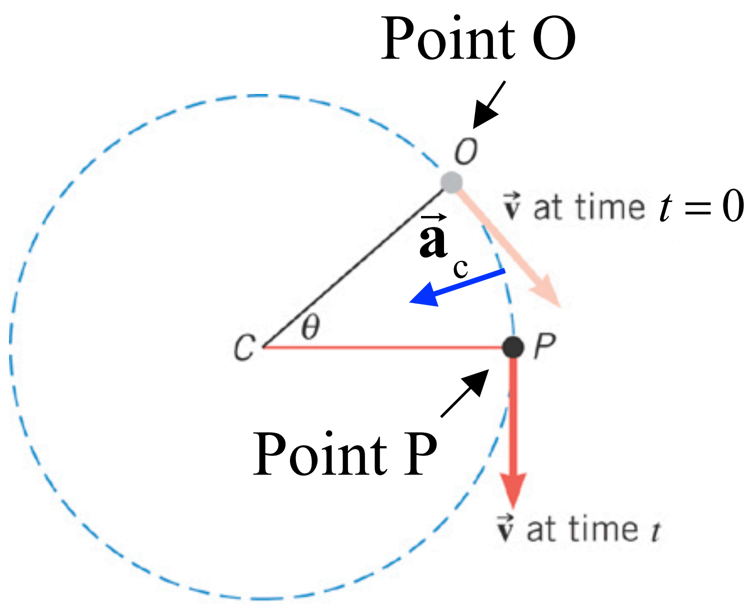
Magnitudes

$$\frac{\Delta v}{v} = \frac{vt}{r}$$

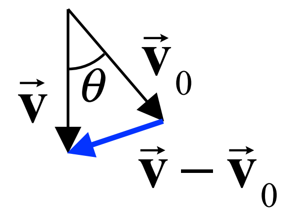
$$a_c = \frac{\Delta v}{t} = \frac{v^2}{r}$$

$$a_c = \frac{v^2}{r}$$

The direction of the centripetal acceleration is towards the center of the circle; in the same direction as the change in velocity.



(a)



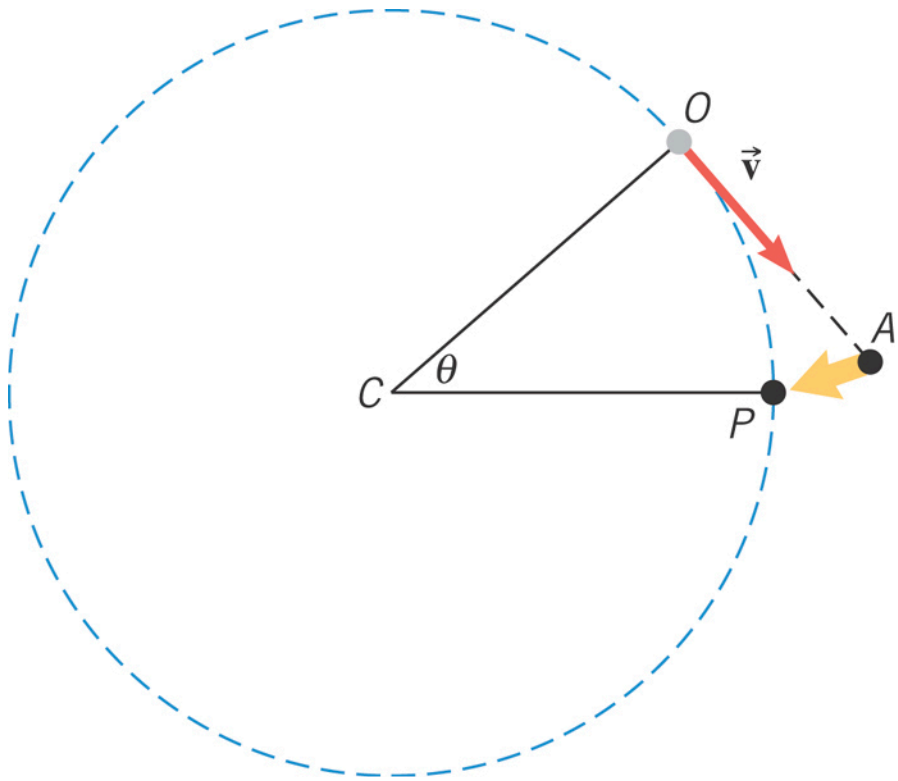
$$a_c = \frac{v^2}{r}$$

Centripetal acceleration
vector points *inward*
at ALL points on the circle

Conceptual Example: Which Way Will the Object Go?

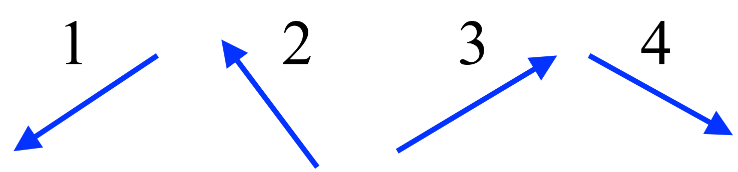
An object (•) is in uniform circular motion. At point *O* it is released from its circular path.

Does the object move along the
(A) Straight path between *O* and *A*
or
(B) Along the circular arc between points *O* and *P* ?



Example: The Effect of Radius on Centripetal Acceleration

The bobsled track contains turns with radii of 33 m and 24 m.
Match the acceleration vector directions below to the points A,B,C,D.

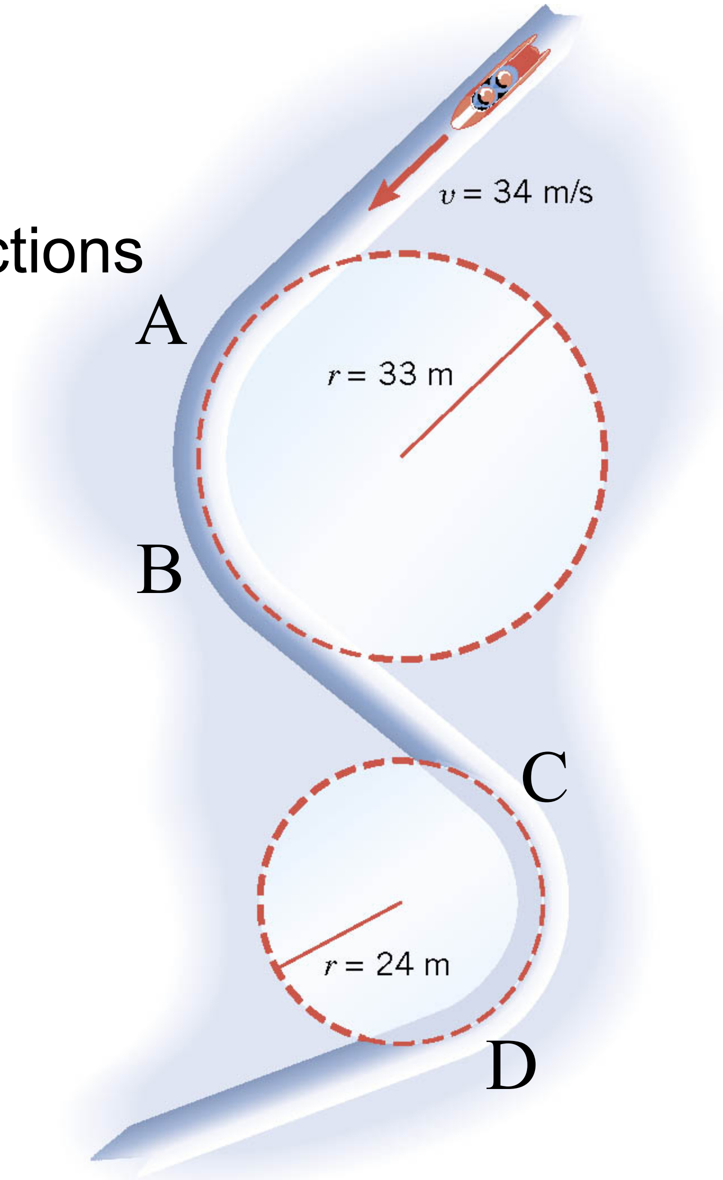


A –

B –

C –

D –



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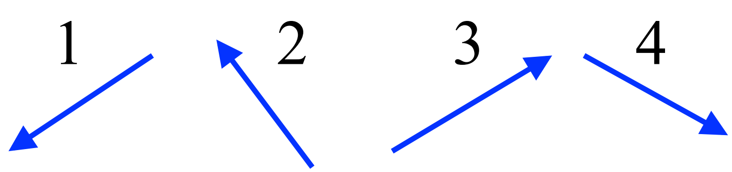
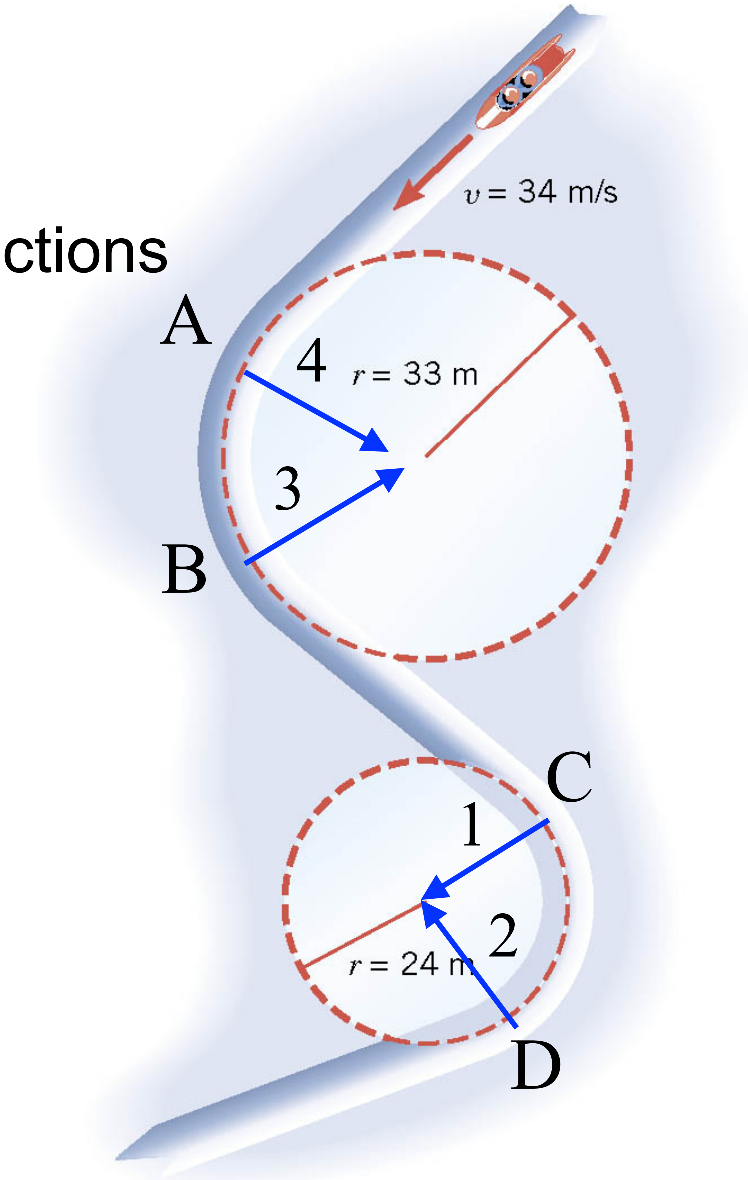




A – 4

B – 3

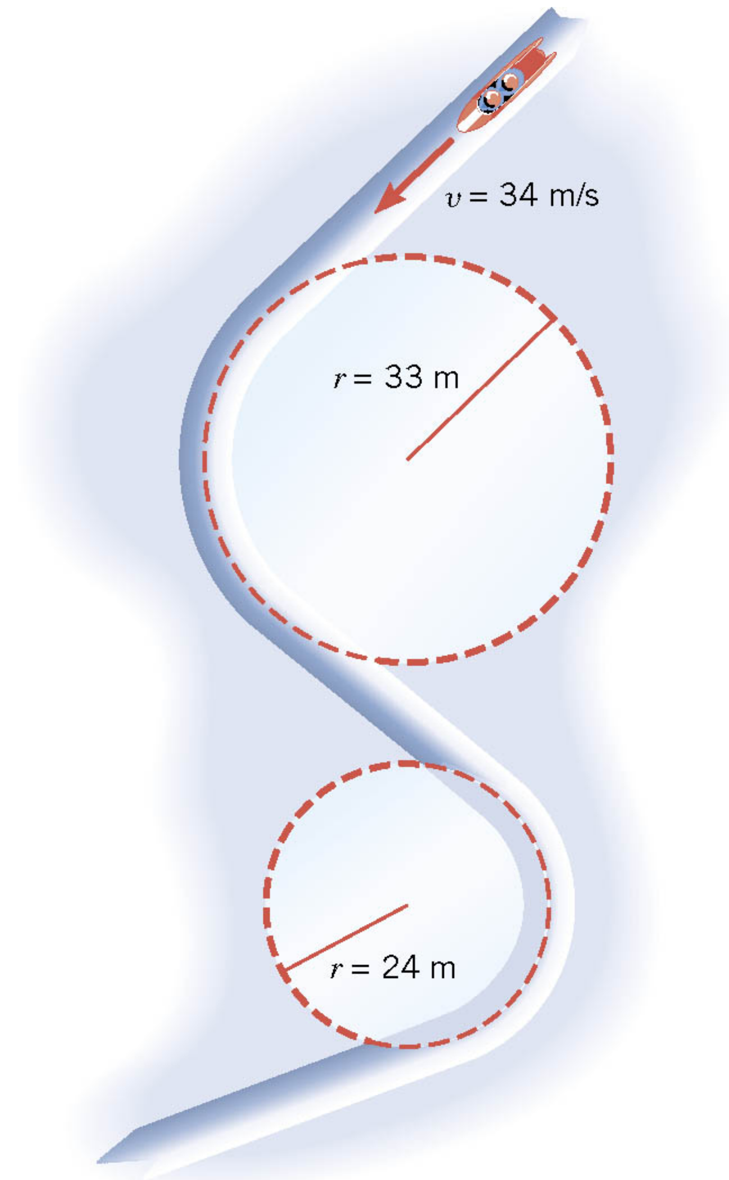
C – 1

D – 2

3.5 Centripetal Acceleration

$$a_C = v^2/r$$

Find the centripetal acceleration at each turn for a speed of 34 m/s. Express answers as multiples of $g = 9.8\,\mathrm{m/s^2}$.



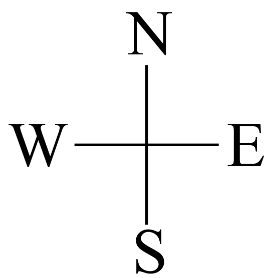
$$a_C = \frac{(34\,\mathrm{m/s})^2}{33\ \mathrm{m}} = 35\,\mathrm{m/s^2} = 3.6g$$

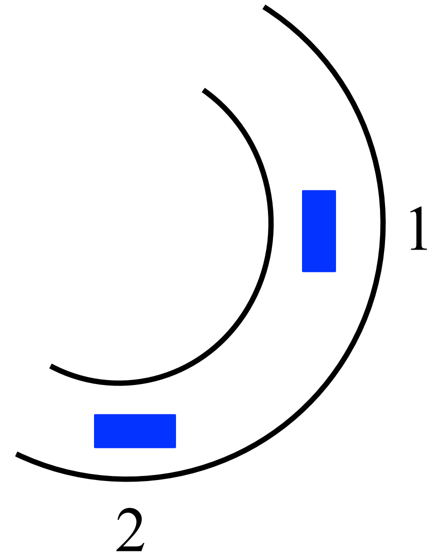
$$a_C = \frac{(34\,\mathrm{m/s})^2}{24\ \mathrm{m}} = 48\,\mathrm{m/s^2} = 4.9g$$

Clicker Question 3.5.1

A car is moving counter-clockwise around a circular section of road at constant speed. What are the directions of its velocity and acceleration at position 1.

	v	**a**
a)	N	S
b)	N	E
c)	N	W
d)	N	N
e)	S	E





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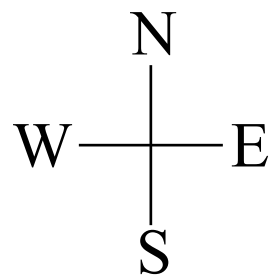
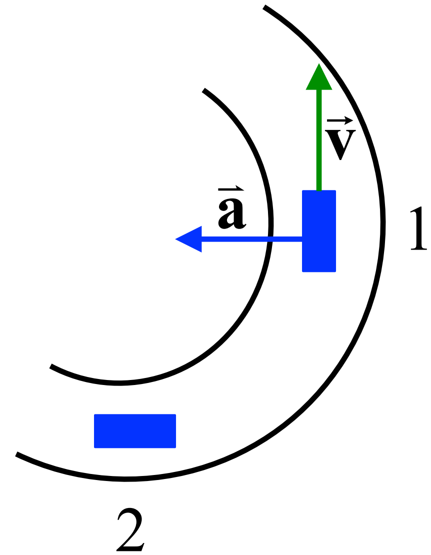


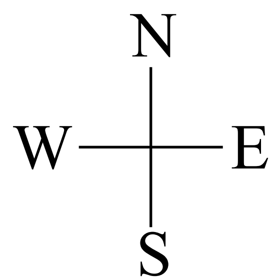


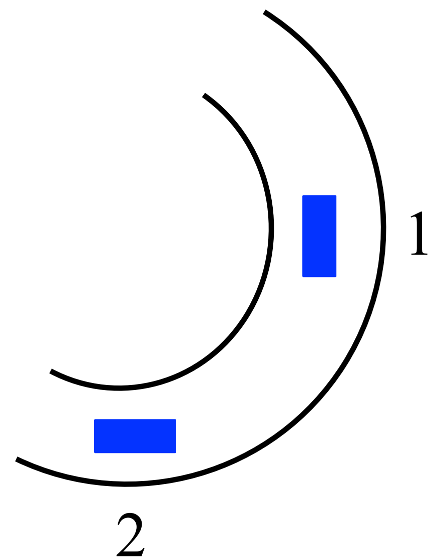
Clicker Question 3.5.2

A car is moving <u>counter-clockwise</u> around a circular section of road at constant speed. What are the directions of its velocity and acceleration

at position 2?

	v	**a**
a)	E	S
b)	E	E
c)	E	N
d)	E	W
e)	W	S





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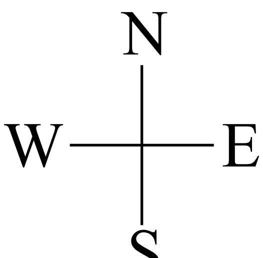
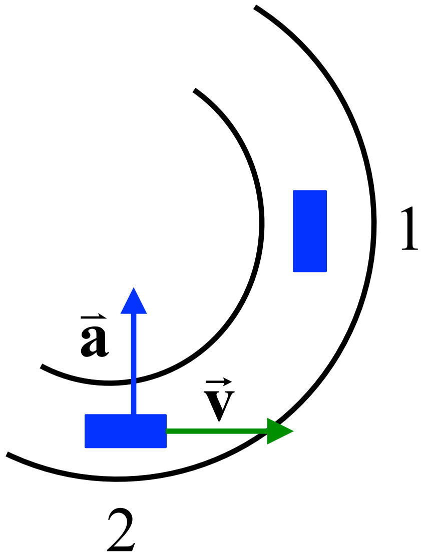
c)	E	N
d)	E	W
e)	W	S





Chapter 8

Accelerated Circular Motion

8.1 Rotational Motion and Angular Displacement

Why are there 360 degrees in a circle?
Why are there 60 minutes in an hour?
Why are there 60 seconds in a minute?

Because the Greeks, who invented these units were enamored with numbers that are divisible by most whole numbers, 12 or below (except 7 and 11).

Strange, because later it was the Greeks who discovered that the ratio of the radius to the circumference of a circle was a number known as 2π.

A new unit, radians, is really useful for angles.

8.1 Rotational Motion and Angular Displacement

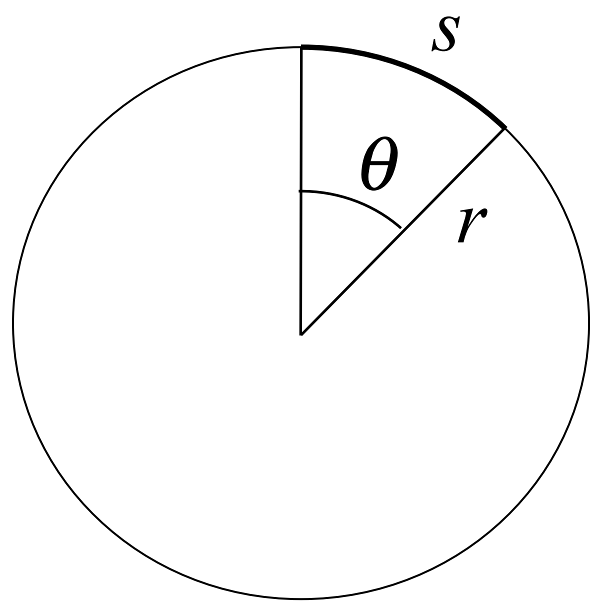
A new unit, radians, is really useful for angles.

Radian measure

$$\theta(\text{radians}) = \frac{s\ (\text{arc length})}{r\ (\text{radius})}$$

$$s = r\theta$$

(s in same units as r)



Full circle

$$\theta = \frac{s}{r} = \frac{2\pi \cancel{r}}{\cancel{r}}$$

$$= 2\pi\ (\text{radians})$$

Conversion of degree to radian measure

$$\theta(\text{rad}) = \theta(\text{deg.})\left(\frac{2\pi}{360}\frac{\text{rad}}{\text{deg.}}\right)$$

$$\left(\frac{2\pi}{360}\frac{\text{rad}}{\text{deg.}}\right) = 1$$