Chapter 7

Simple Harmonic Motion

continued

FREQUENCY OF VIBRATION

$$\sum F_x = ma_x$$

$$-kx = ma_x$$

$$x = A\cos\omega t$$

$$a_x = -A\omega^2\cos\omega t$$

$$-Ak\cos\omega t = -Am\omega^2\cos\omega t$$

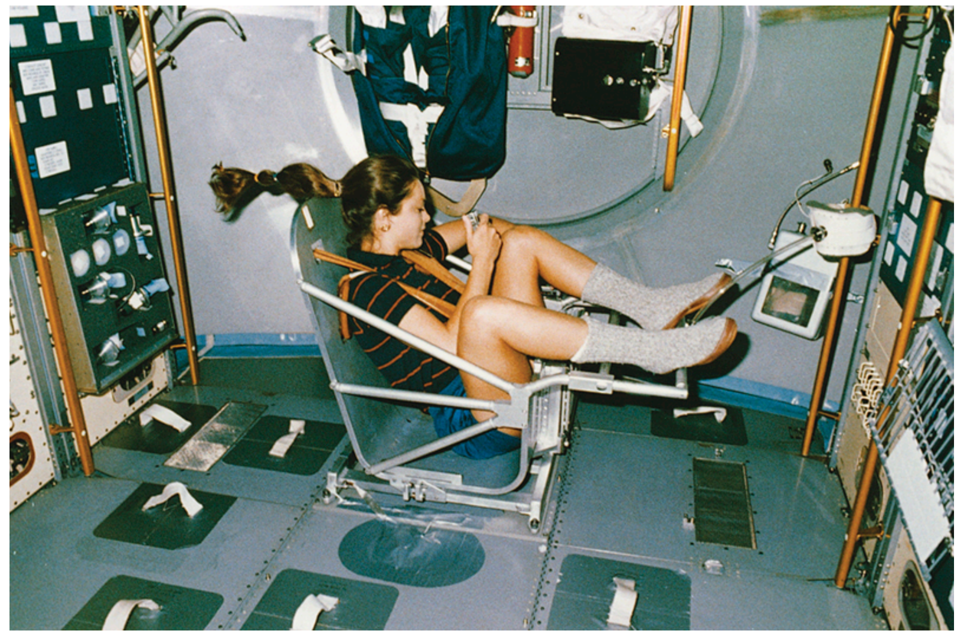
$$k = m\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

angular frequency for oscillations of a mass (m) on a spring (k)

Example A Body Mass Measurement Device

The device consists of a spring-mounted chair in which the astronaut sits. The spring has a spring constant of 606 N/m, and the mass of the chair is 12.0 kg. The measured period is 2.41 s. Find the mass of the astronaut.



spring constant: $k = 606\,\mathrm{N/m}$

chair mass: $m_{\text{chair}} = 12.0\,\mathrm{kg}$

oscillation period: $T = 2.41\,\mathrm{s}$

$$\omega = \sqrt{\frac{k}{m_{\text{total}}}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$m_{\text{total}} = \frac{k}{\omega^2} = \frac{kT^2}{4\pi^2} = 89.2\ \mathrm{kg}$$

$$m_{\text{astro}} = m_{\text{total}} - m_{\text{chair}} = 77.2\,\mathrm{kg}$$

Summary: spring constants & oscillations

Hooke's Law $F_A = kx$ Displacement proportional to applied force

Oscillations $\omega = \sqrt{\frac{k}{m}}$ Angular frequency $(\omega = 2\pi f = 2\pi / T)$

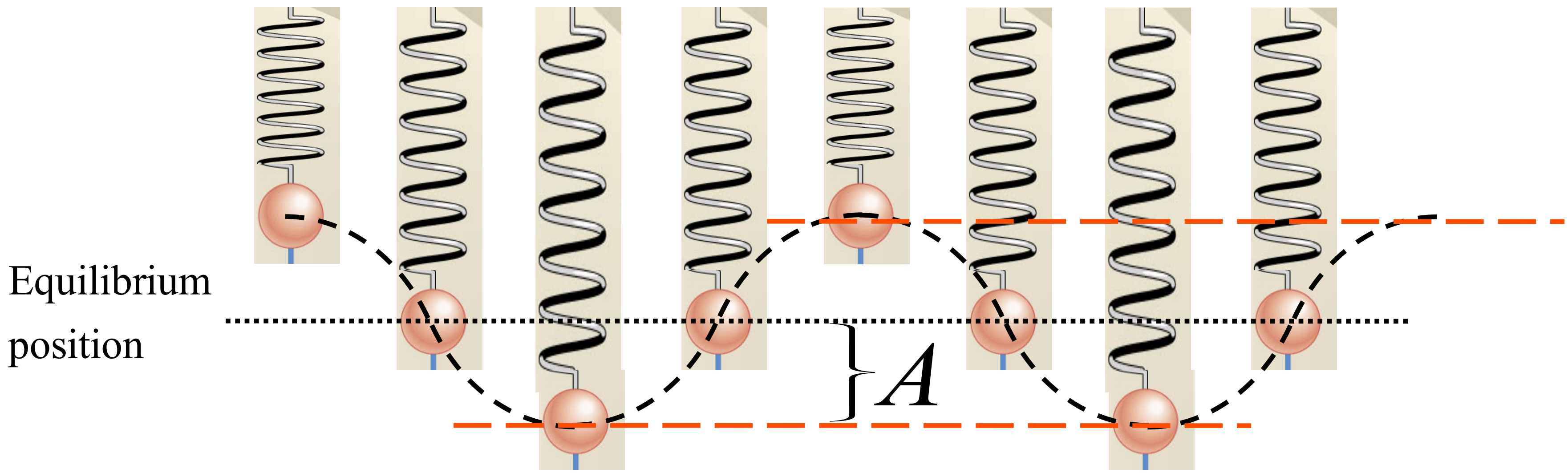
position: $x = A\cos(\omega t)$

velocity: $v_x = -\underbrace{A\omega}_{v_{\max}}\sin(\omega t)$

acceleration: $a_x = -\underbrace{A\omega^2}_{a_{\max}}\cos\omega t$

7.3 Energy in Simple Harmonic Motion

Consider this motion taking place far from the Earth



Speed maximum at equilibrium position

Energy all in kinetic energy: $E = \frac{1}{2} m v_{max}^2 = \frac{1}{2} m A^2 \omega^2$

At highest and lowest point energy is

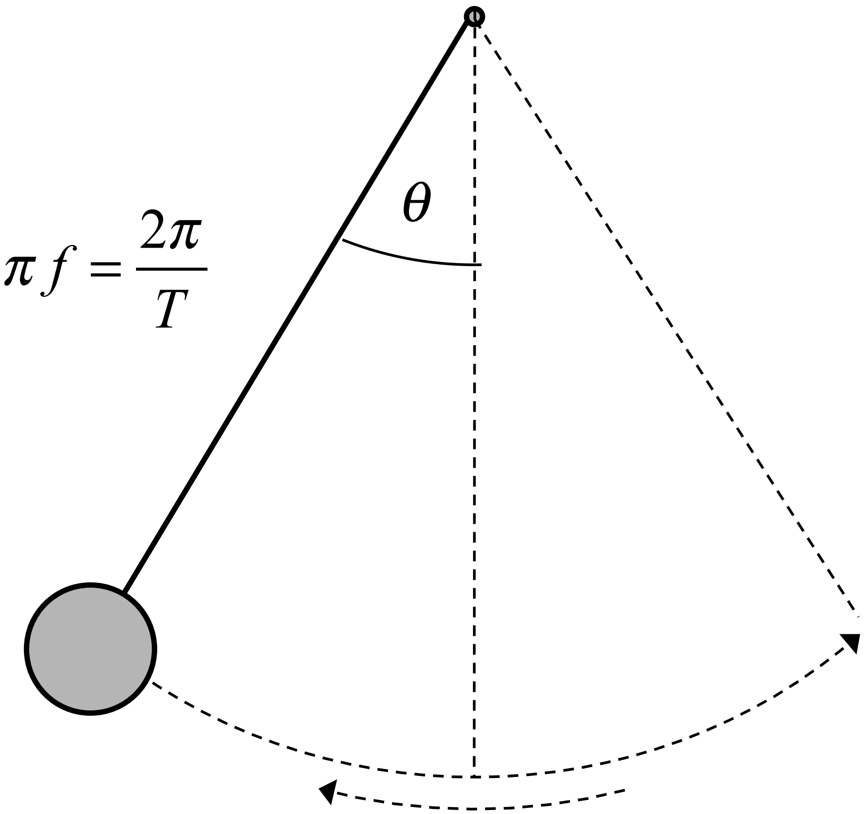
all in spring potential energy: $U_S = \frac{1}{2} k A^2 = E_{Total}$

At intermediate points total energy

$E_{Total} = \frac{1}{2} k A^2 = K + U_S = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$

7.5 The Pendulum

A ***simple pendulum*** consists of
a particle attached to a frictionless
pivot by a cable of negligible mass.



Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\omega = \sqrt{\frac{g}{L}} \quad \text{(small angles only)}$$

$$I = mL^2$$

$$\frac{1}{L} = \frac{mL}{I}$$

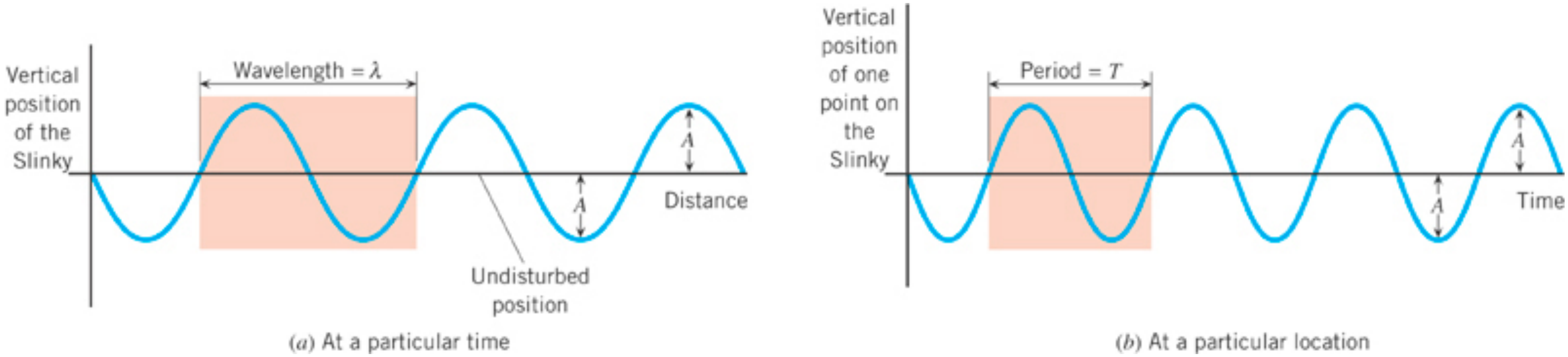
$$\omega = \sqrt{\frac{mgL}{I}} \quad \text{(small angles only)}$$

Works for objects with moment of inertia, I
and distance to center of mass, L_{CM}

Chapter 11

Waves & Sound

11.2 Periodic Waves



In the drawing, one ***cycle*** is shaded in color.

The ***amplitude*** A is the maximum excursion of a particle of the medium from the particles undisturbed position.

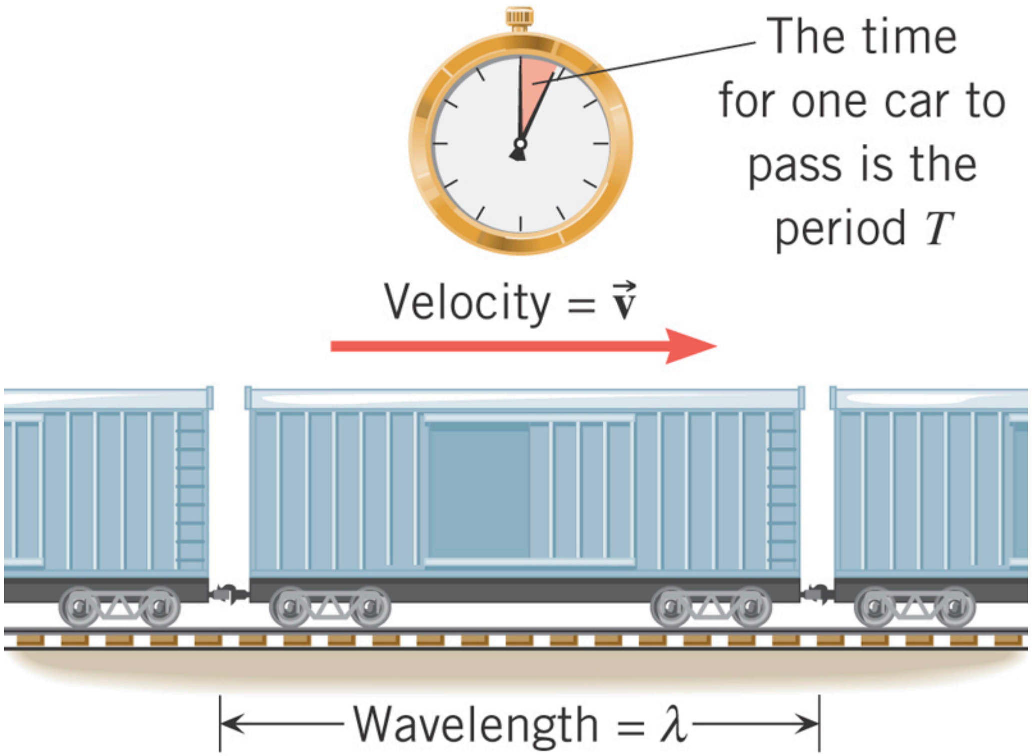
The ***wavelength*** is the horizontal length of one cycle of the wave.

The ***period*** is the time required for one complete cycle.

The ***frequency*** is related to the period and has units of Hz, or s^{-1}.

$$f = \frac{1}{T}$$

11.2 Periodic Waves



$$vT = \lambda; \quad f = \frac{1}{T}$$

$$v = \frac{\lambda}{T} = f\lambda \quad \Rightarrow \quad \lambda = \frac{v}{f}$$

Example: **The Wavelengths of Radio Waves**

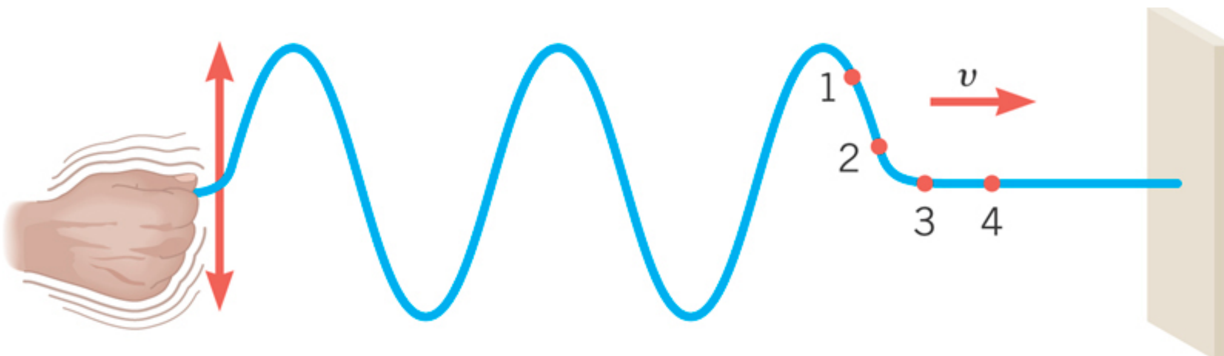
AM and FM radio waves are transverse waves consisting of electric and magnetic field disturbances traveling at a speed of 3.00×10^{8} m/s. A station broadcasts AM radio waves whose frequency is 1230×10^{3} Hz and an FM radio wave whose frequency is 91.9×10^{6} Hz. Find the distance between adjacent crests in each wave.

$$\lambda_{\mathrm{AM}} = \frac{v}{f} = \frac{3.00\times 10^{8}\ \mathrm{m/s}}{1230\times 10^{3}\mathrm{Hz}} = 244\ \mathrm{m}$$

$$\lambda_{\mathrm{FM}} = \frac{v}{f} = \frac{3.00\times 10^{8}\ \mathrm{m/s}}{91.9\times 10^{6}\mathrm{Hz}} = 3.26\ \mathrm{m}$$

11.3 The Speed of a Wave on a String

The speed at which the wave moves to the right depends on how quickly one particle of the string is accelerated upward in response to the net pulling force.



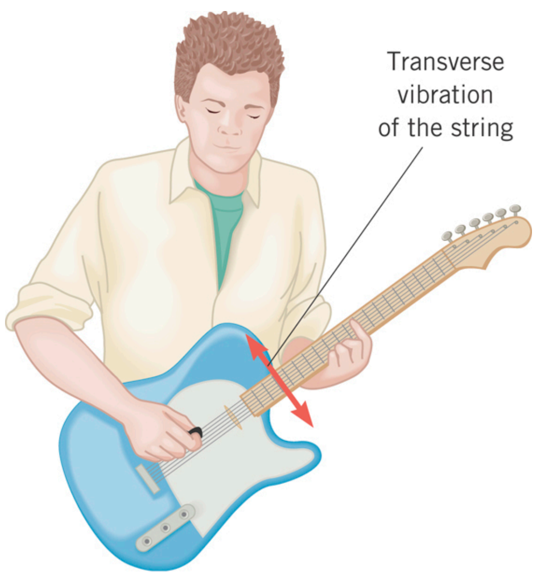
$$v = \sqrt{\frac{T}{\mu}}$$

Tension: T

Linear mass density: $\mu = m/L$

Example: **Waves Traveling on Guitar Strings**

Transverse waves travel on each string of an electric guitar after the string is plucked. The length of each string between its two fixed ends is 0.628 m, and the mass is 0.208 g for the highest pitched E string and 3.32 g for the lowest pitched E string. Each string is under a tension of 226 N. Find the speeds of the waves on the two strings.



High E

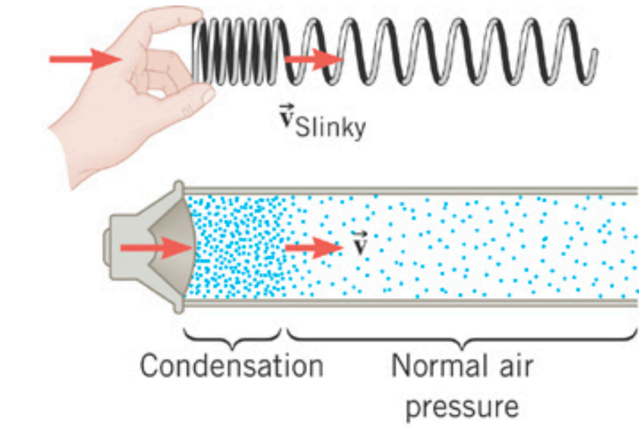
$$v = \sqrt{\frac{T}{m/L}} = \sqrt{\frac{226\text{ N}}{\left(0.208\times 10^{-3}\text{kg}\right)/\left(0.628\text{ m}\right)}} = 826\,\text{m/s}$$

Low E

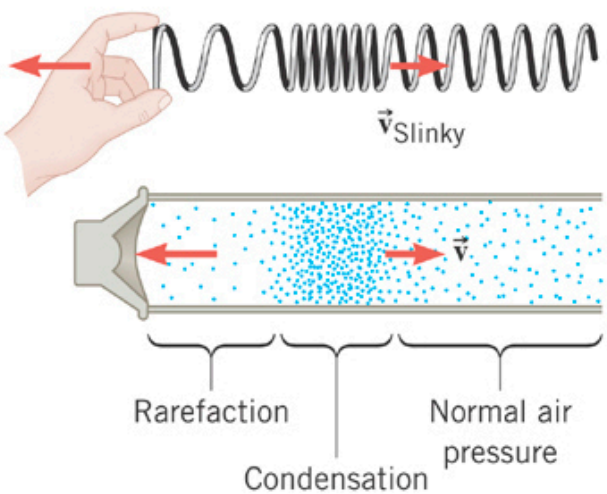
$$v = \sqrt{\frac{T}{m/L}} = \sqrt{\frac{226\text{ N}}{\left(3.32\times 10^{-3}\text{kg}\right)/\left(0.628\text{ m}\right)}} = 207\,\text{m/s}$$

11.3 The Nature of Sound Waves

LONGITUDINAL SOUND WAVES

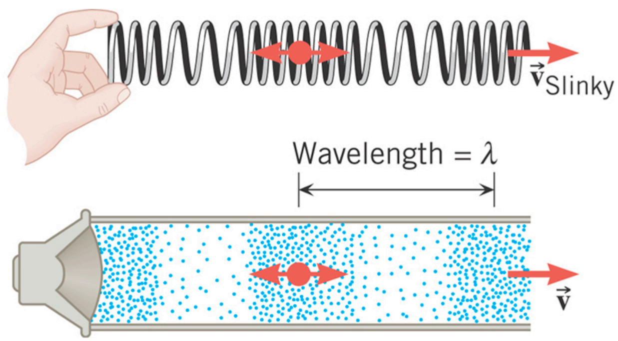


(a)



(b)

The distance between adjacent condensations is equal to the wavelength of the sound wave.



THE FREQUENCY OF A SOUND WAVE

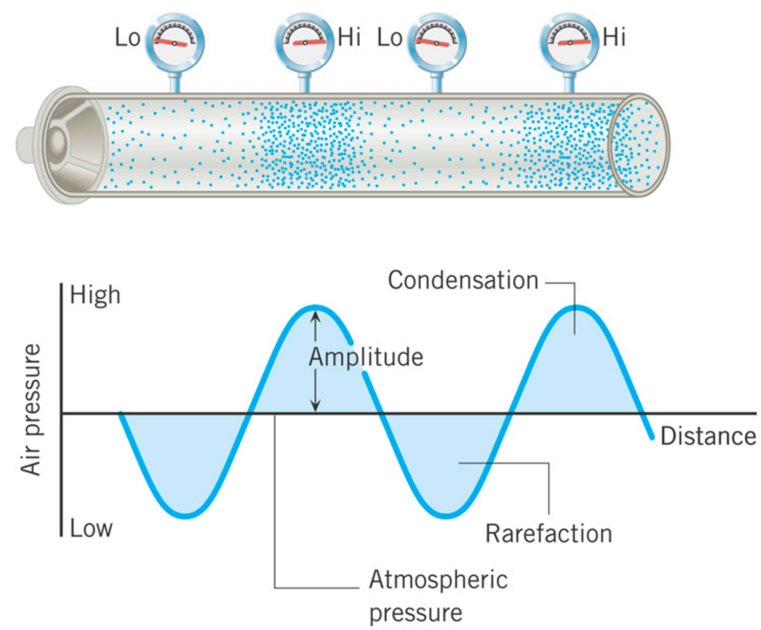
The ***frequency*** is the number of cycles per second.

A sound with a single frequency is called a ***pure tone***.

The brain interprets the frequency in terms of the subjective quality called ***pitch***.

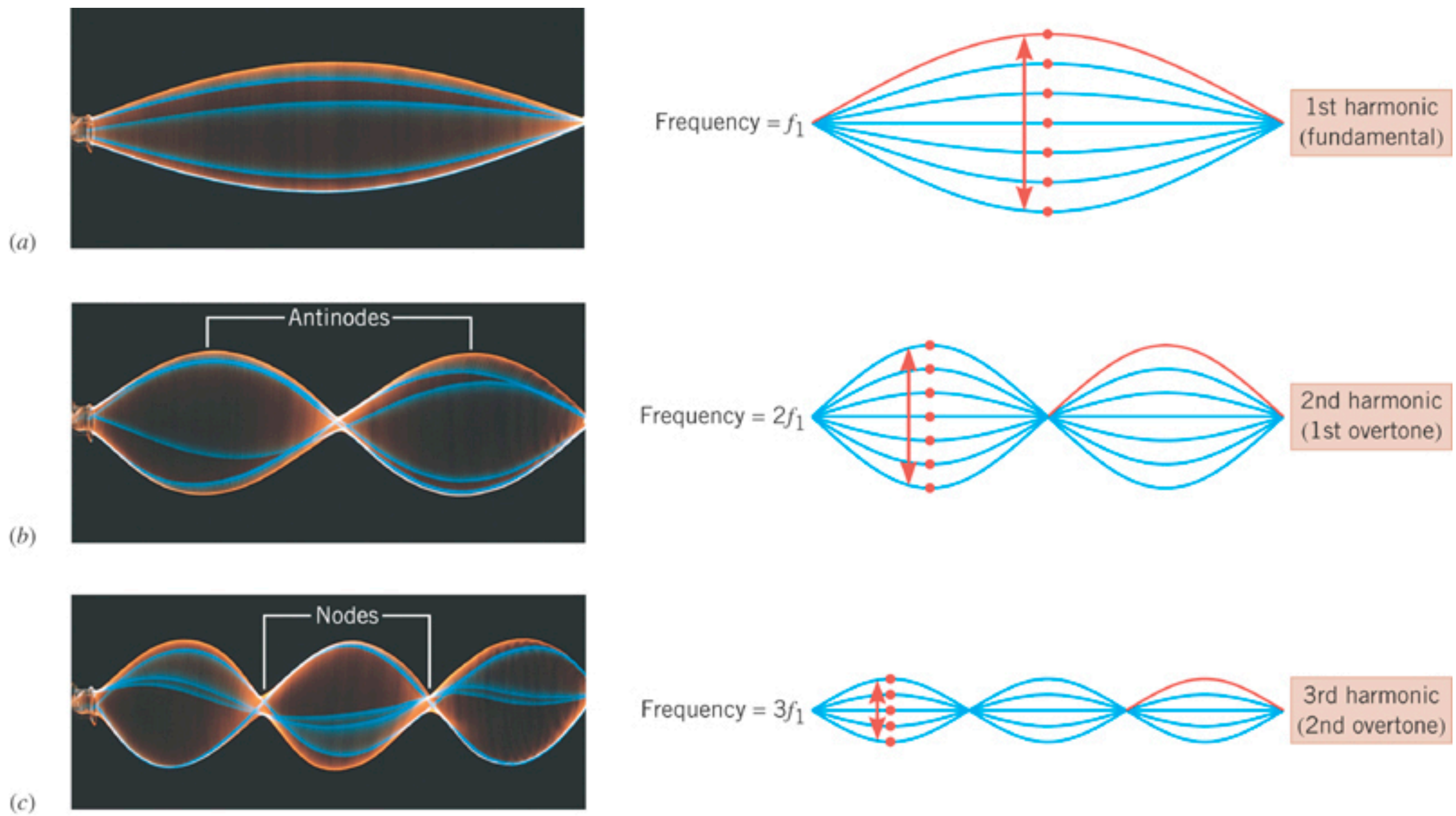
THE AMPLITUDE OF A SOUND WAVE

Loudness is an attribute of a sound that depends primarily on the pressure amplitude of the wave.

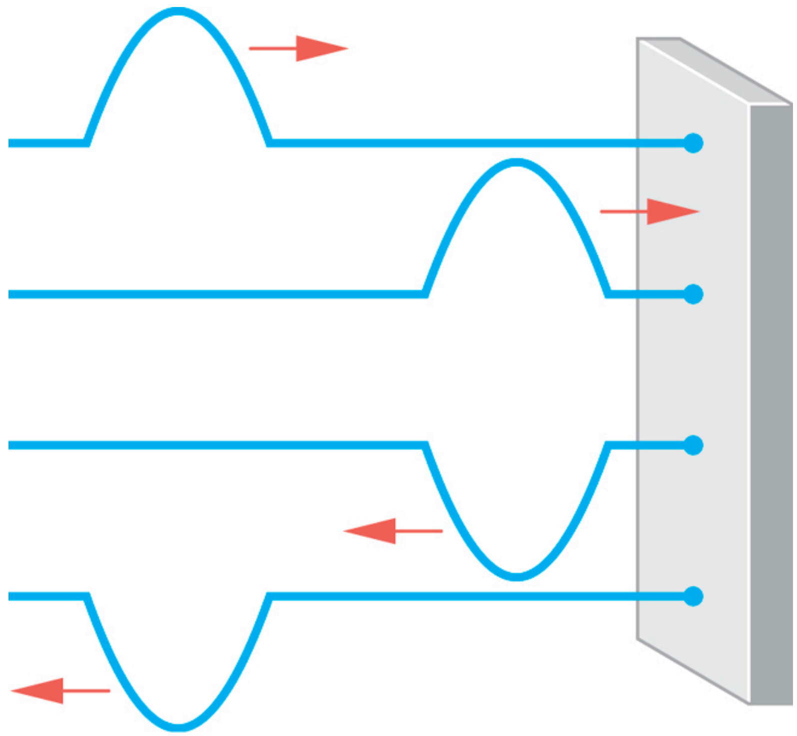


11.3 Transverse Standing Waves

Transverse standing wave patters.



11.3 ***Transverse Standing Waves***

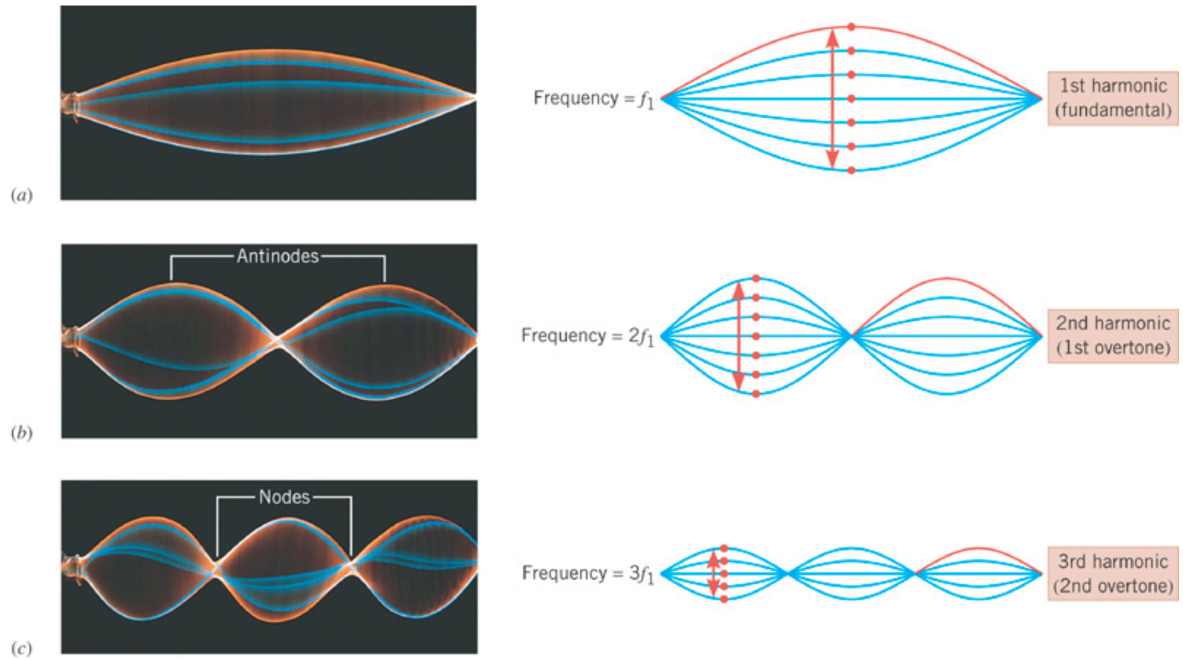


In reflecting from the wall, a forward-traveling half-cycle becomes a backward-traveling half-cycle that is inverted.

Unless the timing is right, the newly formed and reflected cycles tend to offset one another.

Repeated reinforcement between newly created and reflected cycles causes a large amplitude standing wave to develop.

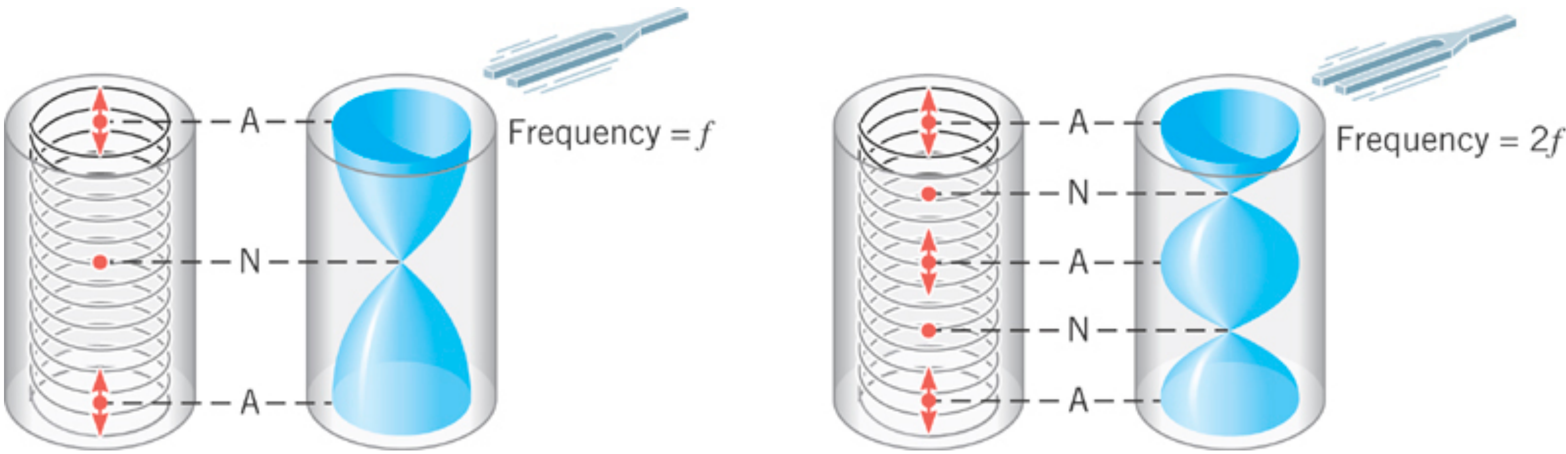
11.3 Transverse Standing Waves



String fixed at both ends

$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

11.3 ***Longitudinal Standing Waves***



Tube open at both ends

$$f_n = n\left(\frac{v}{2L}\right)$$

$$n = 1,2,3,4,\ldots$$

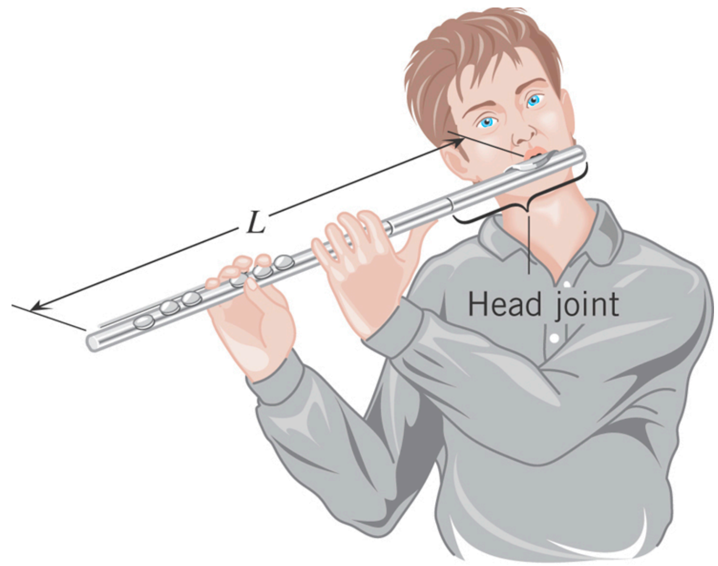
\# of Nodes

Example: Playing a Flute

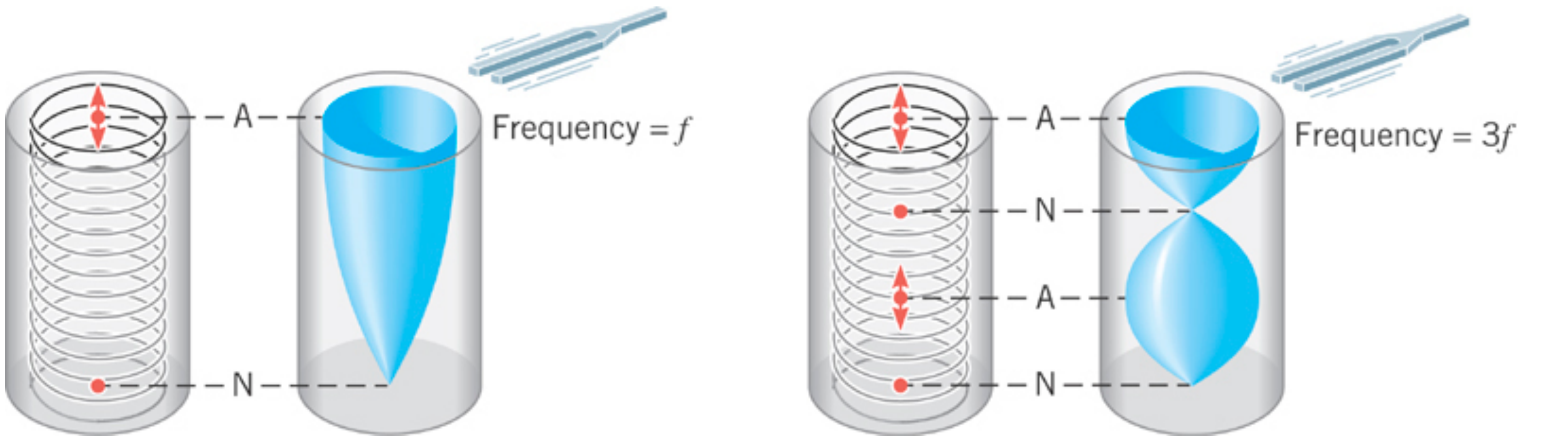
When all the holes are closed on one type of flute, the lowest note it can sound is middle C (261.6 Hz). If the speed of sound is 343 m/s, and the flute is assumed to be a cylinder open at both ends, determine the distance L.

$$f_n = n\left(\frac{v}{2L}\right) \qquad n = 1, 2, 3, 4, \ldots$$

$$L = \frac{nv}{2f_n} = \frac{1(343\,\mathrm{m/s})}{2(261.6\,\mathrm{Hz})} = 0.656\,\mathrm{m}$$



11.3 Longitudinal Standing Waves



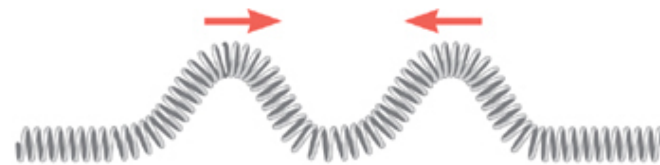
Tube open at one end

$$f_n = n\left(\frac{v}{4L}\right) \qquad n = 1,3,5,\ldots$$

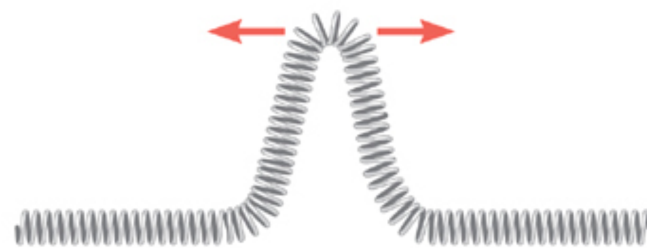
$$n \text{ is } 2\times\text{Nodes} - 1$$

11.3 The Principle of Linear Superposition

When the pulses merge, the Slinky assumes a shape that is the sum of the shapes of the individual pulses.



(*a*) Overlap begins

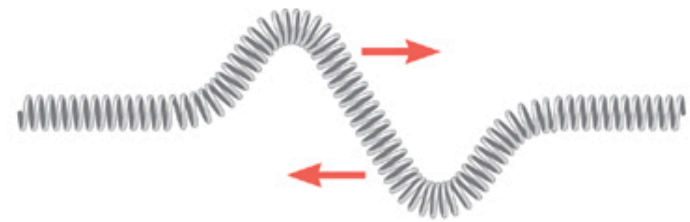


(*b*) Total overlap; the Slinky has twice the height of either pulse

(*c*) The receding pulses

11.3 The Principle of Linear Superposition

When the pulses merge, the Slinky assumes a shape that is the sum of the shapes of the individual pulses.



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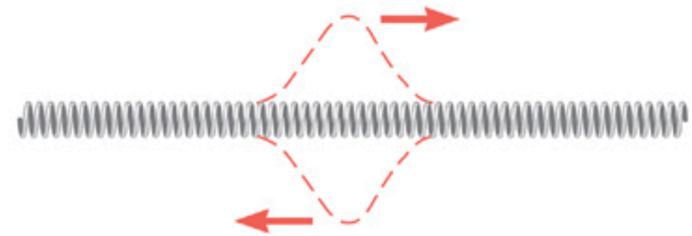
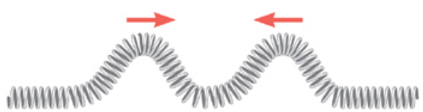


(*b*) Total overlap

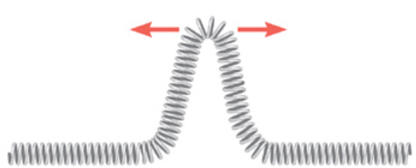
(*c*) The receding pulses

THE PRINCIPLE OF LINEAR SUPERPOSITION

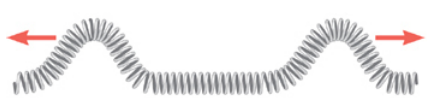
When two or more waves are present simultaneously at the same place, the resultant disturbance is the sum of the disturbances from the individual waves.



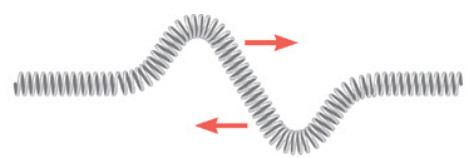
(*a*) Overlap begins



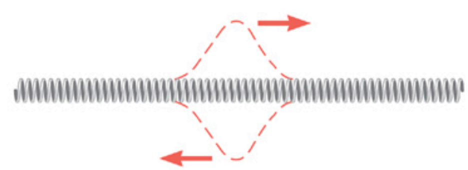
(*b*) Total overlap; the Slinky has twice the height of either pulse



(*c*) The receding pulses



(*a*) Overlap begins

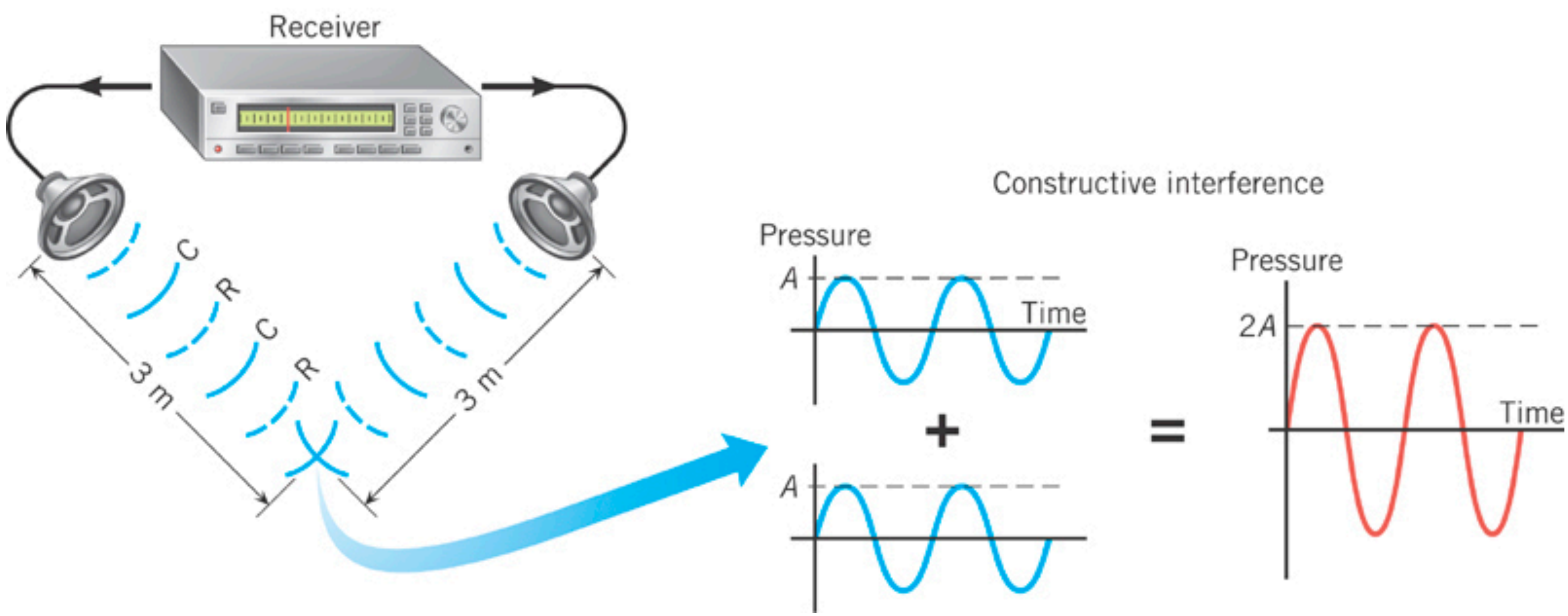


(*b*) Total overlap

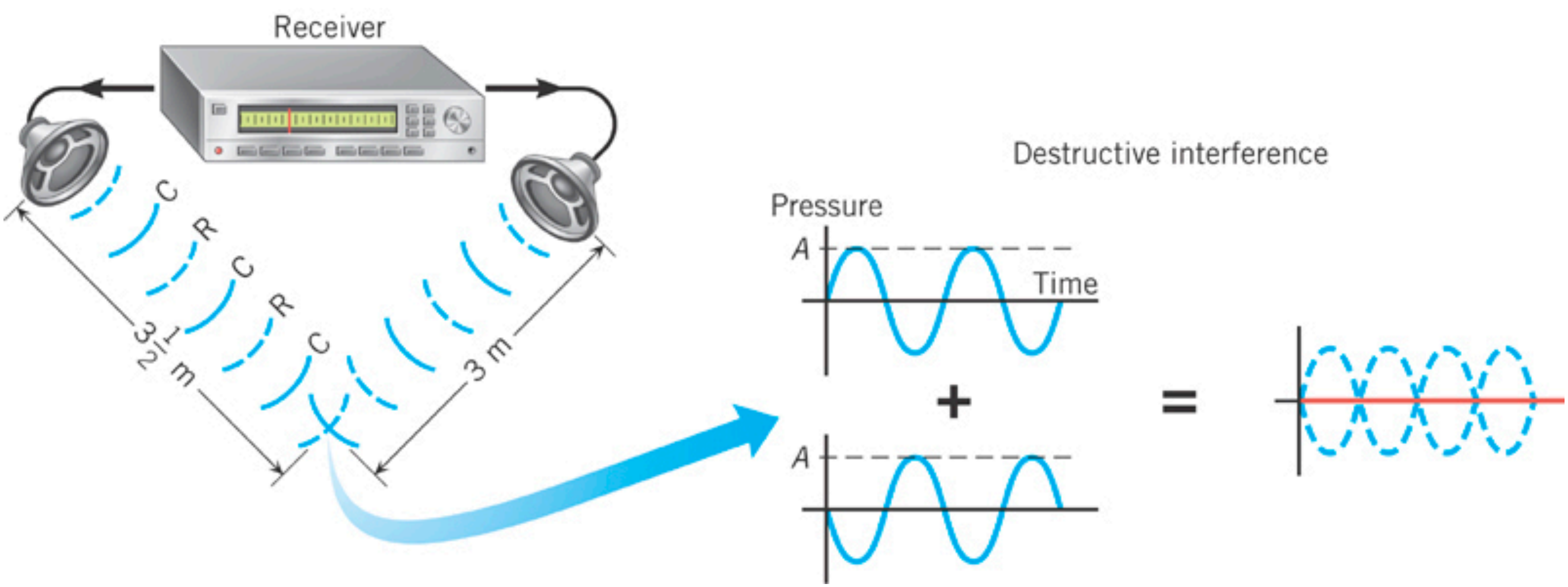
(*c*) The receding pulses

11.3 Constructive and Destructive Interference of Sound Waves

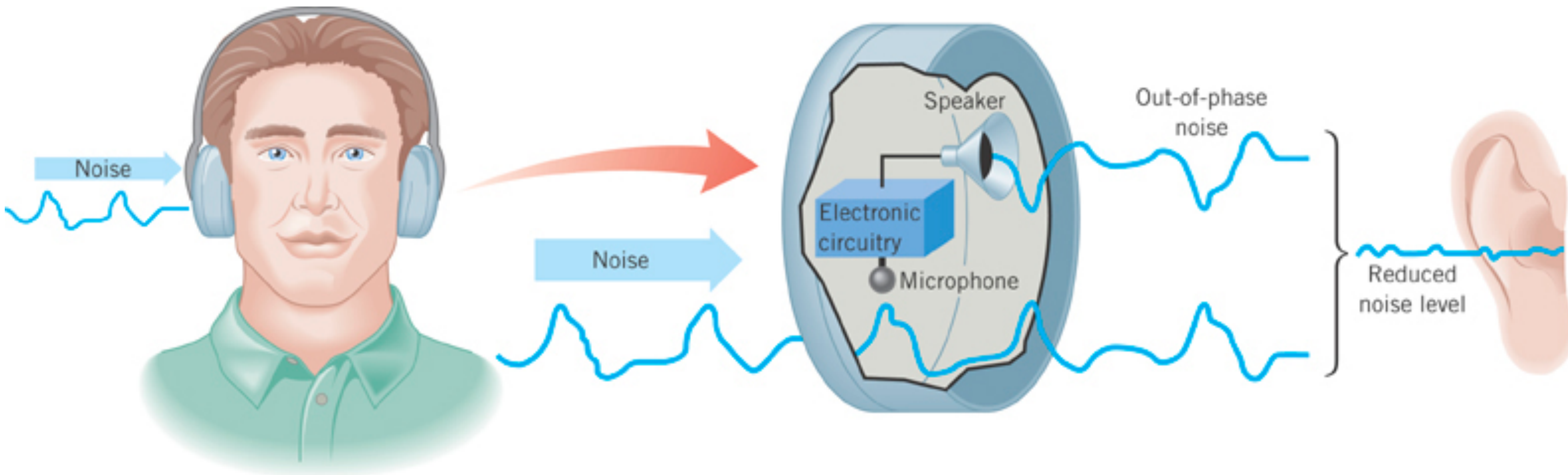
When two waves always meet condensation-to-condensation and rarefaction-to-rarefaction, they are said to be ***exactly in phase*** and to exhibit ***constructive interference.***



When two waves always meet condensation-to-rarefaction, they are said to be ***exactly out of phase*** and to exhibit ***destructive interference.***



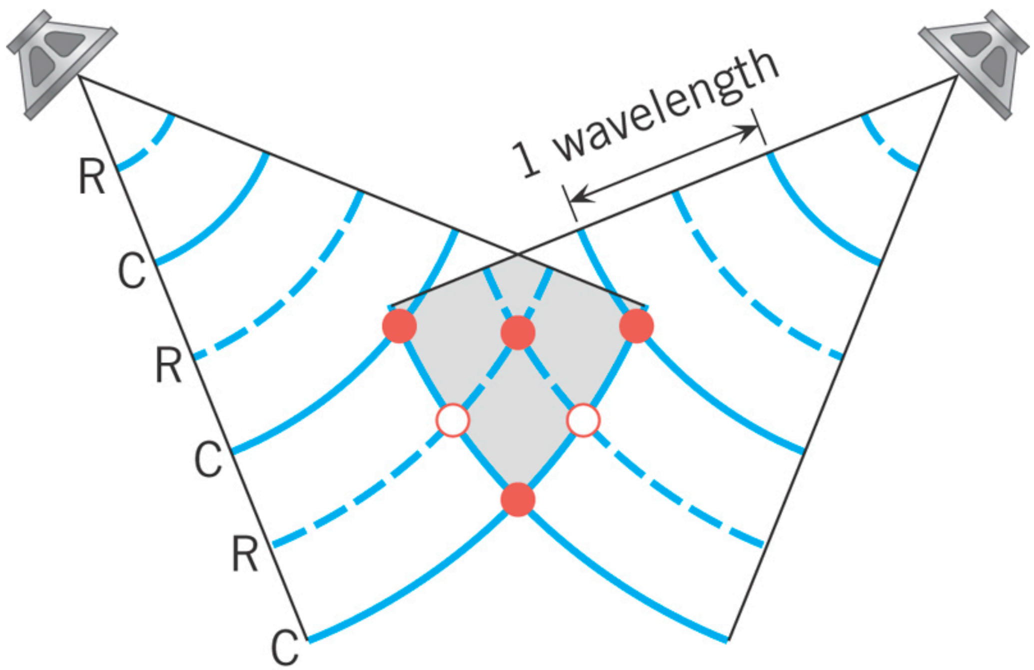
11.3 Constructive and Destructive Interference of Sound Waves



11.3 Constructive and Destructive Interference of Sound Waves

If the wave patters do not shift relative to one another as time passes, the sources are said to be ***coherent***.

For two wave sources vibrating in phase, a difference in path lengths that is zero or an integer number (1, 2, 3, . .) of wavelengths leads to constructive interference; a difference in path lengths that is a half-integer number (½ , 1 ½, 2 ½, . .) of wavelengths leads to destructive interference.



11.3 Sound Intensity

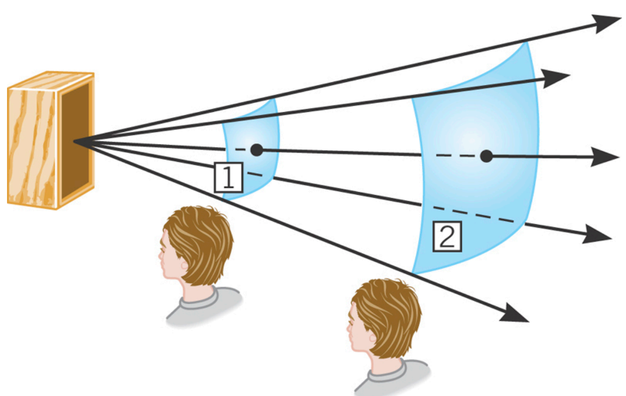
The amount of energy transported per second is called the ***power*** of the wave.

The ***sound intensity*** is defined as the power that passes perpendicularly through a surface divided by the area of that surface.

$$I = P/A; \quad \text{power: } P \ \ \text{(watts)}$$

Example: Sound Intensities

12×10^{-5} W of sound power passed through the surfaces labeled 1 and 2. The areas of these surfaces are $4.0 m^2$ and $12 m^2$. Determine the sound intensity at each surface.



$$I_1 = \frac{P}{A_1} = \frac{12 \times 10^{-5}\,\mathrm{W}}{4.0\,\mathrm{m}^2} = 3.0 \times 10^{-5}\,\mathrm{W/m^2}$$

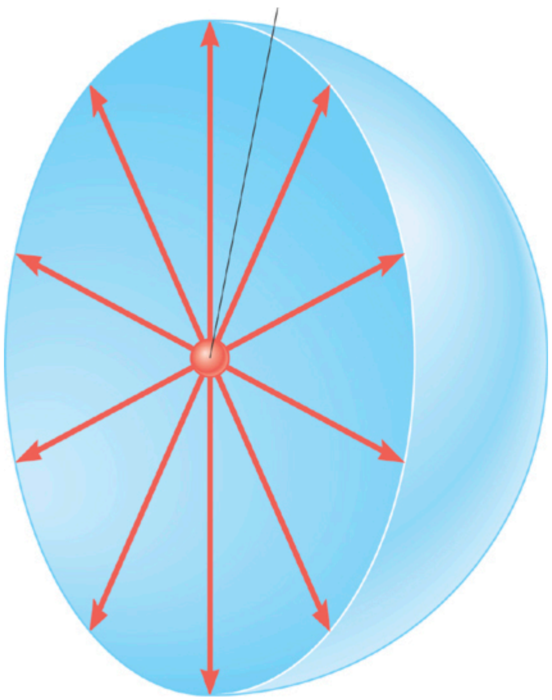
$$I_2 = \frac{P}{A_2} = \frac{12 \times 10^{-5}\,\mathrm{W}}{12\,\mathrm{m}^2} = 1.0 \times 10^{-5}\,\mathrm{W/m^2}$$

11.3 Sound Intensity

For a 1000 Hz tone, the smallest sound intensity that the human ear can detect is about 1×10^{-12} W/m^2. This intensity is called the ***threshold of hearing***.

On the other extreme, continuous exposure to intensities greater than 1W/m^2 can be painful.

If the source emits sound *uniformly in all directions,* the intensity depends on the distance from the source in a simple way.



$$I = \frac{P}{4\pi r^2}$$

Intensity depends inversely on the square of the distance from the source.

11.3 Decibels

The ***decibel*** (dB) is a measurement unit used when comparing two sound Intensities.

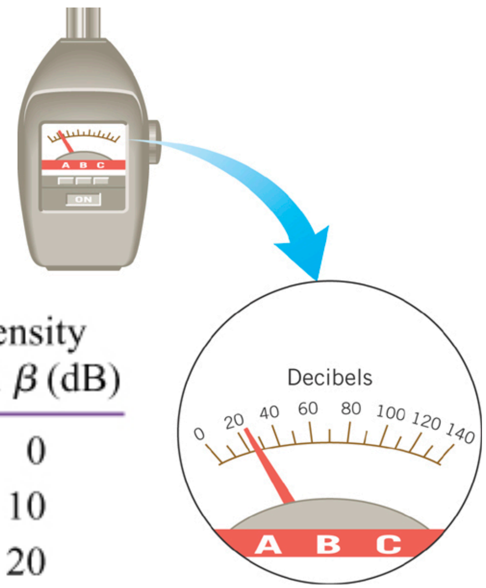
Human hearing mechanism responds to sound ***intensity level***, logarithmically.

$$\beta = (10\ \text{dB})\log\left(\frac{I}{I_o}\right)$$

Note that log(1) = 0

dB (decibel)

$$I_o = 1.00\times 10^{-12}\ \text{W/m}^2$$



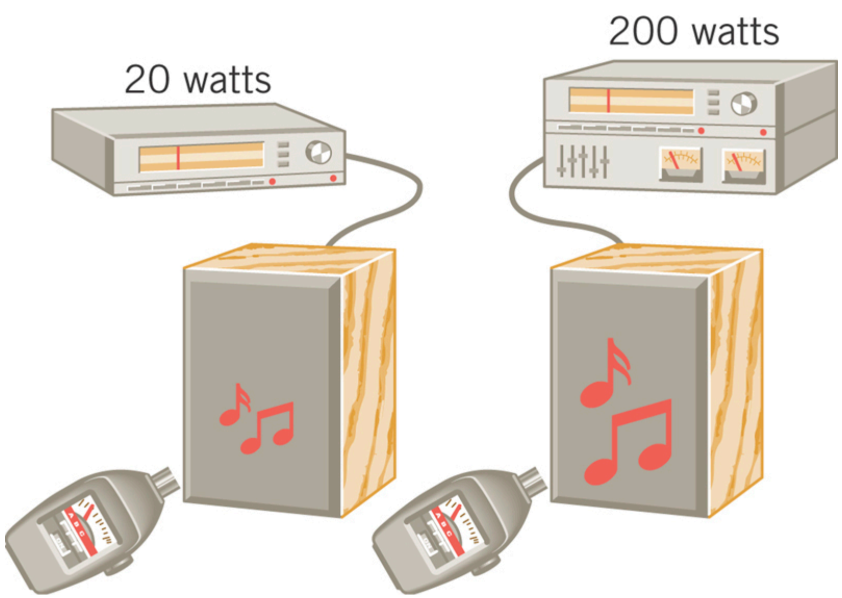
	Intensity I (W/m^2)	Intensity Level β (dB)
Threshold of hearing	1.0×10^{-12}	0
Rustling leaves	1.0×10^{-11}	10
Whisper	1.0×10^{-10}	20
Normal conversation (1 meter)	3.2×10^{-6}	65
Inside car in city traffic	1.0×10^{-4}	80
Car without muffler	1.0×10^{-2}	100
Live rock concert	1.0	120
Threshold of pain	10	130

11.3 Decibels

Example: Comparing Sound Intensities

Audio system 1 produces a sound intensity level of 90.0 dB, and system 2 produces an intensity level of 93.0 dB. Determine the ratio of intensities.

$$\beta = (10\text{ dB})\log\left(\frac{I}{I_o}\right)$$



$$90\text{ dB} = (10\text{ dB})\log(I/I_o)$$
$$\log(I/I_o) = 9;$$
$$I = I_o \times 10^9 = (1\times10^{-12}\text{ W/m}^2)\times10^9$$
$$= 1\times10^{-3}\text{ W/m}^2$$

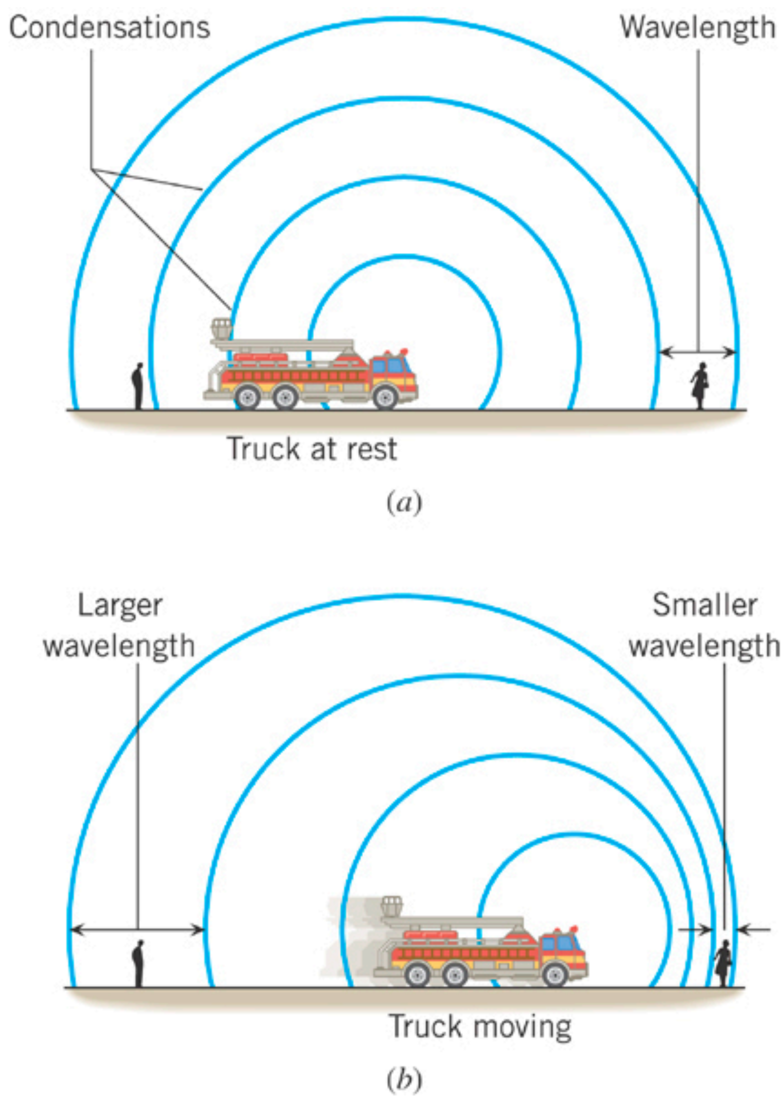
$$93\text{ dB} = (10\text{ dB})\log(I/I_o)$$
$$\log(I/I_o) = 9.3;$$
$$I = I_o \times 10^{9.3} = (1\times10^{-12}\text{ W/m}^2)\times10^{9.3}$$
$$= 1\times10^{-2.7}\text{ W/m}^2 = 1\times10^{-3}(10^{0.3})\text{W/m}^2$$
$$= 1\times10^{-3}(2)\text{W/m}^2 = 2\times10^{-3}\text{W/m}^2$$

$$93\text{dB} = 90\text{dB} + 3\text{dB}$$

Adding 3dB results in a factor of 2

$$3\text{ dB} = (10\text{dB})\log(I_2/I_1)$$
$$0.3 = \log(I_2/I_1);$$
$$I_2 = 10^{0.3} I_1 = 2 I_1$$

11.5 The Doppler Effect



The ***Doppler effect*** is the change in frequency or pitch of the sound detected by an observer because the sound source and the observer have different velocities with respect to the medium of sound propagation.

SOURCE (*s*) MOVING AT v_s TOWARD A STATIONARY OBSERVER (*obs*)

$$f_{obs} = f_s\left(\frac{1}{1 - v_s/v}\right)$$

SOURCE (*s*) MOVING AT v_s AWAY A FROM STATIONARY OBSERVER (*obs*)

$$f_{obs} = f_s\left(\frac{1}{1 + v_s/v}\right)$$

Summary: Waves and Sound

Periodic Waves

$$v = \lambda f$$

v : velocity of wave

λ : wavelength

f : frequency

Standing Waves

$$\lambda = \frac{2L}{n} \quad (n \text{ anti-nodes})$$

String wave speed

$$v = \sqrt{T/\mu}$$

T : Tension

μ : lin. mass density

Sound Intensity

$$I = \frac{P}{A}; \quad \beta = (10\text{dB})\log\frac{I}{I_0} \quad (I_0 = 1.0\times 10^{-12}\ \text{W/m}^2)$$

β units are decibels (dB)

Doppler Effect

(Observer at rest)

$$f_{obs} = f_s\left(\frac{1}{1 \mp v_s/v}\right)$$

− source approaching

+ source receding