Chapter 8

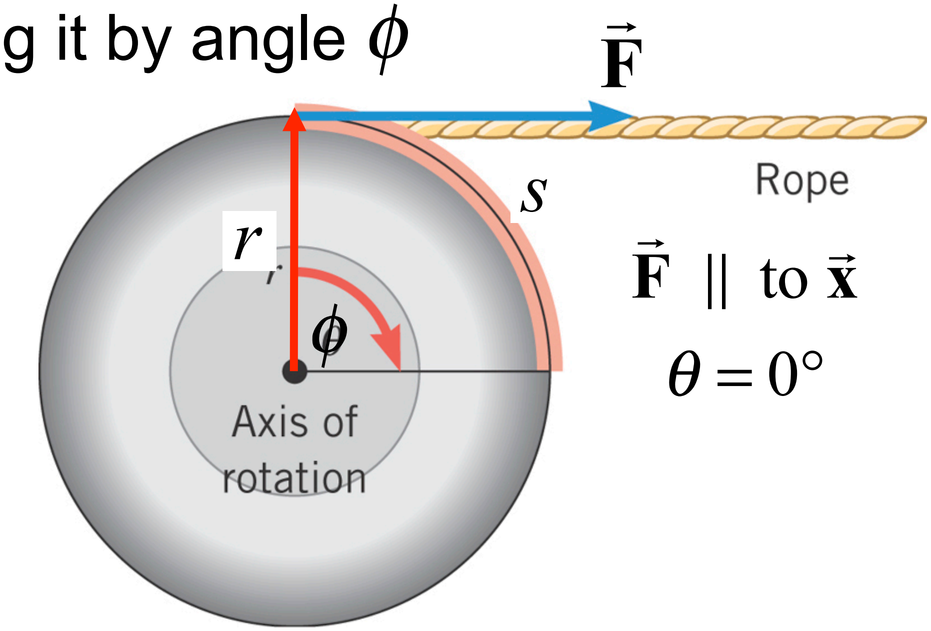
continued

Rotational Dynamics

8.4 Rotational Work and Energy

Work to accelerate a mass rotating it by angle ϕ

$$W = F(\cos\theta)x \qquad x = r\phi$$
$$= Fr\phi \qquad Fr = \tau \ \text{(torque)}$$
$$= \tau\phi$$



$\vec{\mathbf{F}}$ || to $\vec{\mathbf{x}}$

$\theta = 0°$

DEFINITION OF ROTATIONAL WORK

The rotational work done by a constant torque in turning an object through an angle is

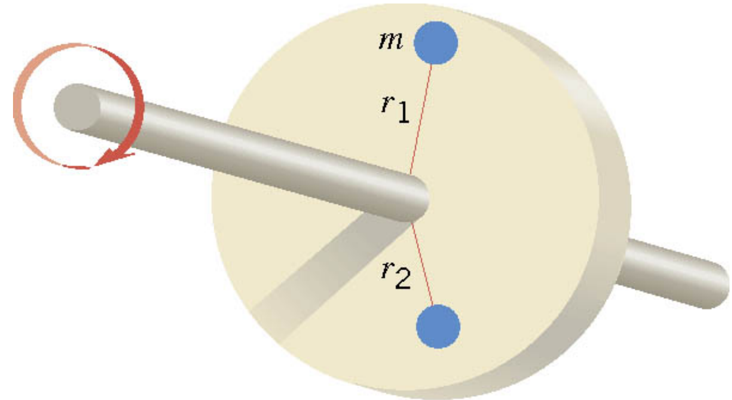
$$\boxed{W_R = \tau\phi}$$

Requirement: The angle must be expressed in radians.

SI Unit of Rotational Work: joule (J)

8.4 Rotational Work and Energy

Kinetic Energy of a rotating one point mass



$$K = \tfrac{1}{2} m v_T^2 \qquad v_T = r\omega$$

$$= \tfrac{1}{2} m r^2 \omega^2$$

Kinetic Energy of many rotating point masses

$$K = \sum\left(\tfrac{1}{2} m r^2 \omega^2\right) = \tfrac{1}{2}\left(\sum m r^2\right)\omega^2 = \tfrac{1}{2} I \omega^2$$

DEFINITION OF ROTATIONAL KINETIC ENERGY

The rotational kinetic energy of a rigid rotating object is

$$\boxed{K_{Rot} = \tfrac{1}{2} I \omega^2} \qquad \boxed{I = \sum_{i=1}^{n} (mr^2)_i}$$

Requirement: The angular speed must be expressed in rad/s.

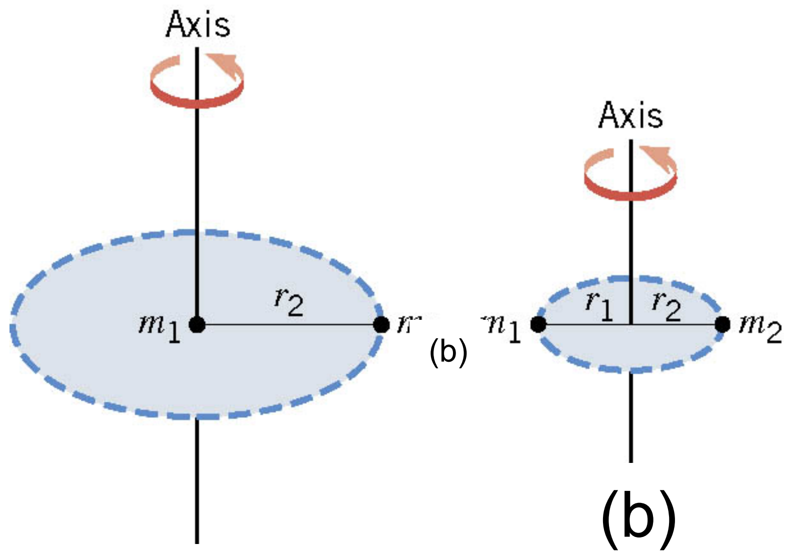
SI Unit of Rotational Kinetic Energy: joule (J)

Moment of Inertia depends on axis of rotation.

Two particles each with mass, m, and are fixed at the ends of a thin rigid rod. The length of the rod is L. Find the moment of inertia when this object rotates relative to an axis that is perpendicular to the rod at (a) one end and (b) the center.

$$m_1 = m_2 = m$$

$$r_1 = 0, \quad r_2 = L$$



$$r_1 = \frac{L}{2}, \quad r_2 = \frac{L}{2}$$

(b)

(a)

(a) $$I = \sum_i (mr^2)_i = m_1 r_1^2 + m_2 r_2^2 = m(0)^2 + m(L)^2 = mL^2$$

(b) $$I = \sum_i (mr^2)_i = m_1 r_1^2 + m_2 r_2^2 = m(L/2)^2 + m(L/2)^2 = \tfrac{1}{2} mL^2$$

8.4 Rotational Work and Energy

Rotational

Kinetic Energy

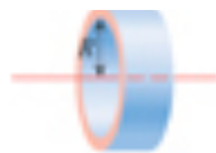
$$\boxed{K_{Rot} = \tfrac{1}{2} I\omega^2}$$

Moments of Inertia I

Rigid Objects Mass M

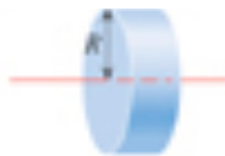
Thin walled hollow cylinder

$$I = MR^2$$



Solid cylinder or disk

$$I = \tfrac{1}{2} MR^2$$



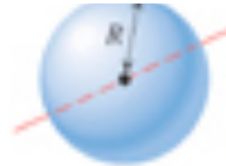
Thin rod length ℓ through center

$$I = \tfrac{1}{12} M\ell^2$$

Thin rod length ℓ through end

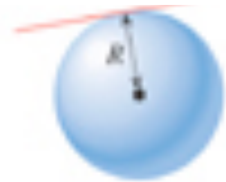
$$I = \tfrac{1}{3} M\ell^2$$

Solid sphere through center



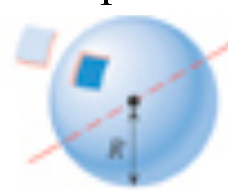
$$I = \tfrac{2}{5} MR^2$$

Solid sphere through surface tangent



$$I = \tfrac{7}{5} MR^2$$

Thin walled sphere through center



$$I = \tfrac{2}{3} MR^2$$

Thin plate width ℓ, through center

$$I = \tfrac{1}{12} M\ell^2$$

Thin plate width ℓ through edge

$$I = \tfrac{1}{3} M\ell^2$$

Example: A flywheel has a mass of 13.0 kg and a radius of 0.300m. What angular velocity (in rev/min) gives it an energy of 1.20 x 10^9 J ?

$$I_{disk} = \tfrac{1}{2} MR^2$$

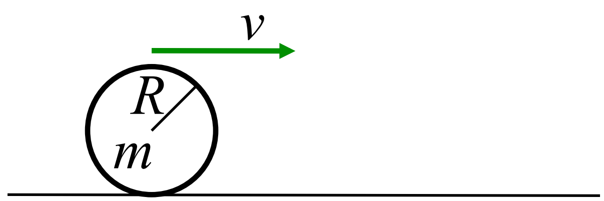
$$K = \tfrac{1}{2} I\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2} MR^2\right)\omega^2$$

$$\omega^2 = \frac{4K}{MR^2} = \frac{4.80\times10^9\,\mathrm{J}}{(13.0\mathrm{kg})(0.300\mathrm{m})^2}$$

$$\begin{aligned}\omega &= \sqrt{4.10\times10^9} = 6.40\times10^4\,\mathrm{rad/s} \\ &= 6.40\times10^4\,\mathrm{rad/s}\left(60\,\mathrm{s/min}\right)\left(1\,\mathrm{rev}/(2\pi\,\mathrm{rad})\right) \\ &= 6.12\times10^5\ \text{rpm} \ \ \text{(revolutions per minute)}\end{aligned}$$

Clicker Question 8.6 Rotational Energy

A bowling ball is rolling without slipping at constant speed toward the pins on a lane. What percentage of the ball's total kinetic energy is translational kinetic energy?



a) 50%

b) 71%

c) 46%

d) 29%

e) 33%

Using $v = \omega R$ and $I_{ball} = \frac{2}{5} mR^2$

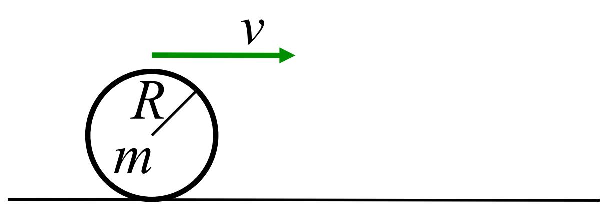
$$K_{lin} = \tfrac{1}{2} mv^2$$

$$K_{rot} = \tfrac{1}{2} I\omega^2$$

Find the value of $\dfrac{K_{lin}}{K_{lin} + K_{rot}}$.

Clicker Question 8.6 Rotational Energy

A bowling ball is rolling without slipping at constant speed toward the pins on a lane. What percentage of the ball's total kinetic energy is translational kinetic energy?



a) 50%

b) 71%

c) 46%

d) 29%

e) 33%

Using $v = \omega R$ and $I_{ball} = \frac{2}{5} mR^2$

$$K_{lin} = \tfrac{1}{2} mv^2 = \tfrac{1}{2} m\omega^2 R^2$$

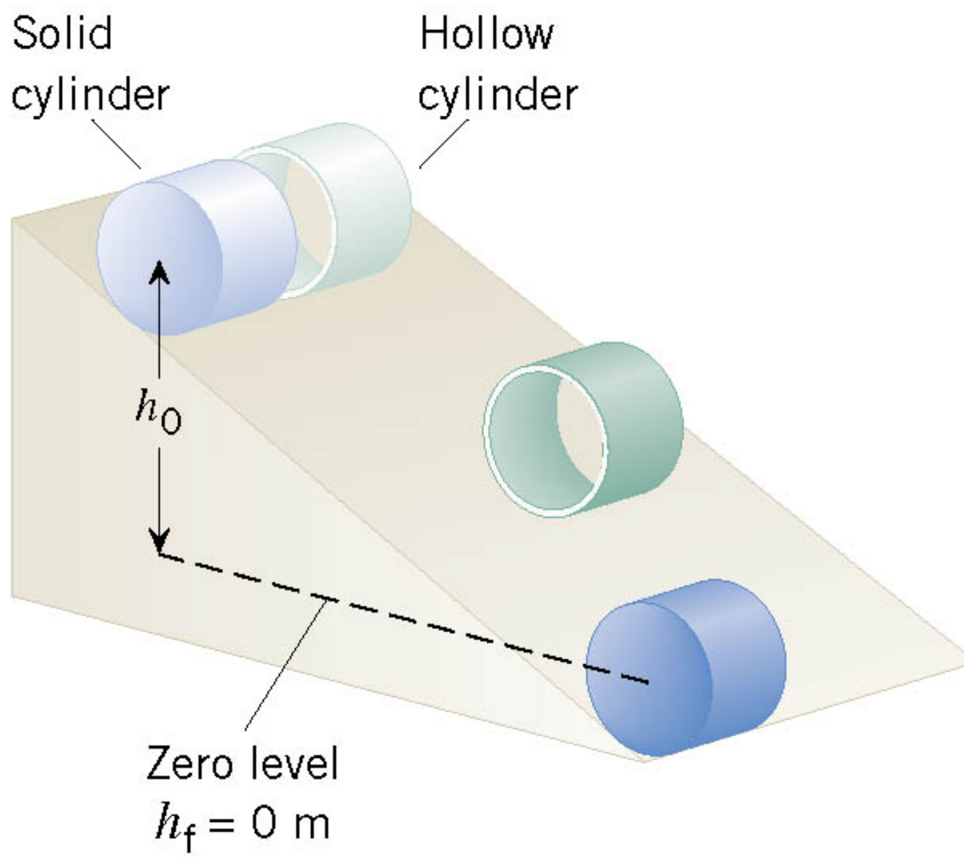
$$K_{rot} = \tfrac{1}{2} I\omega^2 = \tfrac{1}{5} m\omega^2 R^2$$

$$\frac{K_{lin}}{K_{lin} + K_{rot}} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{5}} = \frac{\frac{1}{2}}{\frac{2}{10} + \frac{5}{10}} = \frac{5}{7} = 71\%$$

Example: Rolling Cylinders

A thin-walled hollow cylinder (mass = m_h, radius = r_h) and a solid cylinder (mass = m_s, radius = r_s) start from rest at the top of an incline.

Determine which cylinder has the greatest translational speed upon reaching the bottom.



8.5 ***Rolling Bodies***

Total Energy = (Translational Kinetic + Rotational Kinetic + Potential) Energy

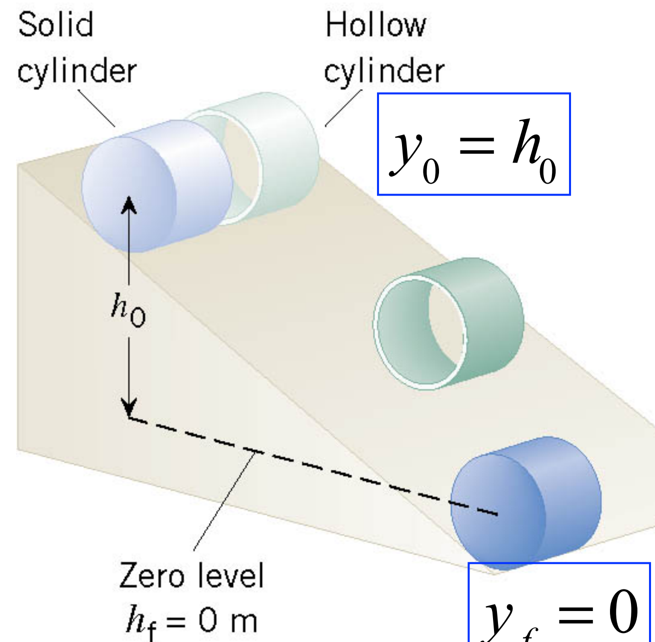
$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgy$$

$$\boxed{v_0 = \omega_0 = 0}$$

ENERGY CONSERVATION $E_f = E_0$

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgy_f = \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgy_0$$

$$\boxed{y_0 = h_0}$$

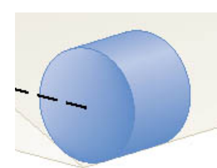


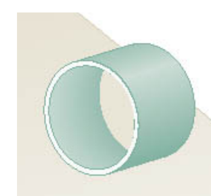
$$\boxed{y_f = 0}$$

$$\boxed{\omega = v/R} \qquad \tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\,v_f^2/R^2 = mgh_0$$

$$v_f^2(m + I/R^2) = 2mgh_0$$

$$\boxed{v_f = \sqrt{\frac{2mgh_0}{m + I/R^2}} = \sqrt{\frac{2gh_0}{1 + I/mR^2}}}$$





$$\boxed{I = \tfrac{1}{2}mR^2} \qquad \boxed{I = mR^2}$$

The cylinder with the smaller moment of inertia will have a greater final translational speed.

8.6 The Action of Forces and Torques on Rigid Objects

Chapter 8 developed the concepts of angular motion.

θ : angles and radian measure for angular variables

ω : angular velocity of rotation (same for entire object)

α : angular acceleration (same for entire object)

$v_T = \omega r$: tangential velocity

$a_T = \alpha r$: tangential acceleration

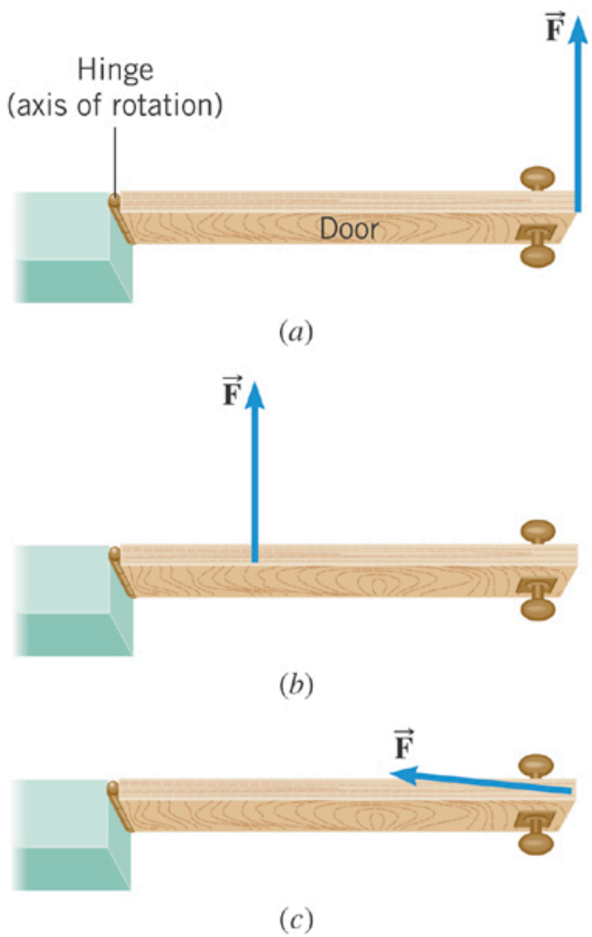
According to Newton's second law, a net force causes an object to have a ***linear acceleration***.

What causes an object to have an ***angular acceleration***?

TORQUE

8.6 The Action of Forces and Torques on Rigid Objects

The amount of torque depends on where and in what direction the force is applied, as well as the location of the axis of rotation.



Maximum rotational effect of the force F.

Smaller rotational effect of the force F.

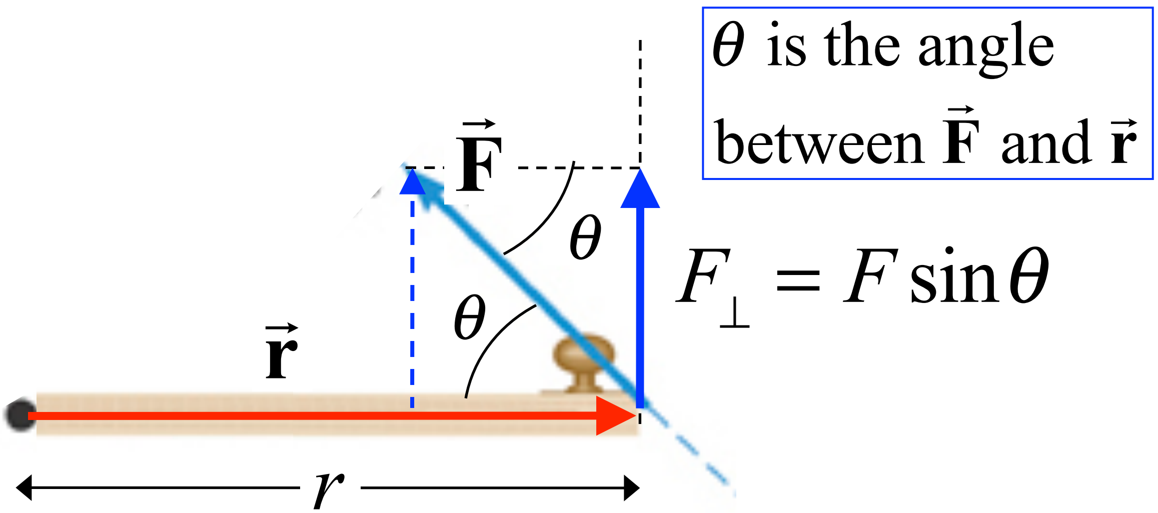
Rotational effect of the force F is minimal; it compresses more than rotates the bar

DEFINITION OF TORQUE

$$\text{Magnitude of Torque} = r \times \left(\text{Component of Force } \perp \text{ to } \vec{\mathbf{r}}\right)$$

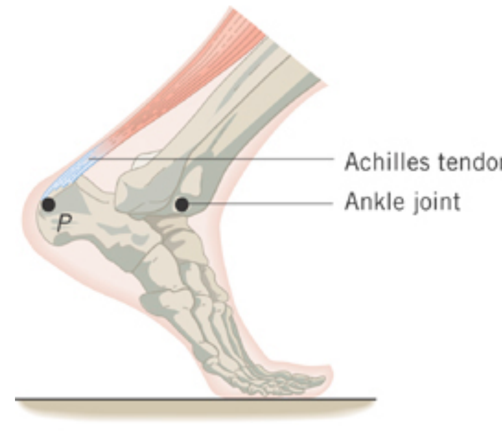
$$\tau = rF_{\perp} = rF\sin\theta$$

Direction: The torque is positive when the force tends to produce a counterclockwise rotation about the axis.
SI Unit of Torque: newton x meter (N·m)



Example: The Achilles Tendon

The tendon exerts a force of magnitude 720 N. Determine the torque (magnitude and direction) of this force about the ankle joint. Assume the angle is 35°.

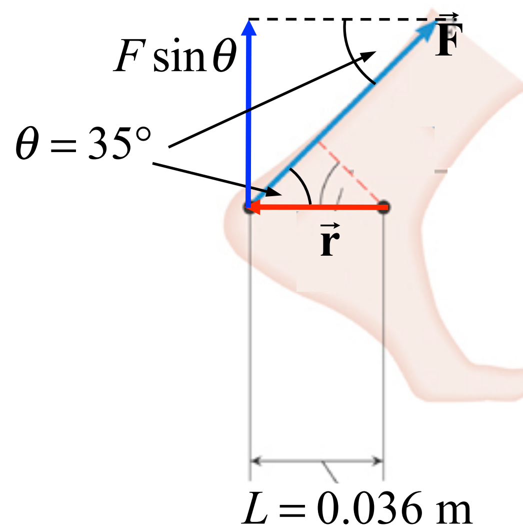


(a)

$$\tau = r\left(F\sin\theta\right) = (.036\ \text{m})(720\ \text{N})(\sin 35°)$$

$$= 15.0\ \text{N}\cdot\text{m}$$

θ is the angle between $\vec{\mathbf{F}}$ and $\vec{\mathbf{r}}$



(b)

Direction is clockwise (–) around ankle joint

Torque vector $\tau = -15.0\ \text{N}\cdot\text{m}$

8.7 Rigid Objects in Equilibrium

If a rigid body is in equilibrium, neither its linear motion nor its rotational motion changes.

$$a_x = a_y = 0 \qquad \alpha = 0$$

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau = 0$$

All equilibrium problems use these equations – no net force and no net torque.

EQUILIBRIUM OF A RIGID BODY

A rigid body is in equilibrium if it has zero translational acceleration and zero angular acceleration. In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \tau = 0$$

Note: constant linear speed or constant rotational speed are allowed for an object in equilibrium.

Reasoning Strategy

1. Select the object to which the equations for equilibrium are to be applied.

2. Draw a free-body diagram that shows all of the external forces acting on the object.

3. Choose a convenient set of *x*, *y* axes and resolve all forces into components that lie along these axes.

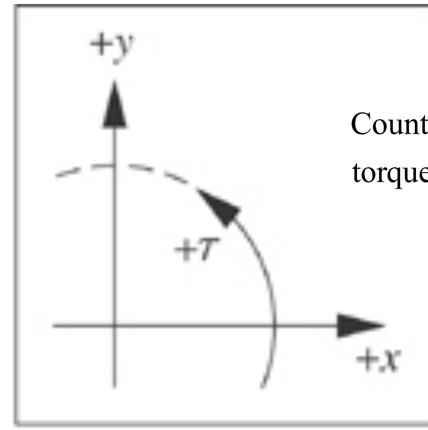
4. Apply the equations that specify the balance of forces at equilibrium. (Set the net force in the *x* and *y* directions equal to zero.)

5. Select a convenient axis of rotation. Set the sum of the torques about this axis equal to zero.

6. Solve the equations for the desired unknown quantities.

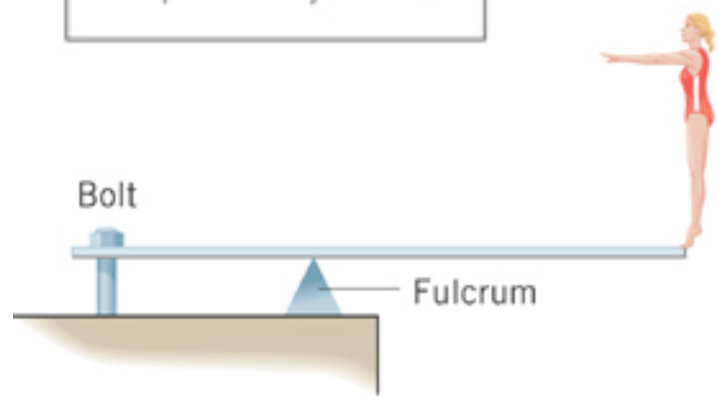
8.7 Rigid Objects in Equilibrium

Example A Diving Board



Counter-clockwise torque is positive

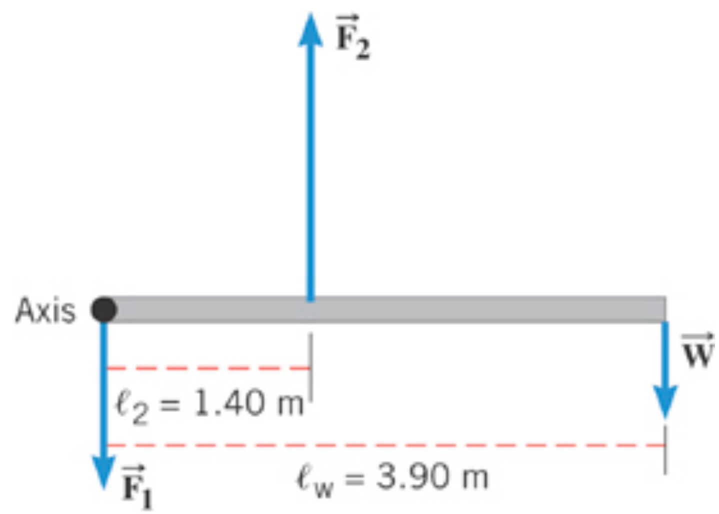
A woman whose weight is 530 N is poised at the right end of a diving board with length 3.90 m. The board has negligible weight and is supported by a fulcrum 1.40 m away from the left end. Find the forces that the bolt and the fulcrum exert on the board.



F_1 acts on rotation axis - produces no torque.

$$\boxed{\begin{aligned}&\sum \tau = 0 = \ell_2 F_2 - \ell_W W\\ &F_2 = (\ell_W/\ell_2)W = (3.9/1.4)530\text{N} = 1480\text{ N}\end{aligned}}$$

$$\boxed{\begin{aligned}&\sum F_y = 0 = -F_1 + F_2 - W\\ &F_1 = F_2 - W = (1480 - 530)\text{N} = 950\text{ N}\end{aligned}}$$



8.7 Rigid Objects in Equilibrium

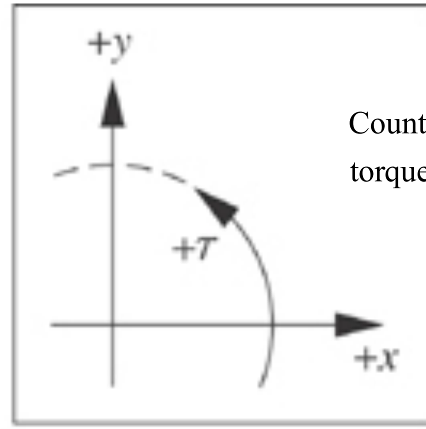
Choice of pivot is arbitary (most convenient)

Pivot at fulcum: F_2 produces no torque.

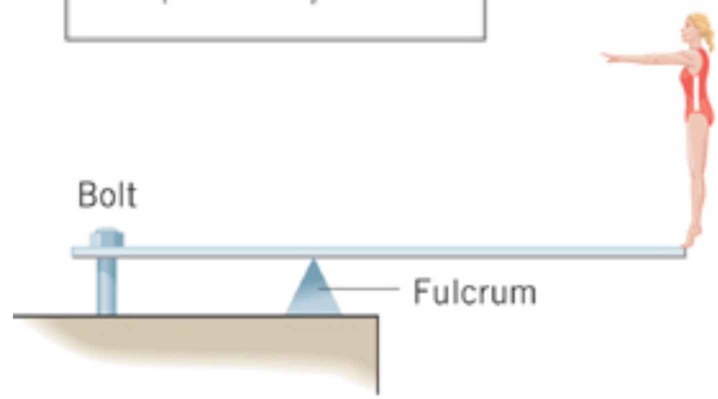
$$\sum \tau = 0 = F_1 \ell_2 - W(\ell_W - \ell_2)$$
$$F_1 = W(\ell_W / \ell_2 - 1) = (530\,\mathrm{N})(1.8) = 950\,\mathrm{N}$$

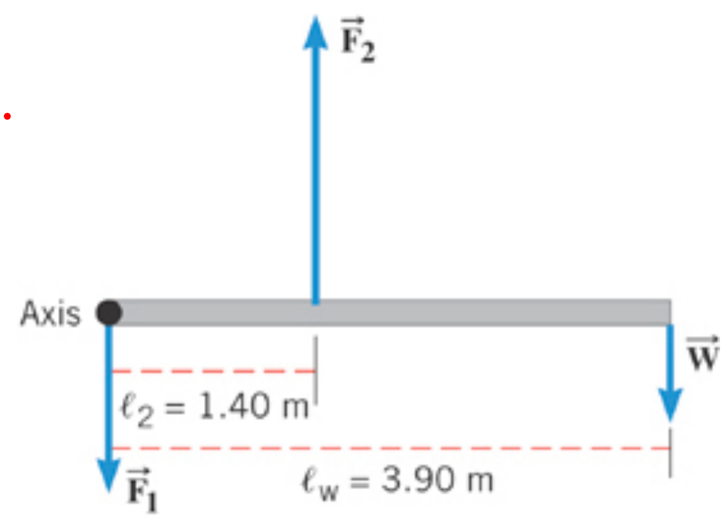
$$\sum F_y = 0 = -F_1 + F_2 - W$$
$$F_2 = F_1 + W = (950 + 530)\,\mathrm{N} = 1480\ \mathrm{N}$$

Yields the same answers as with pivot at Bolt.



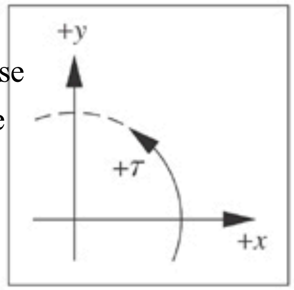
Counter-clockwise torque is positive



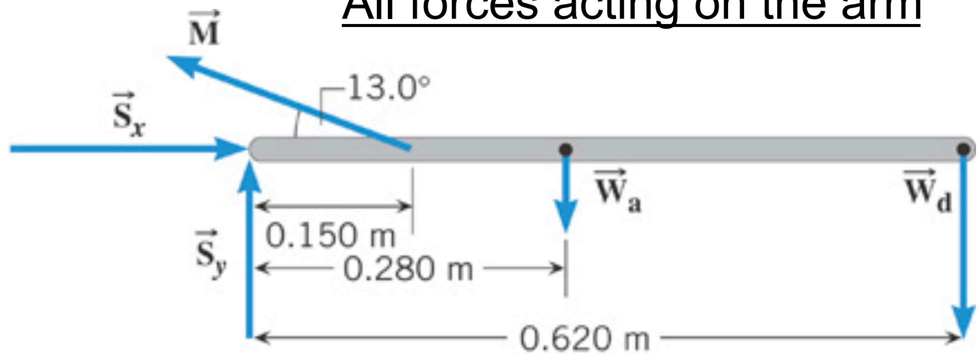


8.7 Rigid Objects in Equilibrium

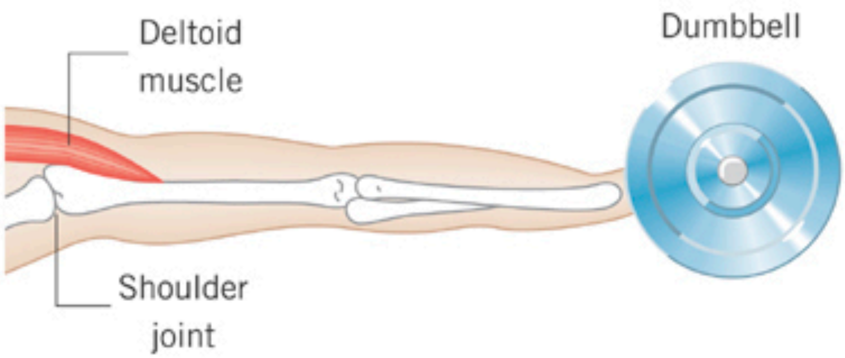
Counter-clockwise torque is positive



All forces acting on the arm

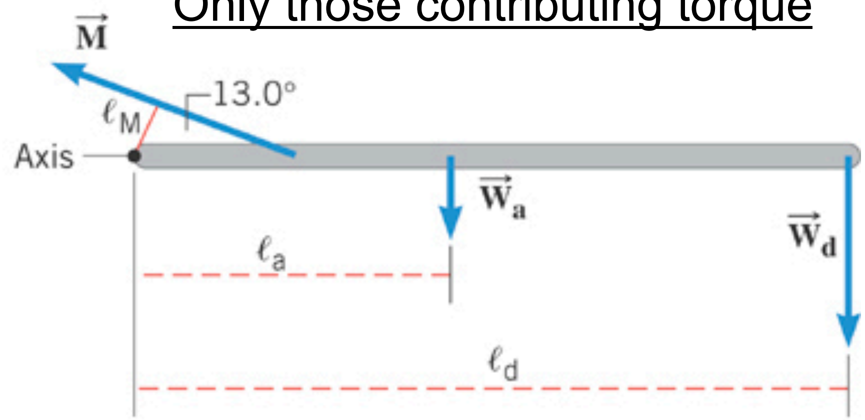


(*b*) Free-body diagram of the arm



(*a*)

Only those contributing torque

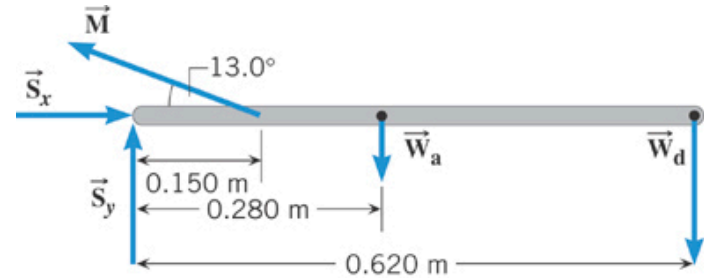


(*c*)

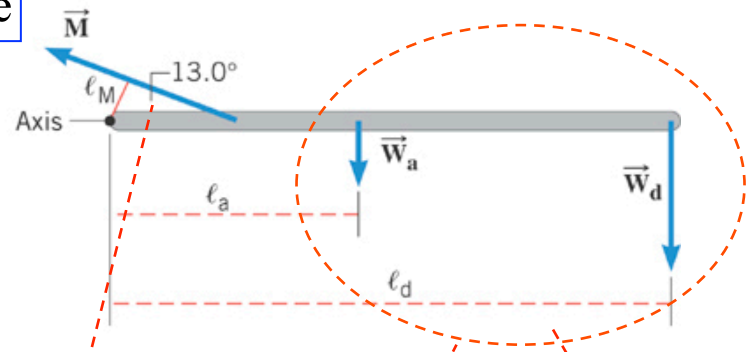
Example 5 Bodybuilding

The arm is horizontal and weighs 31.0 N. The deltoid muscle can supply 1840 N of force. What is the weight of the heaviest dumbell he can hold?

8.7 Rigid Objects in Equilibrium



positive torque



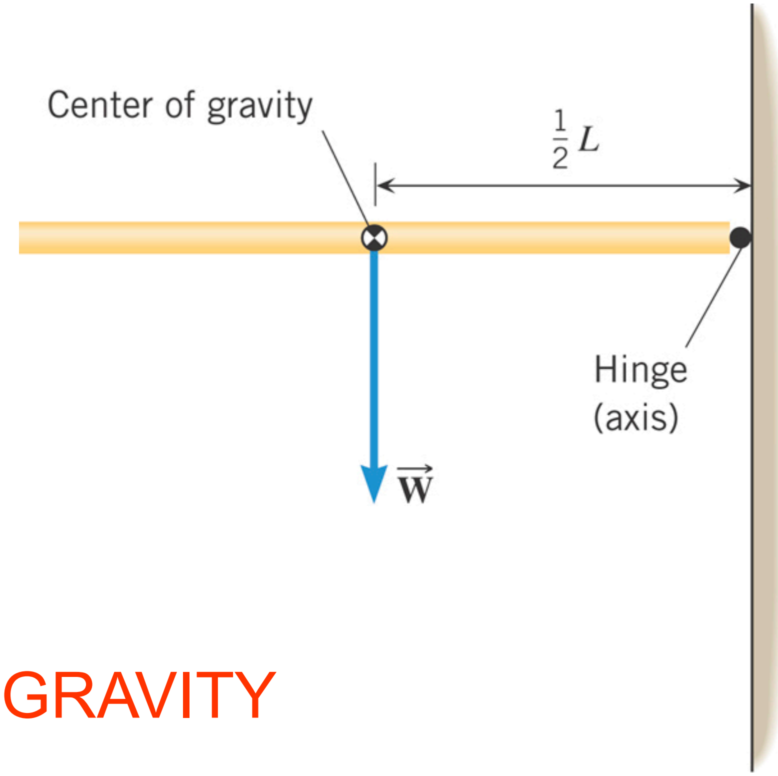
negative torques

$$\sum \tau = M\left(\sin 13°\right)\ell_M - W_a\ell_a - W_d\ell_d = 0$$

$$W_d = \left[+M\left(\sin 13°\right)\ell_M - W_a\ell_a\right]/\ell_d$$

$$= \left[1840\text{N}(.225)(0.15\text{m}) - 31\text{N}(0.28\text{m})\right]/0.62\text{m}$$

$$= 86.1\text{N}$$

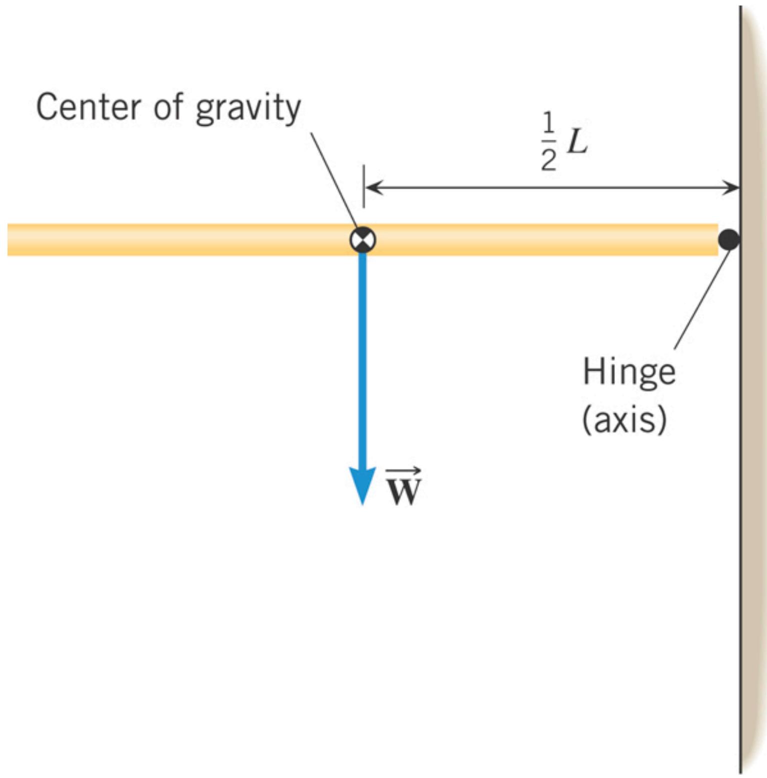


DEFINITION OF CENTER OF GRAVITY

The center of gravity of a rigid body is the point at which its weight can be considered to act when the torque due to the weight is being calculated.

8.7 Center of Gravity

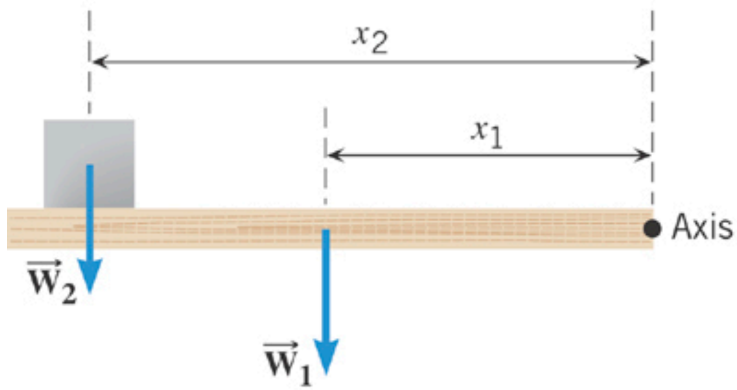
When an object has a symmetrical shape and its weight is distributed uniformly, the center of gravity lies at its geometrical center.



8.7 Center of Gravity

General Form of x_{cg}

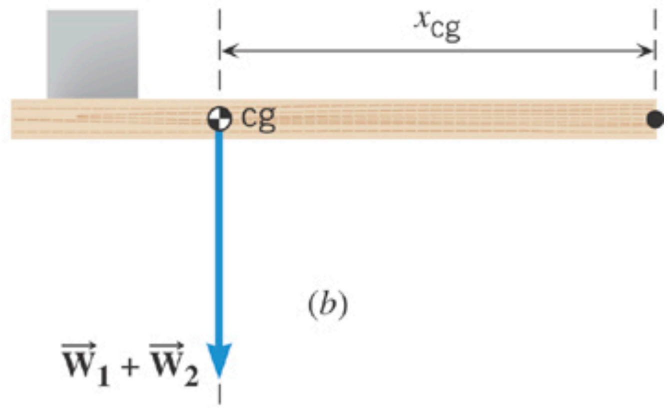
$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + \cdots}{W_1 + W_2 \ + \cdots}$$



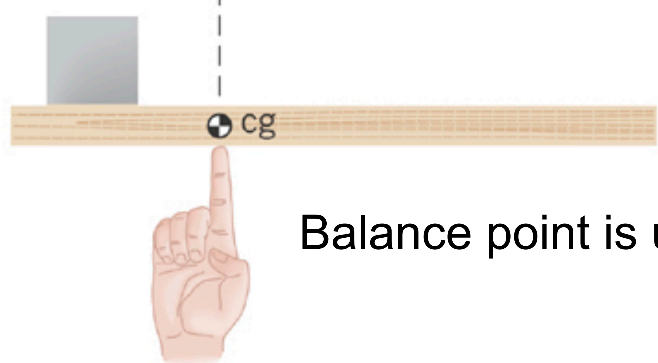
(a)

Center of Gravity, x_{cg} , for 2 masses

$$x_{cg} = \frac{W_1 x_1 + W_2 x_2}{W_1 + W_2}$$



(b)



Balance point is under x_{cg}

(c)