Chapter 9

Gravitation

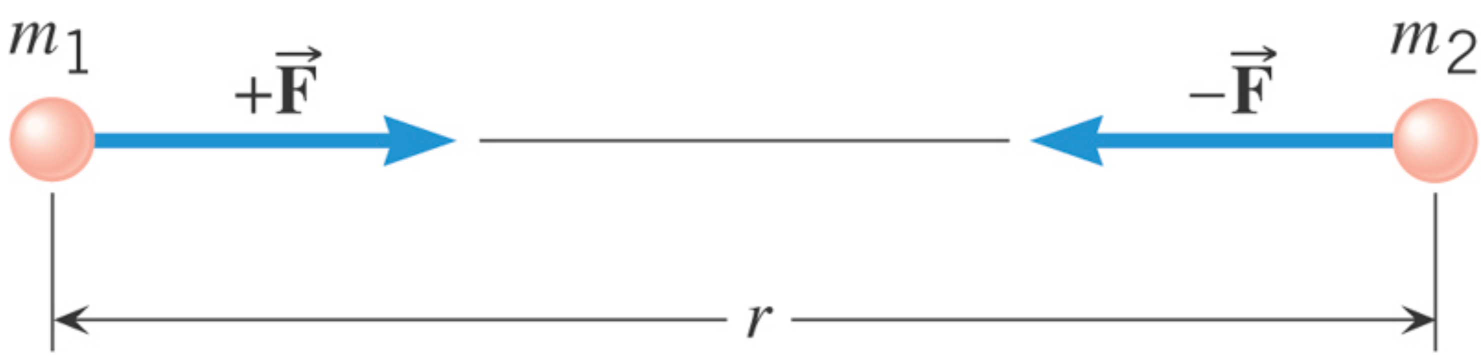
Beyond Earth's surface

9.1 The Gravitational Force

For two particles that have masses m_1 and m_2 and are separated by a distance r, the force has a magnitude given by

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.673\times 10^{-11}\ \mathrm{N\cdot m^2/kg^2}$$



the same magnitude of force acts on each mass, no matter what the values of the masses.

Near the earth's surface

Earth's Gravitational force on mass *m*.

$$F = G\frac{m_1 m_2}{r^2}$$

Radius of the earth

$$r = R_E = 6.38\times 10^6 \text{ m}$$

$$F = mg = G\frac{mM_E}{R_E^2} \Rightarrow$$

$$g = G\frac{M_E}{R_E^2} = \left(6.67\times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(5.97\times 10^{24}\text{ kg}\right)}{\left(6.37\times 10^{6}\text{ m}\right)^2}$$

$$= 9.81 \text{ m/s}^2$$

This is why acceleration due to gravity is this value on the earth.

Your WEIGHT on the earth

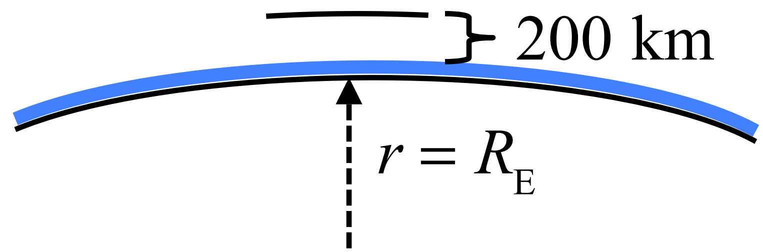
$$W = mg$$

for example: $m = 80.0\text{ kg}$,

$$W = mg = 784\text{ N}$$

Near the earth's surface

In orbit at altitude = 250 km



$$g = 9.81\ \text{m/s}^2$$

At radius of the earth

At 250 km above the earth

$$r' = R_E + 250\ \text{km} = 6.37\times10^6 + 0.25\times10^6\ \text{m}$$

$$= 6.62\times10^6\ \text{m}$$

$$g' = 9.20\ \text{m/s}^2$$

$$g' = \frac{GM_E}{r'^2} = 9.09\ \text{m/s}^2$$

In low-earth orbit,
your weight is 8% less than on earth. NOT ZERO!

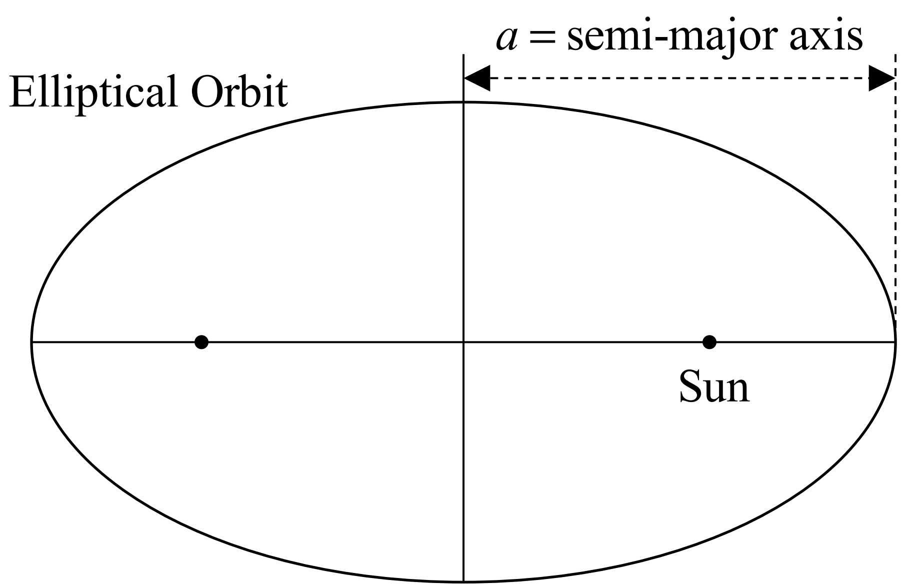
9.2 Elliptical Orbits

In general, the orbit of a satellite (around a planet) or planet (around a star) is an ellipse. Kepler was the first to describe this motion for planets around the sun that are a consequence of Newton's Universal Gravitational Force.

Kepler's Laws for planetary orbits (in homework)

1. Orbits are elliptical with the Sun at one focus.
2. In a given time a planet covers the same area anywhere in the orbit.
3. If T is the orbital period and a the semi-major axis of the orbit, then

$$\frac{a^3}{T^2} = C \text{ (constant)} \qquad C_{\text{Sun}} = 3.36 \times 10^{18} \text{ m}^3/\text{s}^2$$



(for circular orbit, a = radius)

$$\left[\begin{array}{l}\text{For an elliptical orbit, } a = \dfrac{R_1 + R_2}{2}, \\ \text{where } R_1 = \text{aphelion}, R_2 = \text{perihelion}\end{array}\right]$$

9.3 Gravitational Potential Energy

The Gravitational force gets smaller for large distances above the earth.

Gravitational Potential Energy for large distances above the earth requires that the Potential Energy be defined as zero at very large distances. The closer one gets to the earth the lower (or the more negative) the potential energy becomes.

Near the surface of the earth we use:

$$U = mgy \quad (\text{puts } U = 0 \text{ at y} = 0)$$

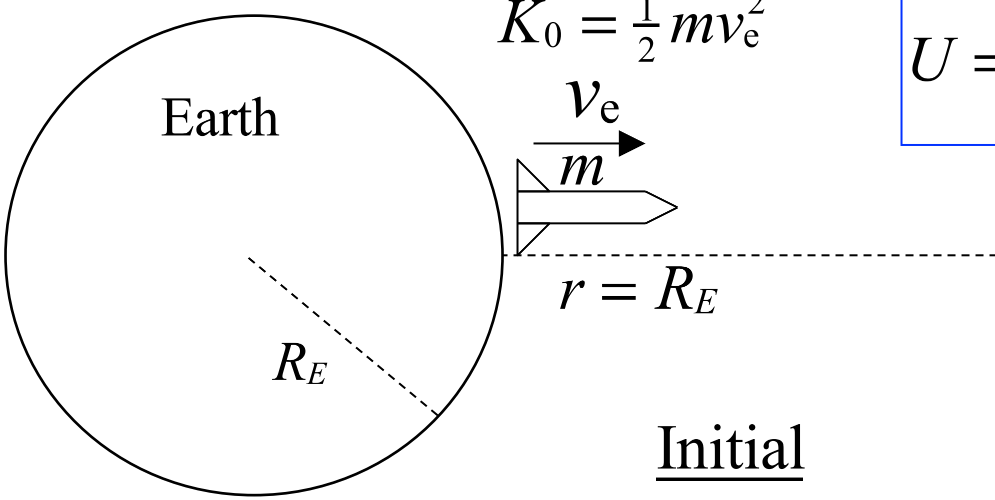
But for extreme heights above the earth:

$$U = -G\frac{mM}{r} \quad (\text{puts } U = 0 \text{ at } r = \infty)$$

Only need changes in Potential Energy to be correct.

Example: Escape velocity (Energy Conservation)

There is a velocity above which an object fired from the surface of the earth will never return to the earth. A rocket fires just long enough to get above the atmosphere and reach the escape speed.



$K_0 = \frac{1}{2} m v_e^2$

$U = -G\frac{mM_E}{r}$

$K = 0$

$U = 0$

$r = R_E$

$r \rightarrow \infty$

	Initial	Final
Kinetic Energy:	$K_0 = \frac{1}{2} m v_e^2$	$K = 0$
Potential Energy:	$U_0 = -G\frac{mM_E}{R_E}$ ($r = R_E$)	$U = -G\frac{mM_E}{r} \rightarrow 0$ ($r \rightarrow \infty$)

Example: Escape velocity (Energy Conservation)

$$K_0 + U_0 = K + U = 0 \quad (K = 0, \ \ U = 0)$$

$$\tfrac{1}{2} m v_e^2 + \left[-G \frac{m M_E}{R_E} \right] = 0$$

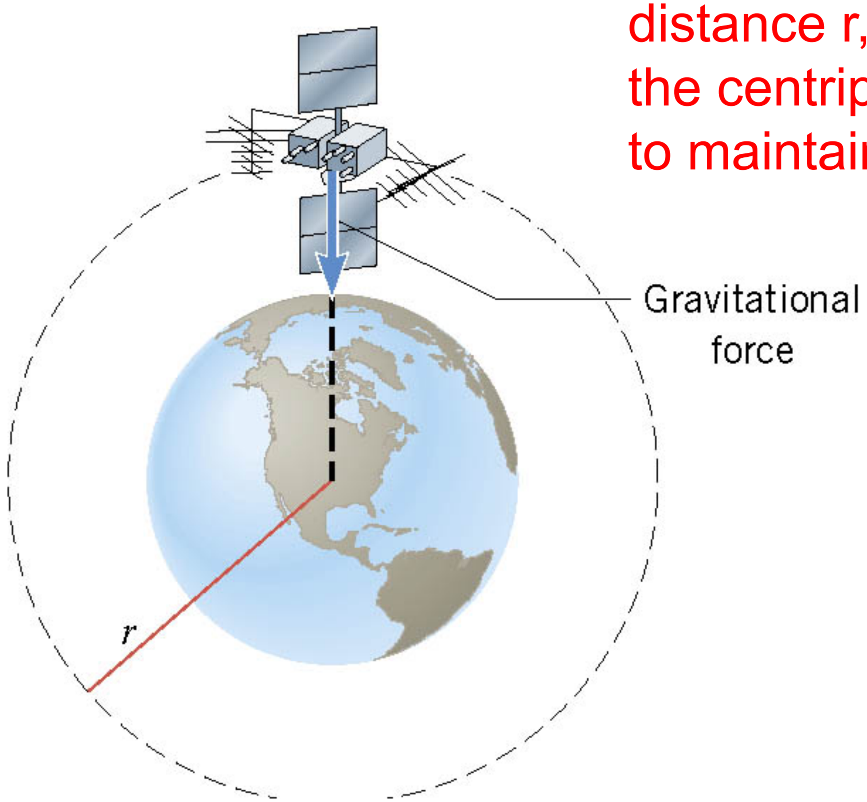
$$v_e^2 = 2G \frac{M_E}{R_E} \qquad \text{earlier:} \ \ g = G \frac{M_E}{R_E^2}$$

$$v_e = \sqrt{2 g R_E} = \sqrt{2(9.81 \mathrm{m/s^2})(6.37 \times 10^6 \mathrm{m})} =$$

$$= 11.2 \mathrm{km/s} = 40.3 \times 10^3 \ \mathrm{km/hr} \ \ (\sim 25{,}000 \ \mathrm{miles/hr})$$

9.4 Satellites in Circular Orbits

There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.



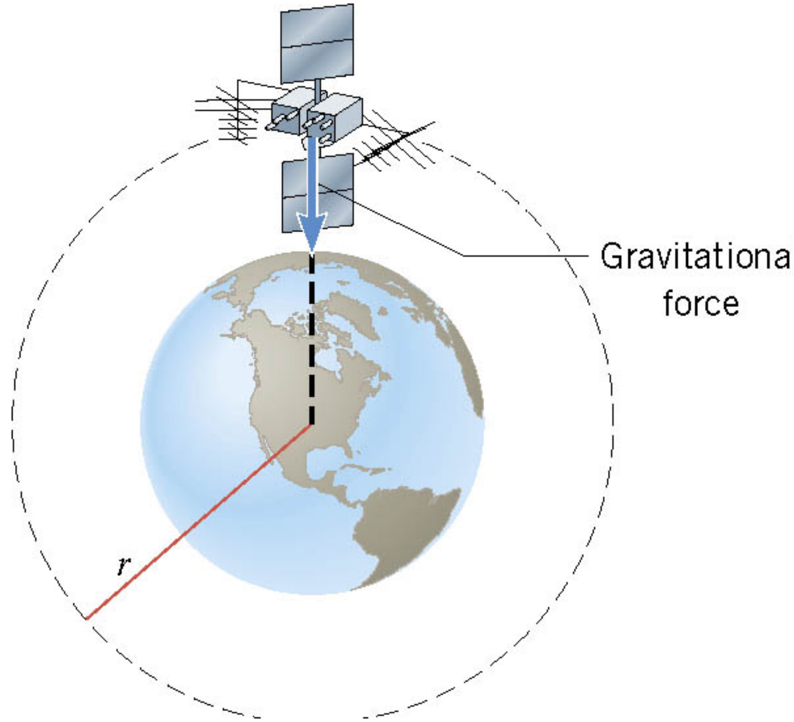
Gravitational force at the distance r, is the source of the centripetal force necessary to maintain the circular orbit.

9.4 Satellites in Circular Orbits

Gravitational force
at the distance r

Centripetal force

$$F_C = G\frac{mM_E}{r^2} = m\frac{v^2}{r} \quad (ma_C)$$



$$v = \sqrt{\frac{GM_E}{r}}$$

Speed to keep satellite in the orbit with radius r.

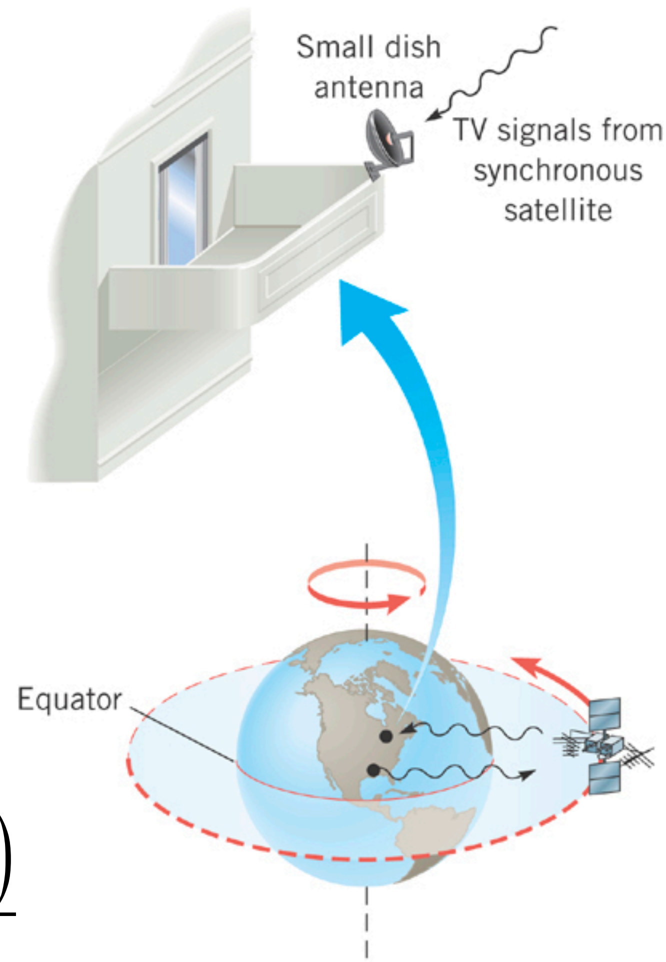
9.4 *Satellites in Circular Orbits*

There is a radius where the speed will make the satellite go around the earth in exactly 24 hours. This keeps the satellite at a fixed point in the sky.

$$\omega = \frac{2\pi}{(3600\,\text{s/hr})(24\,\text{hr})}$$

same for Earth & satellite

$$= 72.7\times 10^{-6}\,\text{rad/s}$$



$$\omega r_s = v_T = \sqrt{\frac{GM_E}{r_s}}$$

$$r_s^3 = \frac{GM_E}{\omega^2}$$

$$= \frac{\left(6.67\times 10^{-11}\,\text{Nm}^2/\text{kg}^2\right)\left(5.97\times 10^{24}\,\text{kg}\right)}{(72.7\times 10^{-6}\,\text{rad/s})^2}$$

$$r_s = 42,200\,\text{km (synchronous orbit)}$$

9.5 Apparent Weightlessness

Can you feel gravity? Remember, we discussed that you can't !

1) Hanging from a 100 m high diving board
– your arms feel stretched by the upward force of bent board.

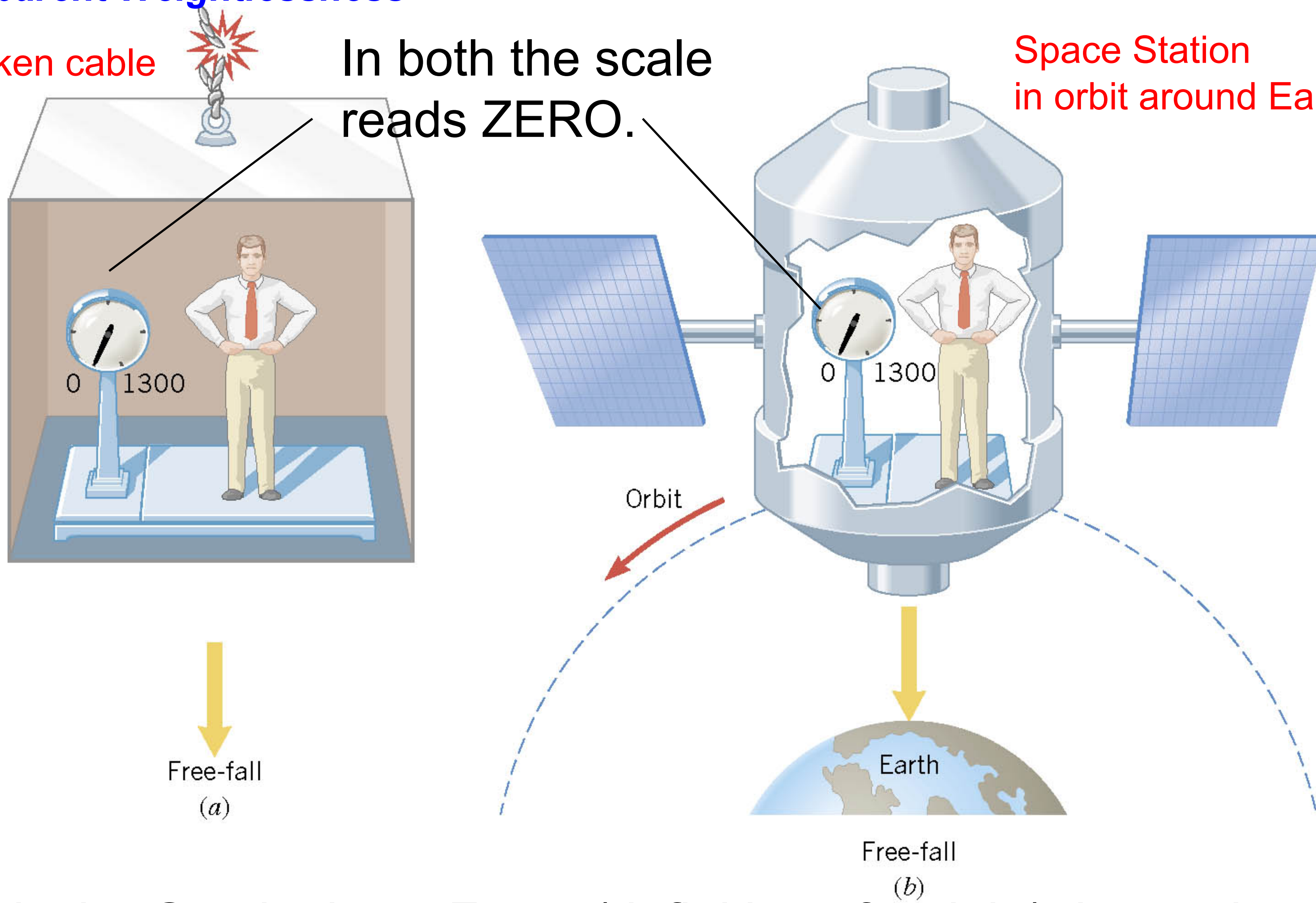
2) Standing on a bed – your legs feel compressed by the compressed springs in the mattress.

The bent diving board or the compressed springs provide the force to balance the gravitational force on your whole body.

When you let go of the diving board and before you hit the ground the ONLY force on you is gravity. It makes you accelerate downward, but it does not stretch or compress your body.

In free fall one cannot feel the force of gravity!

9.5 *Apparent Weightlessness*



But it is the Gravitational Force (definition of weight) that makes both the elevator and the body free-fall with the same acceleration. FEELING weightless and BEING weightless are VERY different.

Chapter 8
continued

Rotational Dynamics

8.3 Angular Variables and Tangential Variables (REVIEW)

$\omega =$ angular velocity - same at all radii (radians/s)

$\alpha =$ angular acceleration - same at all radii (radians/s^2)

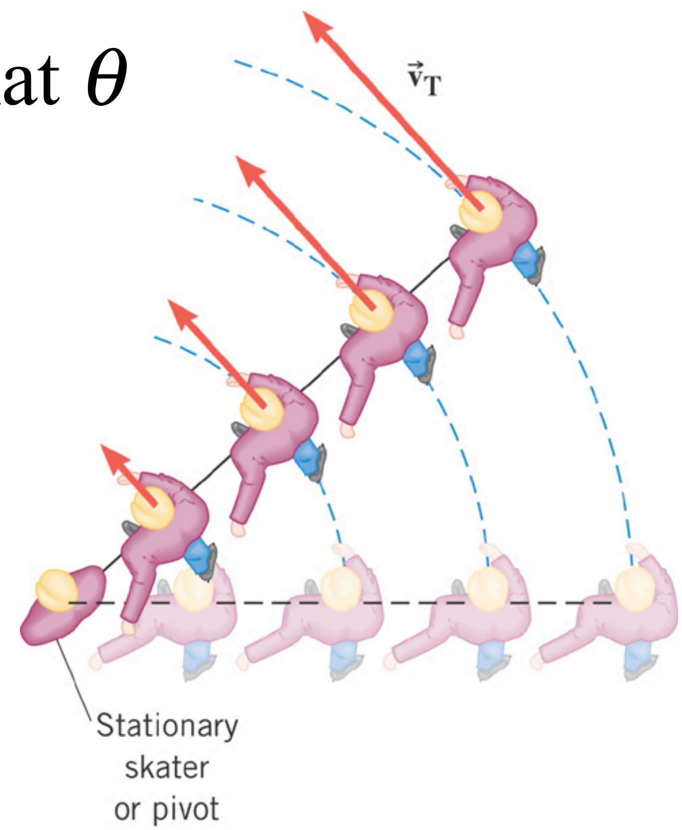
$\vec{\mathbf{v}}_T =$ tangential velocity - different at each radius

$\vec{\mathbf{a}}_T =$ tangential acceleration - different at each radius

Direction is tangent to circle at that θ

$$\vec{\mathbf{v}}_T = \omega r \qquad \vec{\mathbf{a}}_T = \alpha r$$

$\vec{\mathbf{v}}_T$ (m/s)	$\vec{\mathbf{a}}_T$ (m/s^2)
ω (rad/s)	α (rad/s^2)
r (m)	r (m)



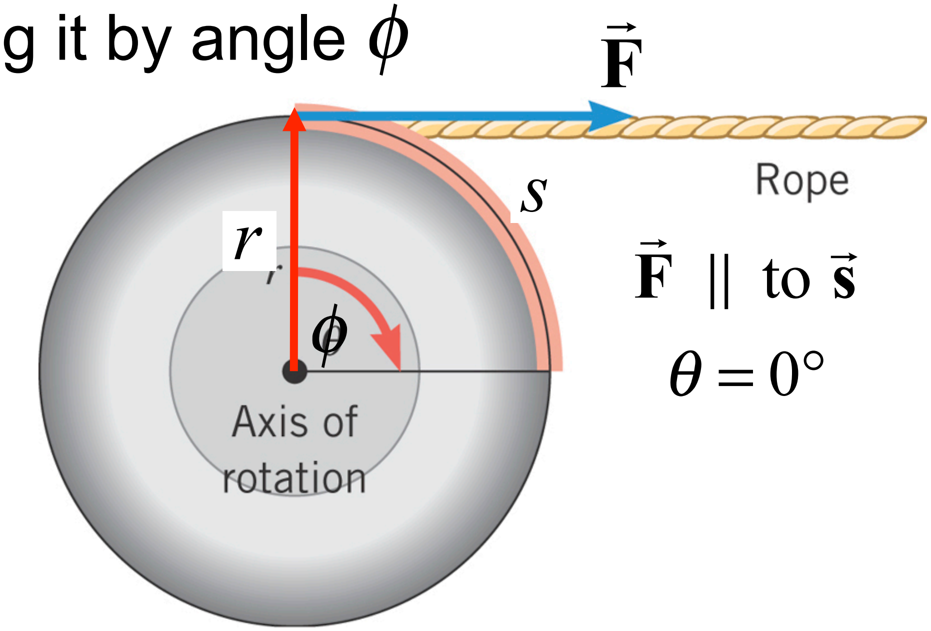
8.4 Rotational Work and Energy

Work to accelerate a mass rotating it by angle ϕ

$$W = F(\cos\theta)x \qquad x = s = r\phi$$

$$= Fr\phi \qquad Fr = \tau \ \text{(torque)}$$

$$= \tau\phi$$



$\vec{F} \parallel$ to $\vec{s}$

$\theta = 0°$

DEFINITION OF ROTATIONAL WORK

The rotational work done by a constant torque in turning an object through an angle is

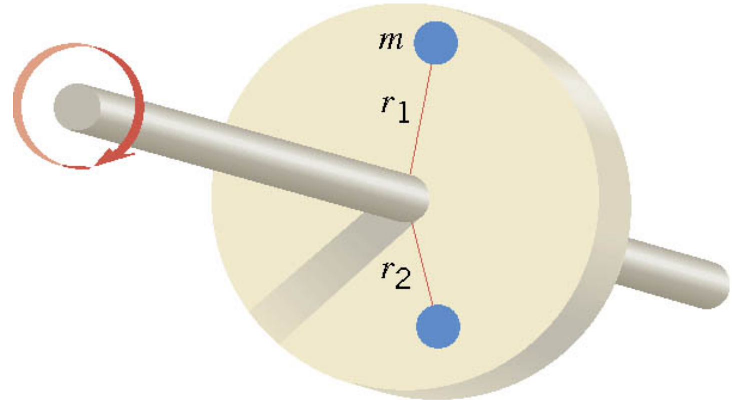
$$\boxed{W_R = \tau\phi}$$

Requirement: The angle must be expressed in radians.

SI Unit of Rotational Work: joule (J)

8.4 *Rotational Work and Energy*

Kinetic Energy of a rotating one point mass



$$K = \tfrac{1}{2} m v_T^2 \qquad v_T = r\omega$$

$$= \tfrac{1}{2} m r^2 \omega^2$$

Kinetic Energy of many rotating point masses

$$K = \sum \left(\tfrac{1}{2} m r^2 \omega^2 \right) = \tfrac{1}{2} \left(\sum m r^2 \right) \omega^2 = \tfrac{1}{2} I \omega^2$$

DEFINITION OF ROTATIONAL KINETIC ENERGY

The rotational kinetic energy of a rigid rotating object is

$$K_{Rot} = \tfrac{1}{2} I \omega^2$$

$$I = \sum_{i=1}^{n} (m r^2)_i$$

Requirement: The angular speed must be expressed in rad/s.

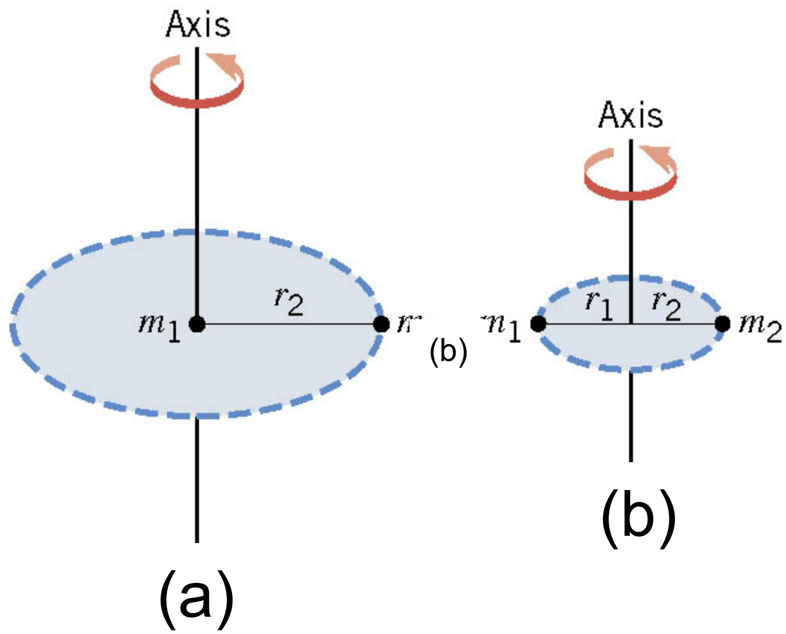
SI Unit of Rotational Kinetic Energy: joule (J)

Moment of Inertia depends on axis of rotation.

Two particles each with mass, m, and are fixed at the ends of a thin rigid rod. The length of the rod is L. Find the moment of inertia when this object rotates relative to an axis that is perpendicular to the rod at (a) one end and (b) the center.

$$m_1 = m_2 = m$$

$$r_1 = 0, \quad r_2 = L$$



$$r_1 = \frac{L}{2}, \quad r_2 = \frac{L}{2}$$

(a) $I = \sum_i (mr^2)_i = m_1 r_1^2 + m_2 r_2^2 = m(0)^2 + m(L)^2 = mL^2$

(b) $I = \sum_i (mr^2)_i = m_1 r_1^2 + m_2 r_2^2 = m(L/2)^2 + m(L/2)^2 = \frac{1}{2} mL^2$