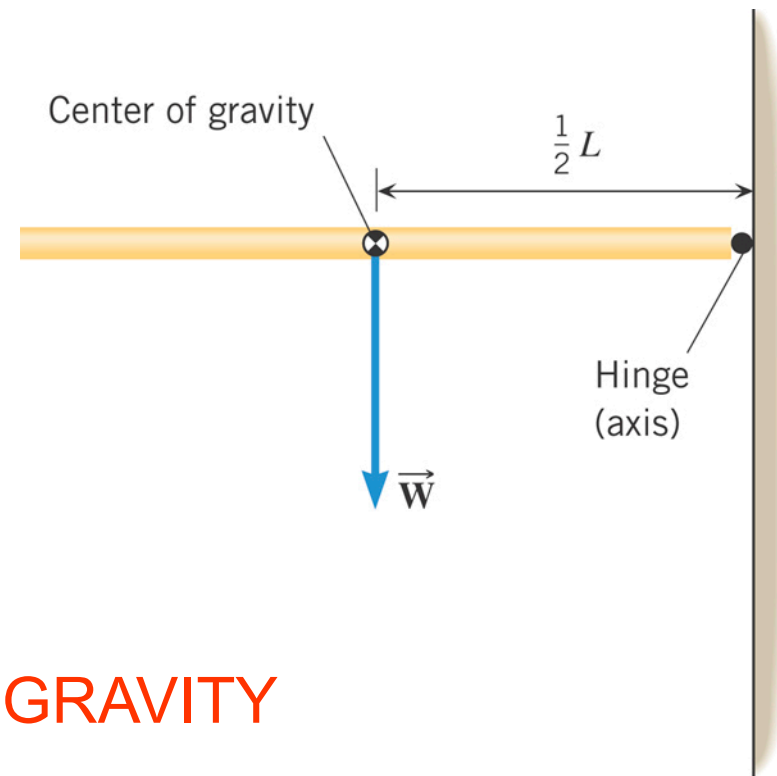


Chapter 8

Rotational Dynamics ***continued***

8.7 Center of Gravity

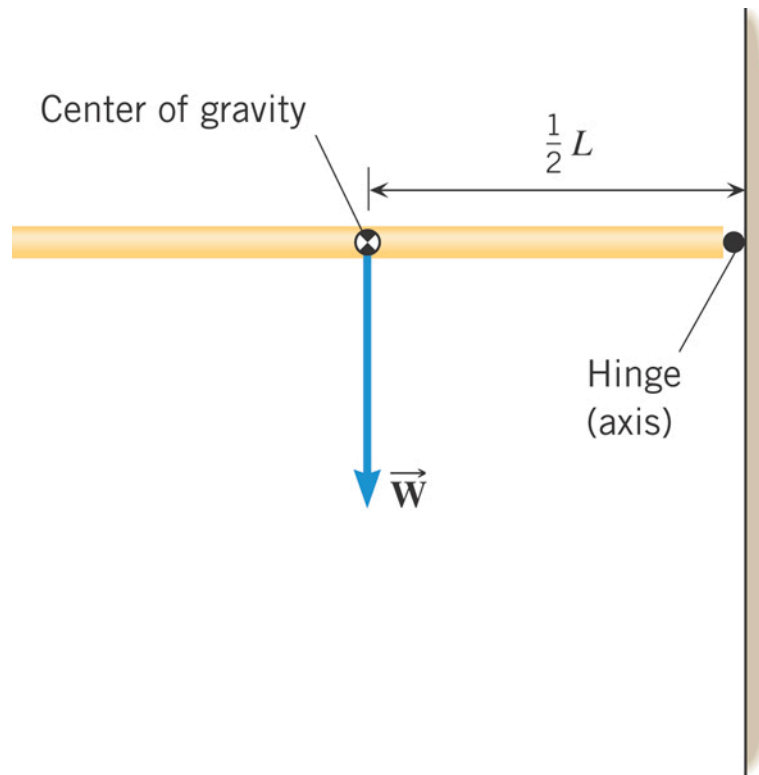


DEFINITION OF CENTER OF GRAVITY

The center of gravity of a rigid body is the point at which its weight can be considered to act when the torque due to the weight is being calculated.

8.7 Center of Gravity

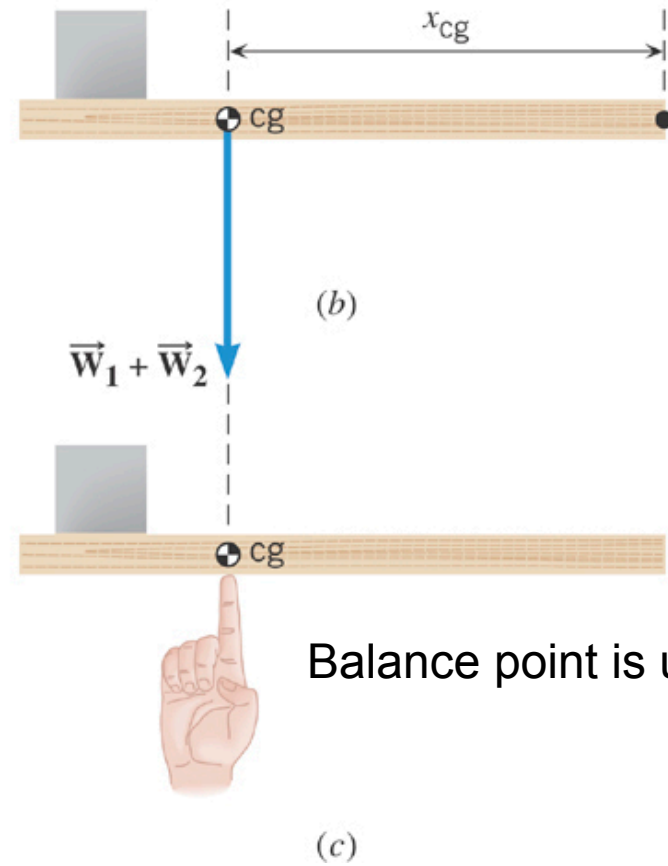
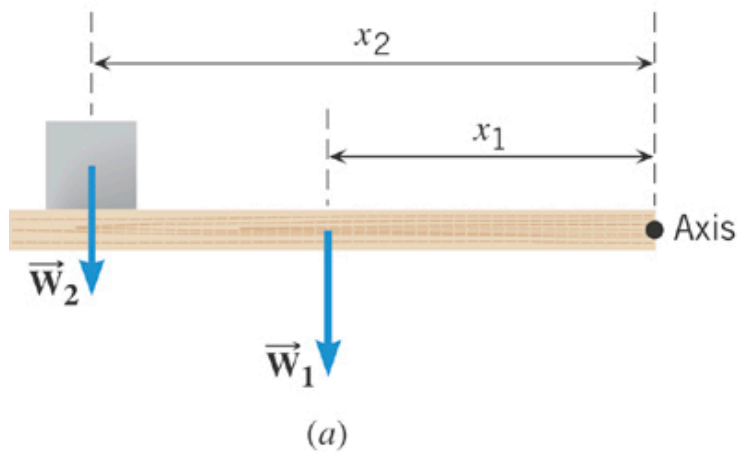
When an object has a symmetrical shape and its weight is distributed uniformly, the center of gravity lies at its geometrical center.



8.7 Center of Gravity

General Form of x_{cg}

$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + \dots}{W_1 + W_2 + \dots}$$



Balance point is under x_{cg}

Center of Gravity, x_{cg} , for 2 masses

$$x_{cg} = \frac{W_1 x_1 + W_2 x_2}{W_1 + W_2}$$

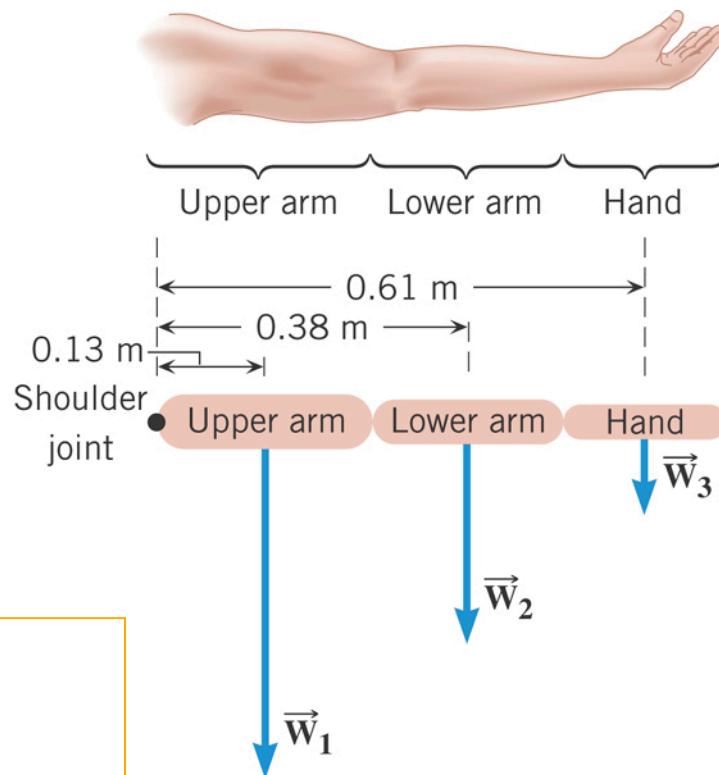
8.7 Center of Gravity

Example: The Center of Gravity of an Arm

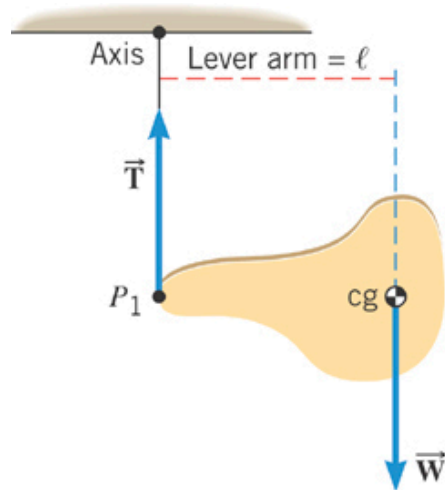
The horizontal arm is composed of three parts: the upper arm (17 N), the lower arm (11 N), and the hand (4.2 N).

Find the center of gravity of the arm relative to the shoulder joint.

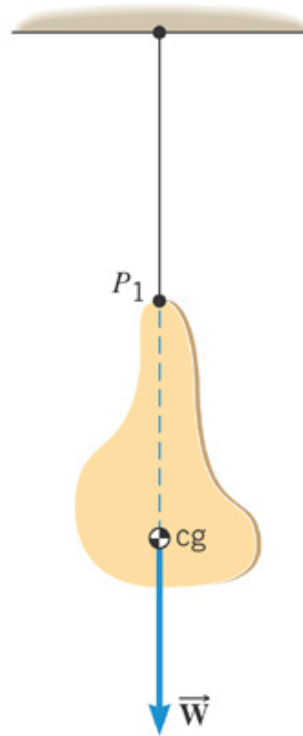
$$\begin{aligned}x_{cg} &= \frac{W_1x_1 + W_2x_2 + W_3x_3}{W_1 + W_2 + W_3} \\&= \frac{[17(0.13) + 11(0.38) + 4.2(0.61)] \text{ N} \cdot \text{m}}{(17 + 11 + 4.2) \text{ N}} = 0.28 \text{ m}\end{aligned}$$



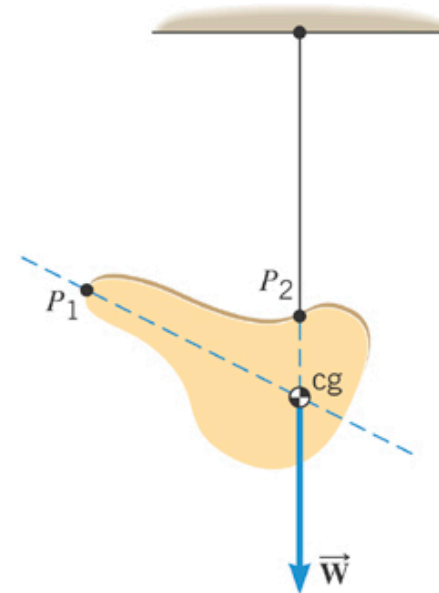
8.7 Center of Gravity



(a)



(b)



(c)

Finding the center of gravity of an irregular shape.

8.7 Rigid Objects in Equilibrium

EQUILIBRIUM OF A RIGID BODY

A rigid body is in equilibrium if it has **zero translational acceleration** and **zero angular acceleration**. In equilibrium, the sum of the externally applied forces is zero, and the sum of the externally applied torques is zero.

$$\sum F_x = 0$$

$$\sum F_y = 0$$

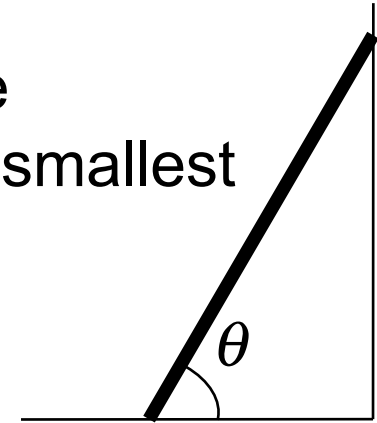
$$\sum \tau = 0$$

Note: **constant linear speed** or **constant rotational speed** are allowed for an object in equilibrium.

8.7 Rigid Objects in Equilibrium

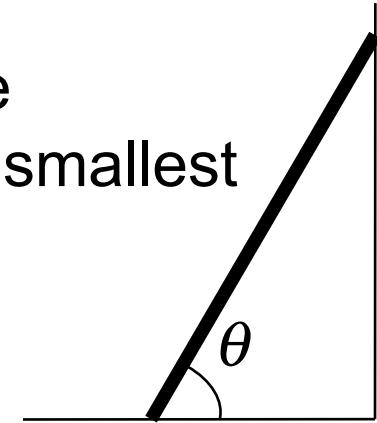
Example: A board length L lies against a wall. The coefficient of friction with ground is 0.650. What is smallest angle the board can be placed without slipping?

1. Determine the forces acting on the board.



8.7 Rigid Objects in Equilibrium

Example: A board length L lies against a wall. The coefficient of friction with ground is 0.650. What is smallest angle the board can be placed without slipping?



1. Determine the forces acting on the board.

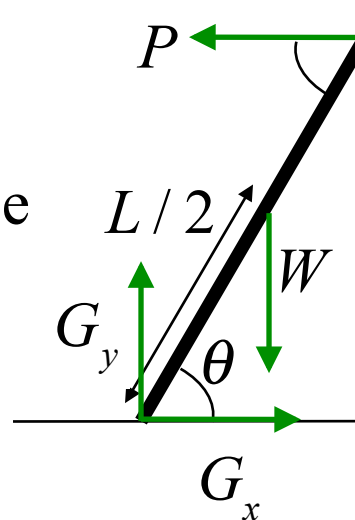
Forces

G_y ground normal force

G_x ground static frictional force

P wall normal force

W gravitational force



Forces:

$$G_y = W$$

$$P = G_x = \mu G_y = \mu W$$

2. Choose pivot point at ground.

8.7 Rigid Objects in Equilibrium

Example: A board length L lies against a wall. The coefficient of friction with ground is 0.650. What is smallest angle the board can be placed without slipping?

1. Determine the forces acting on the board.

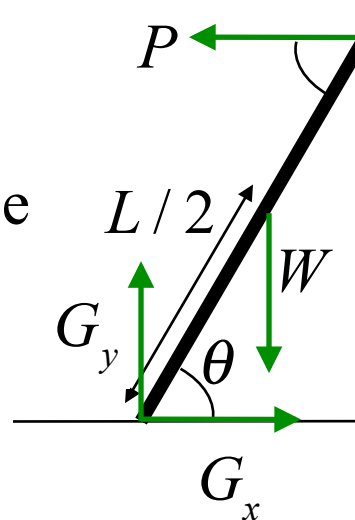
Forces

G_y ground normal force

G_x ground static frictional force

P wall normal force

W gravitational force



Forces:

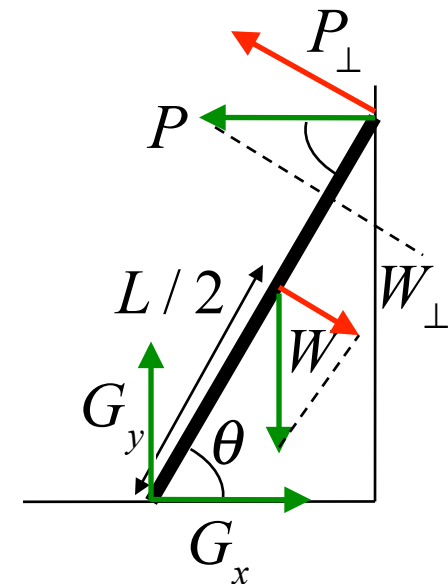
$$G_y = W$$

$$P = G_x = \mu G_y = \mu W$$

2. Choose pivot point at ground.

3. Find components normal to board

P_{\perp} and W_{\perp} are forces producing torque



8.7 Rigid Objects in Equilibrium

Example: A board length L lies against a wall. The coefficient of friction with ground 0.650. What is smallest angle the board can be placed without slipping?

4. Net torque must be zero for equilibrium

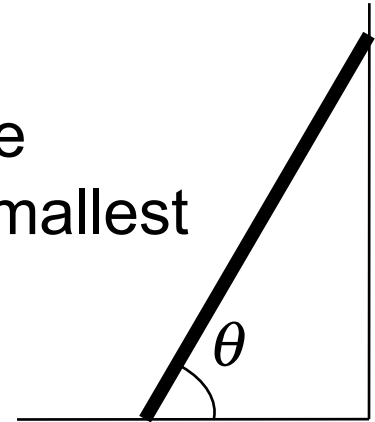
Torque:

$$\tau_P = +P_{\perp} L = (P \sin \theta) L$$

$$\tau_W = -W_{\perp} (L/2) = -(W \cos \theta) (L/2)$$

$$\tau_W + \tau_P = 0 \Rightarrow (P \sin \theta) = (W \cos \theta) / 2$$

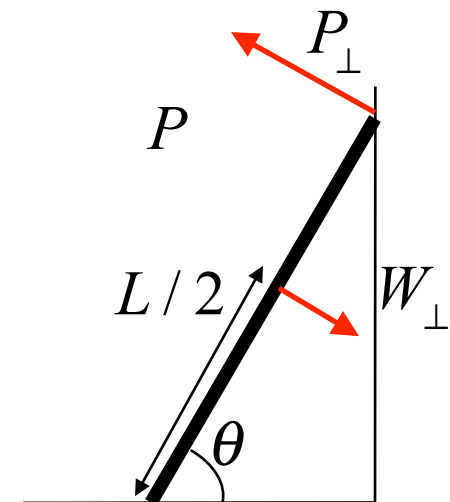
$$W = 2P \sin \theta / \cos \theta$$



Forces:

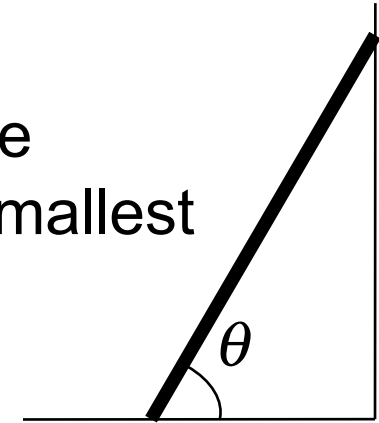
$$G_y = W$$

$$P = G_x = \mu G_y = \mu W$$



8.7 Rigid Objects in Equilibrium

Example: A board length L lies against a wall. The coefficient of friction with ground 0.650. What is smallest angle the board can be placed without slipping?



4. Net torque must be zero for equilibrium

Torque:

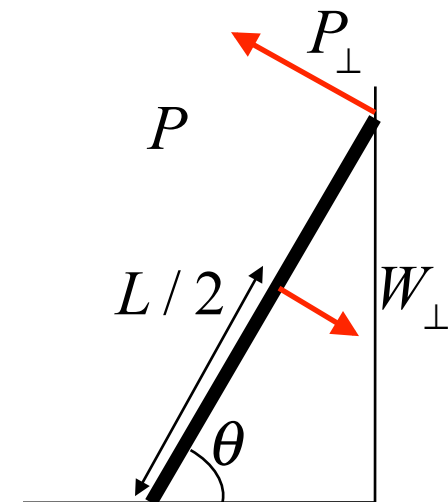
$$\begin{aligned}\tau_P &= +P_{\perp} L = (P \sin \theta) L \\ \tau_W &= -W_{\perp} (L/2) = -(W \cos \theta) (L/2) \\ \tau_W + \tau_P &= 0 \Rightarrow (P \sin \theta) = (W \cos \theta) / 2 \\ W &= 2P \sin \theta / \cos \theta\end{aligned}$$

Forces:

$$\begin{aligned}G_y &= W \\ P &= G_x = \mu G_y = \mu W\end{aligned}$$

5. Combine torque and force equations

$$\begin{aligned}W &= 2P \sin \theta / \cos \theta = 2\mu W \tan \theta \\ \tan \theta &= 1/(2\mu) = 1/(1.3) = 0.77 \\ \theta &= 37.6^\circ\end{aligned}$$



Chapter 8 Rotational Kinematics/Dynamics

Angular motion variables

(with the usual motion equations)

displacement $\theta = s / r$ (rad.)

velocity $\omega = v / r$ (rad./s)

acceleration $\alpha = a / r$ (rad./s²)

torque
(θ : \angle between \vec{F} & \vec{r}) $\tau = rF \sin \theta$

Newton's 2nd Law $\tau_{\text{Net}} = I \alpha$

rot. kinetic energy $K_{\text{rot}} = \frac{1}{2} I \omega^2$

angular momentum $L = I \omega$

Uniform circular motion

centripetal acceleration $a_c = \frac{v^2}{r} = \omega^2 r$

centripetal force $F_c = ma_c = \frac{mv^2}{r}$

I Moment of Inertia

From Ch.8.4 Rotational Kinetic Energy

Moments of Inertia
of Rigid Objects Mass M

Thin walled hollow cylinder
 $I = MR^2$

Solid cylinder or disk
 $I = \frac{1}{2} MR^2$

Thin rod length ℓ through center
 $I = \frac{1}{12} M \ell^2$

Thin rod length ℓ through end
 $I = \frac{1}{3} M \ell^2$

Solid sphere through center

$I = \frac{2}{5} MR^2$

Solid sphere through surface tangent

$I = \frac{7}{5} MR^2$

Thin walled sphere through center

$I = \frac{2}{3} MR^2$

Thin plate width ℓ , through center

$I = \frac{1}{12} M \ell^2$

Thin plate width ℓ through edge

$I = \frac{1}{3} M \ell^2$

8.8 Newton's Second Law for Rotational Motion About a Fixed Axis

$$\tau = F_T r$$

$$= m a_T r$$

$$= m \alpha r^2$$

$$= (m r^2) \alpha$$

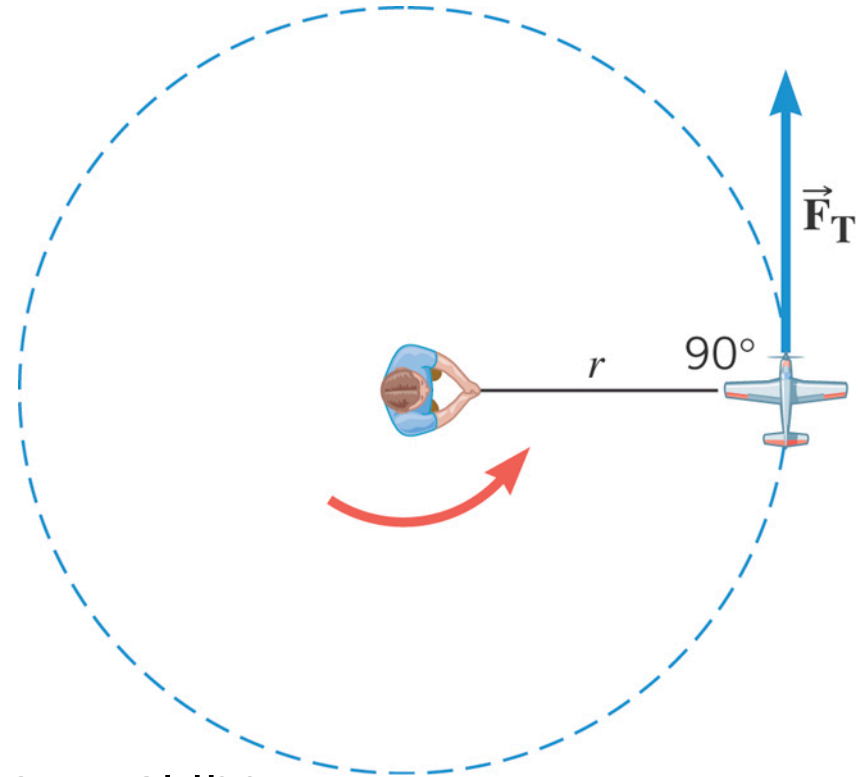
$$= I \alpha$$

$$F_T = m a_T$$

$$a_T = r \alpha$$

$$I = m r^2$$

Moment of Inertia
of the airplane

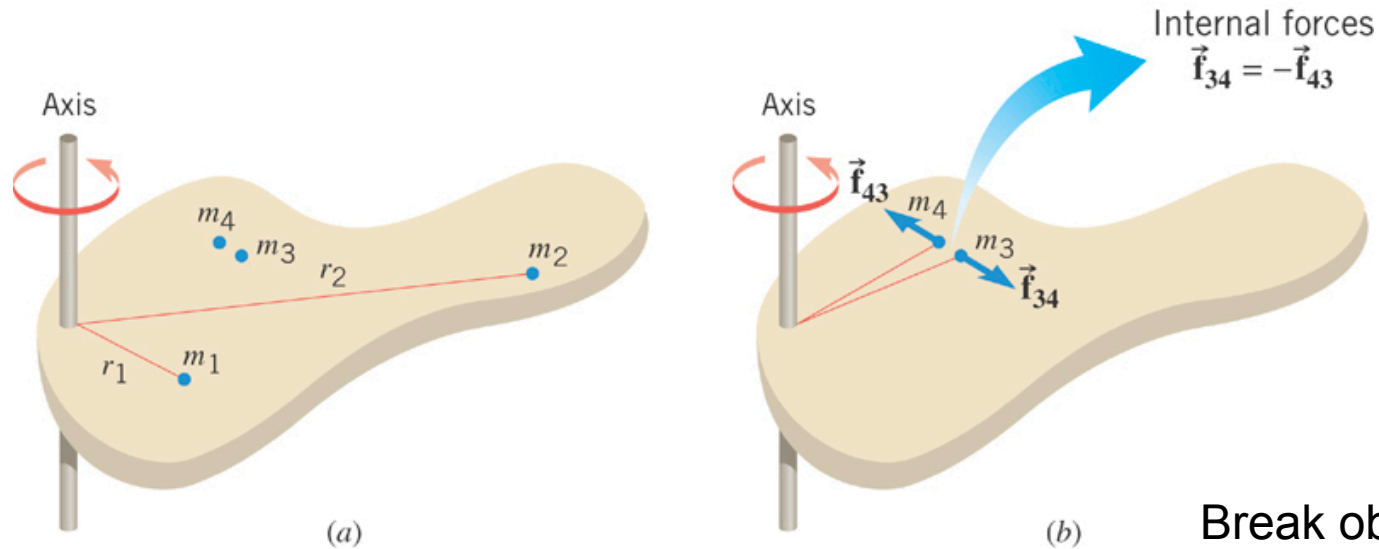


Moment of Inertia, $I = m r^2$, for a point-mass, m , at the end of a massless arm of length, r .

$$\tau = I \alpha$$

Newton's 2nd Law for rotations

8.8 Newton's Second Law for Rotational Motion About a Fixed Axis



Break object into N individual masses

$$\tau_{\text{Net}} = \sum_i (mr^2)_i \alpha$$

Net external
torque

Moment of
inertia

$$\tau_1 = (m_1 r_1^2) \alpha$$

$$\tau_2 = (m_2 r_2^2) \alpha$$

\vdots

$$\tau_N = (m_N r_N^2) \alpha$$

8.8 *Newton's Second Law for Rotational Motion About a Fixed Axis*

ROTATIONAL ANALOG OF NEWTON'S SECOND LAW FOR A RIGID BODY ROTATING ABOUT A FIXED AXIS

$$\text{Net external torque} = \left(\begin{array}{c} \text{Moment of} \\ \text{inertia} \end{array} \right) \times \left(\begin{array}{c} \text{Angular} \\ \text{acceleration} \end{array} \right)$$

$$\tau_{\text{Net}} = I \alpha$$

$$I = \sum_i (mr^2)_i$$

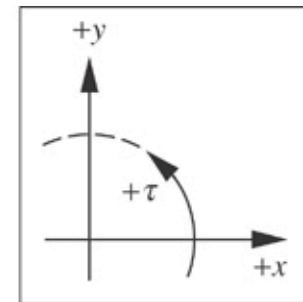
Requirement: Angular acceleration must be expressed in radians/s².

8.8 Newton's Second Law for Rotational Motion About a Fixed Axis

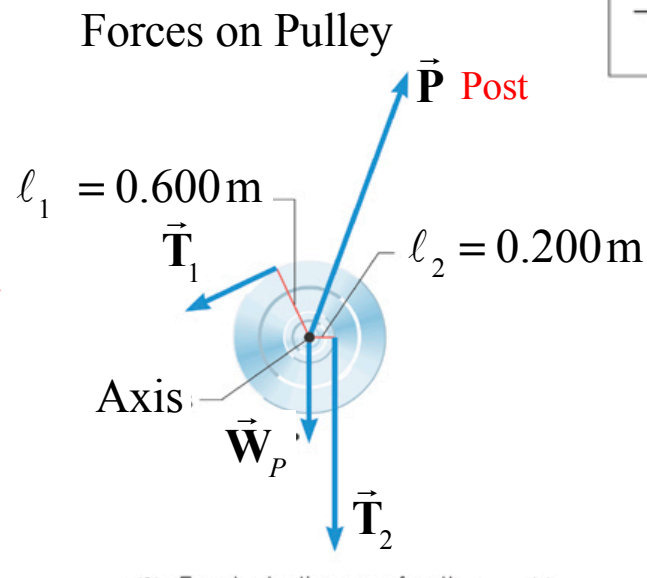
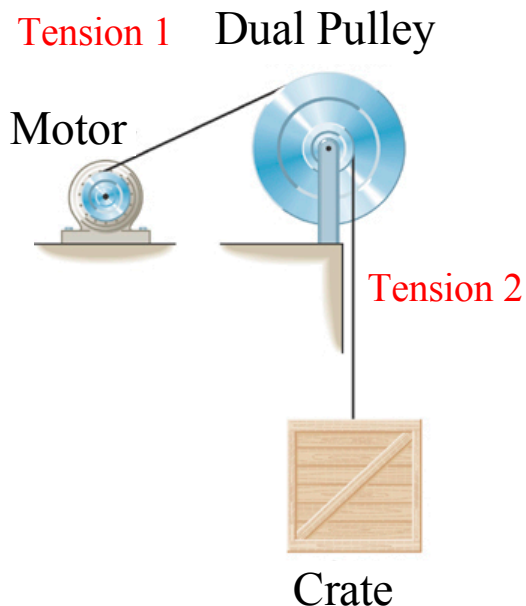
Example: Hoisting a Crate

The combined moment of inertia of the dual pulley is $50.0 \text{ kg}\cdot\text{m}^2$. The crate weighs 4420 N . A tension of 2150 N is maintained in the cable attached to the motor. Find the angular acceleration of the dual Pulley (radius-1 = 0.600 m , radius-2 = 0.200 m).

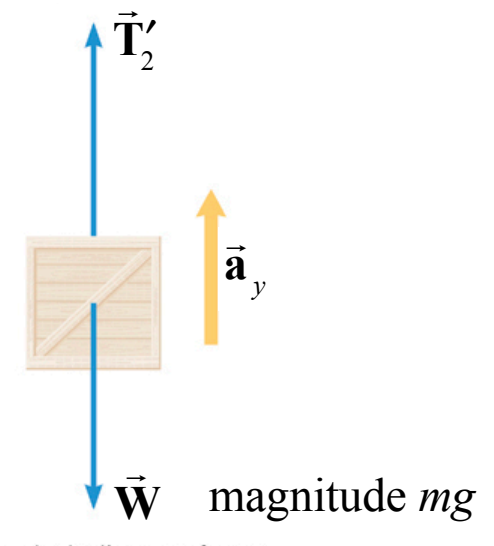
dual pulley



up is positive



Forces on Crate



8.8 Newton's Second Law for Rotational Motion About a Fixed Axis

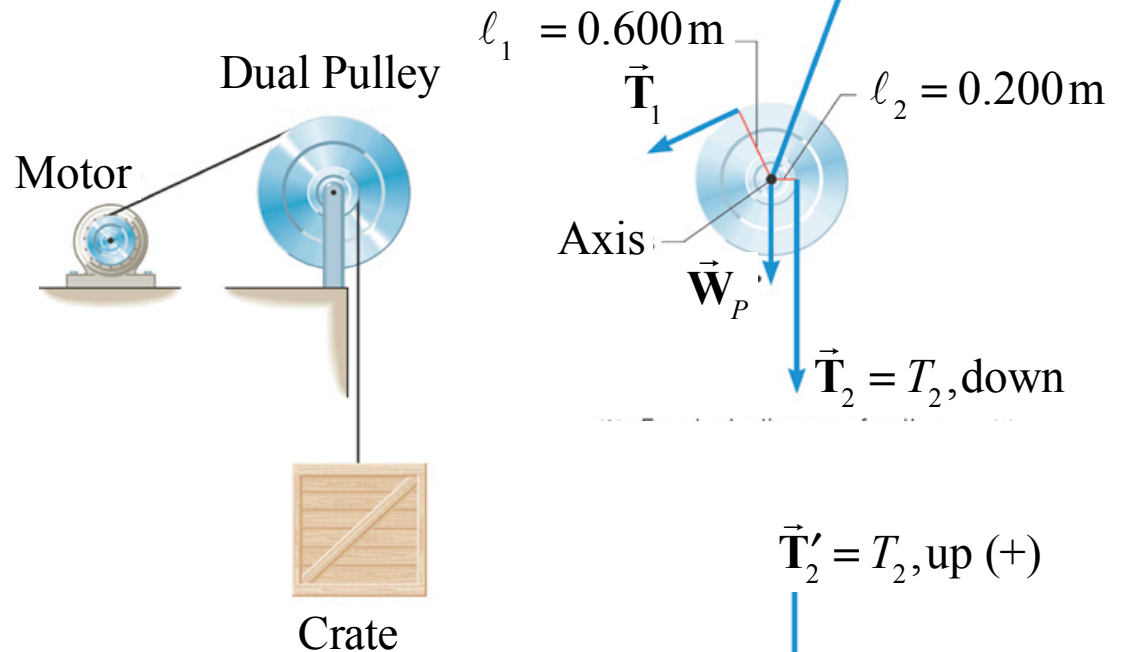
2nd law for linear motion of crate

$$\sum F_y = T_2 - mg = ma_y \quad a_y = \ell_2 \alpha$$

$$\begin{aligned} I &= 46 \text{ kg} \cdot \text{m}^2 \\ mg &= 4420 \text{ N} \\ T_1 &= 2150 \text{ N} \end{aligned}$$

2nd law for rotation of the pulley

$$\sum \tau = T_1 \ell_1 - T_2 \ell_2 = I \alpha$$



$$T_2 - mg = m\ell_2 \alpha$$

$$T_1 \ell_1 - T_2 \ell_2 = I \alpha$$

$$T_2 = m\ell_2 \alpha + mg \quad \& \quad T_2 = \frac{T_1 \ell_1 - I \alpha}{\ell_2}$$

$$m\ell_2^2 \alpha + I \alpha = T_1 \ell_1 - mg \ell_2$$

$$\alpha = \frac{T_1 \ell_1 - mg \ell_2}{m\ell_2^2 + I} = 6.3 \text{ rad/s}^2$$

$$T_2 = m\ell_2 \alpha + mg = (451(.6)(6.3) + 4420) \text{ N} = 6125 \text{ N}$$

8.9 *Angular Momentum*

DEFINITION OF ANGULAR MOMENTUM

The angular momentum L of a body rotating about a fixed axis is the product of the body's moment of inertia and its angular velocity with respect to that axis:

$$L = I\omega$$

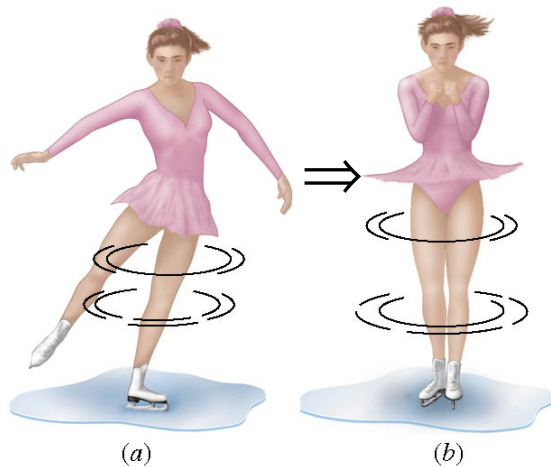
Requirement: The angular speed must be expressed in rad/s.

SI Unit of Angular Momentum: $\text{kg}\cdot\text{m}^2/\text{s}$

8.9 Angular Momentum

PRINCIPLE OF CONSERVATION OF ANGULAR MOMENTUM

The angular momentum of a system remains constant (is conserved) if the net external torque acting on the system is zero.



Moment of Inertia
decreases

$$I = \sum mr^2, \quad r_f < r_i$$
$$I_f < I_i$$
$$\frac{I_i}{I_f} > 1$$

Angular momentum, L

$$L_i = I_i \omega_i; \quad L_f = I_f \omega_f$$

No external torque

\Rightarrow Angular momentum conserved

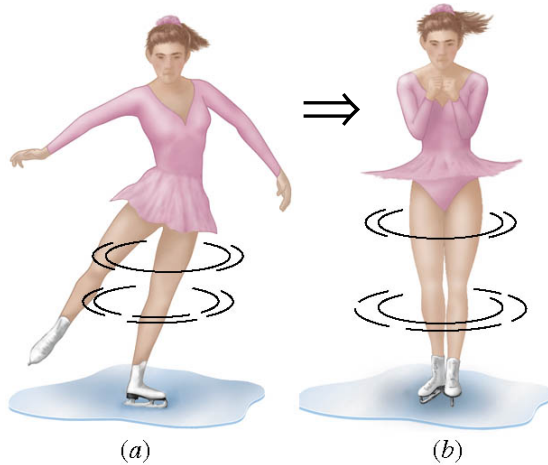
$$L_f = L_i$$

$$I_f \omega_f = I_i \omega_i$$

$$\omega_f = \frac{I_i}{I_f} \omega_i; \quad \frac{I_i}{I_f} > 1$$

$\omega_f > \omega_i$ (angular speed increases)

8.9 Angular Momentum



From Angular Momentum Conservation

$$\omega_f = \left(I_i / I_f \right) \omega_i$$

because $I_i / I_f > 1$

Angular velocity increases

Is Energy conserved?

$$K_f = \frac{1}{2} I_f \omega_f^2$$

$$= \frac{1}{2} I_f \left(I_i / I_f \right)^2 \omega_i^2$$

$$= \left(I_i / I_f \right) \left(\frac{1}{2} I_i \omega_i^2 \right) \quad K_i = \frac{1}{2} I_i \omega_i^2;$$

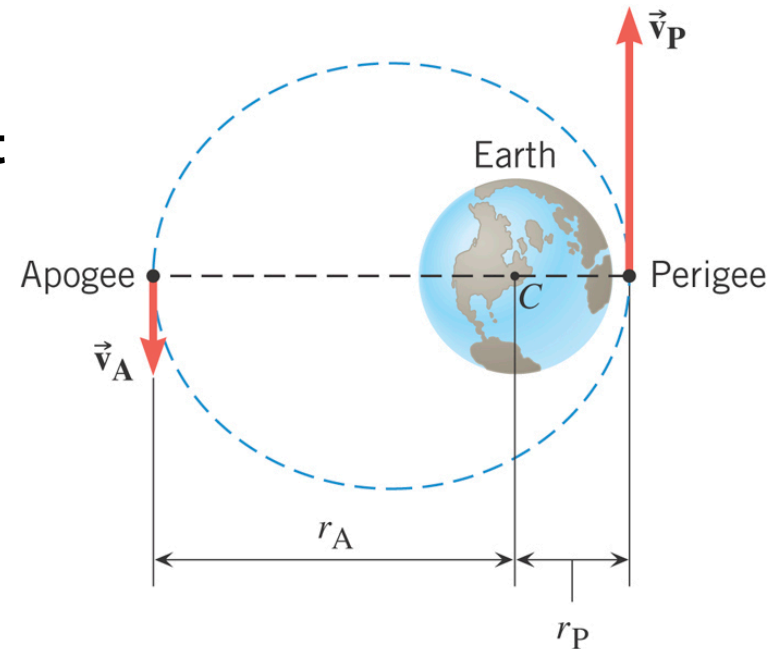
$$= \left(I_i / I_f \right) K_i \Rightarrow \text{Kinetic Energy increases}$$

Energy is **NOT conserved** because pulling in the arms does (NC) work on the mass of each arm and **increases the kinetic energy** of rotation.

8.9 Angular Momentum

Example: A Satellite in an Elliptical Orbit

An artificial satellite is placed in an elliptical orbit about the earth. Its point of closest approach is 8.37×10^6 m from the center of the earth, and its point of greatest distance is 25.1×10^6 m from the center of the earth. The speed of the satellite at the perigee is 8450 m/s. Find the speed at the apogee.



$$I_A = mr_A^2; \quad I_P = mr_P^2$$
$$\omega_A = v_A / r_A; \quad \omega_P = v_P / r_P$$

Gravitational force along r (no torque) \Rightarrow Angular momentum conserved

$$I_A \omega_A = I_P \omega_P$$

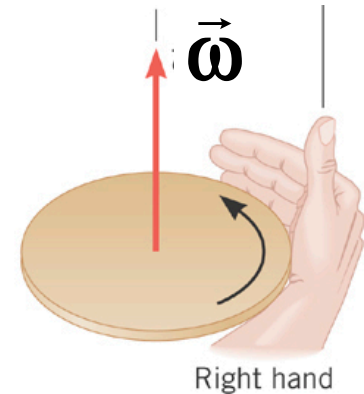
$$mr_A^2 (v_A / r_A) = mr_P^2 (v_P / r_P) \Rightarrow r_A v_A = r_P v_P$$

$$v_A = (r_P / r_A) v_P = \left[(8.37 \times 10^6) / (25.1 \times 10^6) \right] (8450 \text{ m/s}) = 2820 \text{ m/s}$$

8.9 The Vector Nature of Angular Variables

Right-Hand Rule: Grasp the axis of rotation with your right hand, so that your fingers circle the axis in the same sense as the rotation.

Your extended thumb points along the axis in the direction of the angular velocity.

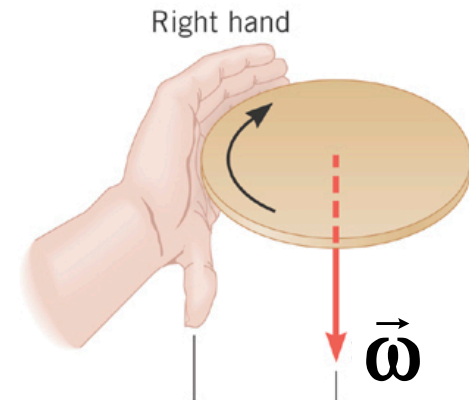


Vector Quantities in Rotational Motion

Angular Acceleration $\vec{\alpha} = \frac{\Delta \vec{\omega}}{\Delta t}$

Angular Momentum $\vec{L} = I \vec{\omega}$

Torque $\vec{\tau} = I \vec{\alpha} = \frac{\Delta \vec{L}}{\Delta t}$ Changes Angular Momentum



If torque is perpendicular to the angular momentum, only the direction of the angular momentum changes (precession) – no changes to the magnitude.

Rotational/Linear Dynamics Summary

<u>linear</u>		<u>rotational</u>	<u>linear</u>	<u>rotational</u>
x	displacement	θ	$\vec{\mathbf{F}} = m\vec{\mathbf{a}}$	$\vec{\tau} = I\vec{\alpha}$
v	velocity	ω	$W = F(\cos\theta)x$	$W_{rot} = \tau\phi$
a	acceleration	α	$K = \frac{1}{2}mv^2$	$K_{rot} = \frac{1}{2}I\omega^2$
m	point m inertia	$I = mr^2$	$W \Rightarrow \Delta K$	$W_{rot} \Rightarrow \Delta K_{rot}$
F	force/torque	$\tau = Fr \sin\theta$	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$	$\vec{L} = I\vec{\omega}$
<u>Uniform circular motion</u>			$\vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}}$	$\vec{\tau}\Delta t = \Delta\vec{L}$
$a_C = v_T^2/r$		$F_C = mv_T^2/r$		

Potential Energies

$$U_G = mgy$$

$$\text{or } U_G = -GmM_E/r$$

$$U_S = \frac{1}{2}kx^2$$

Conservation laws

Conserved:	If $W_{NC} = 0,$	If $\mathbf{F}_{ext} = 0,$	If $\tau_{ext} = 0,$
	$E = K + U$	$\mathbf{P}_{system} = \sum \mathbf{p}$	$\vec{L} = I\vec{\omega}$