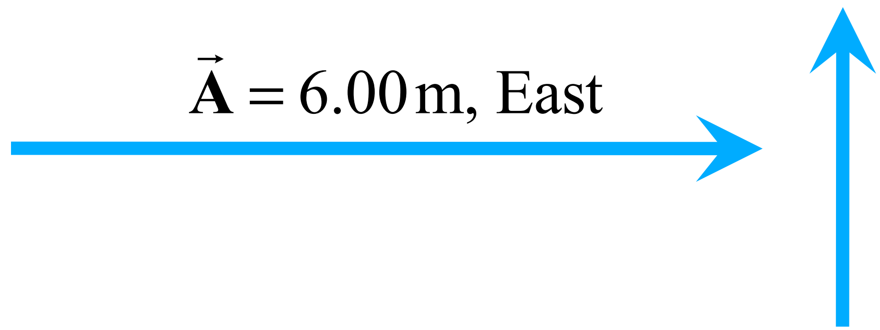
Chapter 3

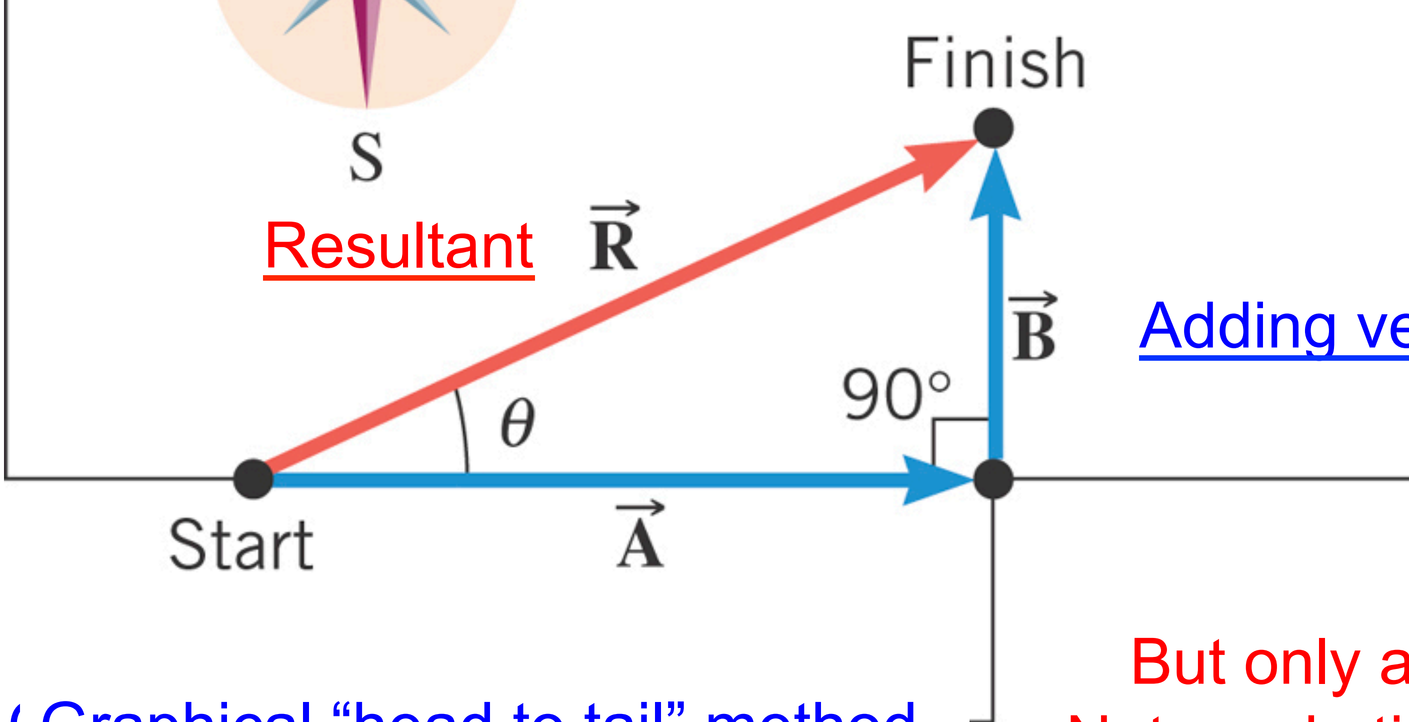
Kinematics in Two Dimensions

continued

3.2 Scalars and Vectors   (Vector Addition and Subtraction)

Two displacement vectors





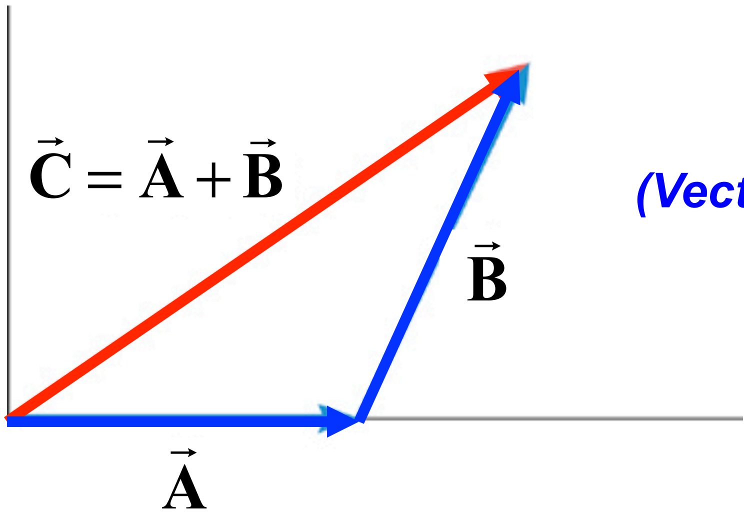
Resultant

Adding vectors together

Graphical “head to tail” method

But only a visual aid
Not a substitute for math

3.2 Scalars and Vectors (Vector Addition and Subtraction)



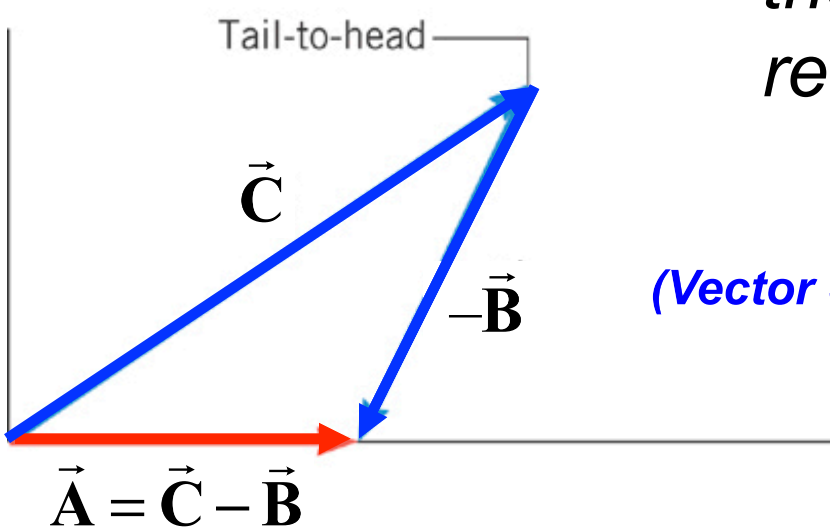
(a)

(Vector Addition)

Move B to other side

$$\vec{C} - \vec{B} = \vec{A}$$

When a vector is multiplied by -1, the magnitude of the vector remains the same, but the direction of the vector is reversed.



(b)

(Vector Subtraction)

Vector addition is associative.
Order of addition doesn't matter

3.2 Scalars and Vectors (Vector Addition and Subtraction)

Apply Pythagorean Theroem to get *R*

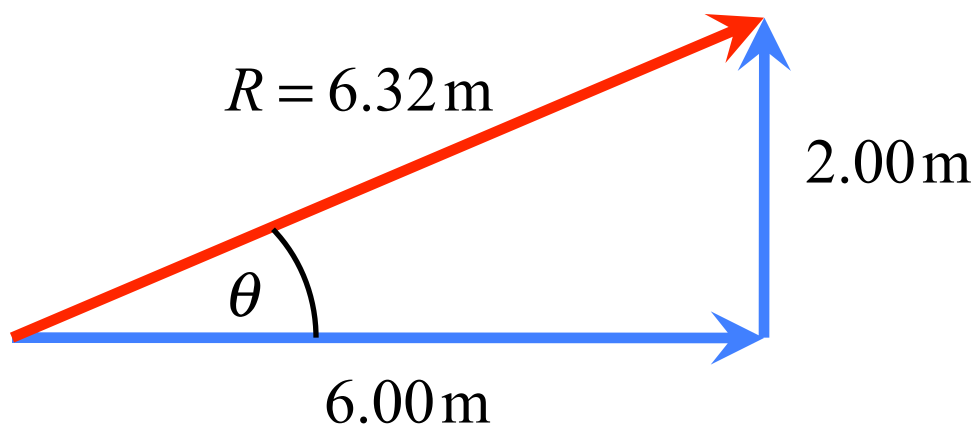
$$R = \sqrt{(2.00\,\text{m})^2 + (6.00\,\text{m})^2} = 6.32\text{m}$$

Use trigonometry to determine the angle

$$\tan\theta = 2.00/6.00$$

$$\text{tangent (angle)} = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\theta = \tan^{-1}(2.00/6.00) = 18.4^\circ$$



Also

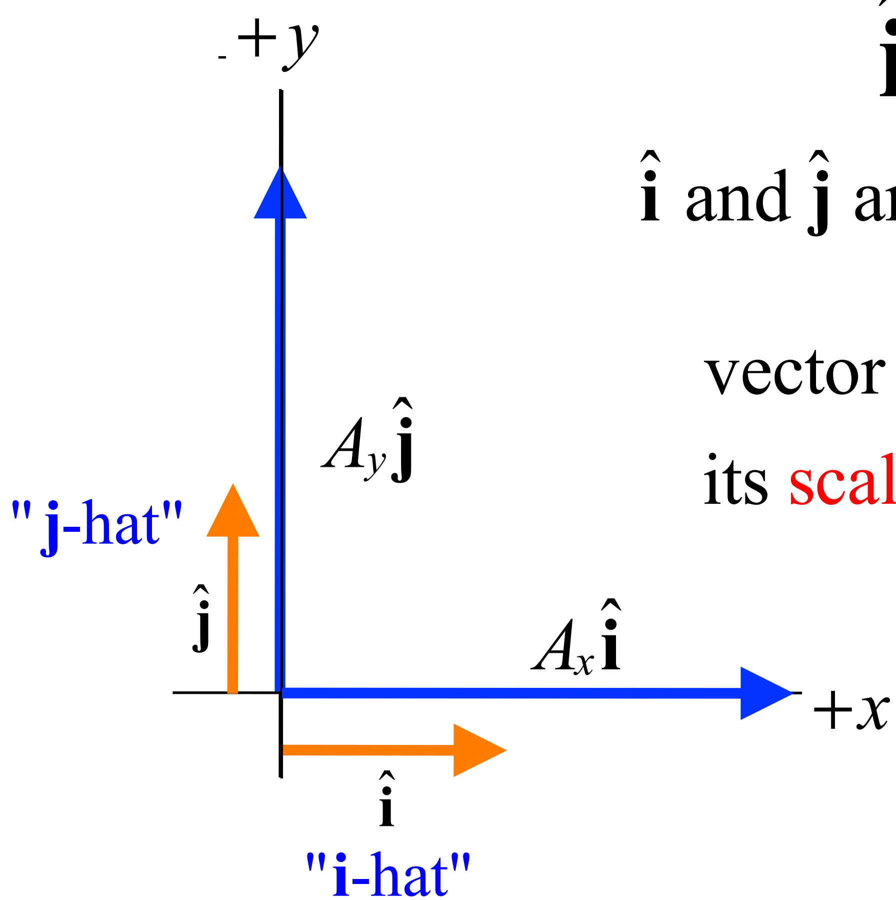
$$\theta = \sin^{-1}(2.00/6.32) = 18.4^\circ$$

Also

$$\theta = \cos^{-1}(6.00/6.32) = 18.4^\circ$$

3.2 Vector Addition and Subtraction (The Components of a Vector)

It is often easier to work with the **scalar components** of the vectors, rather than the component vectors themselves.



"**i**-hat" "**j**-hat"

$\hat{\mathbf{i}}$ $\hat{\mathbf{j}}$

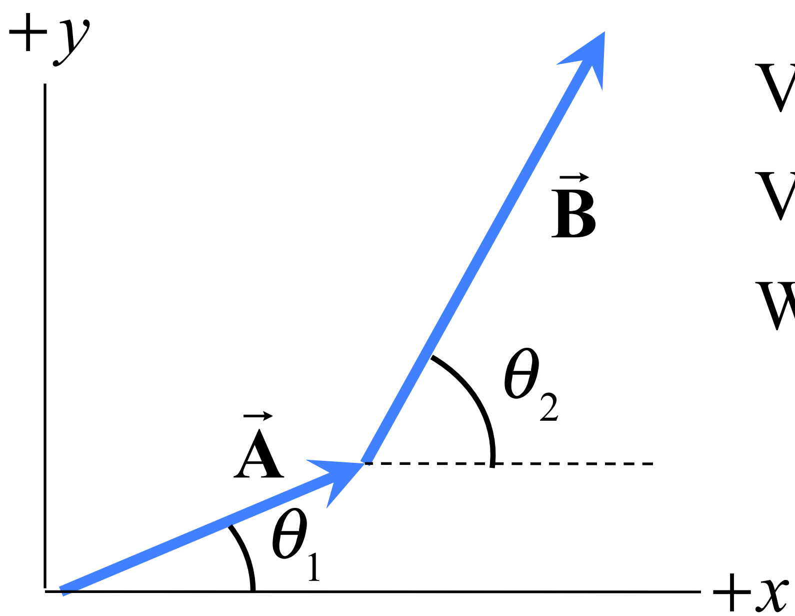
$\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are "unit vectors", magnitude is 1.

vector $\vec{\mathbf{A}}$ expressed in terms of its scalar components: $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$

A_x and A_y are the scalar components of the vector, $\vec{\mathbf{A}}$.

3.2 Addition of Vectors by Means of Components

Adding two vectors (not at right angles)



Vector $\vec{\mathbf{A}}$ has magnitude A and angle θ_1

Vector $\vec{\mathbf{B}}$ has magnitude B and angle θ_2

What is the vector $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$?

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} \qquad \vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + \mathrm{B}_y\hat{\mathbf{j}}$$

1) Determine components of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$: A_x, A_y and B_x, B_y

2) Add x-components to find $C_x = A_x + B_x$

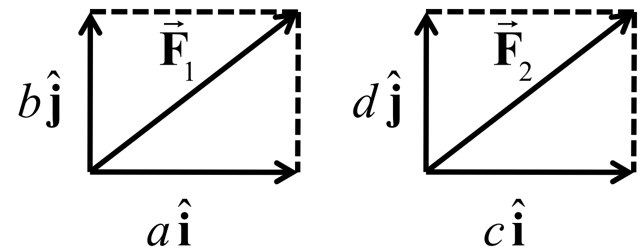
3) Add y-components to find $C_y = A_y + B_y$

4) Determine the magnitude and angle of vector $\vec{\mathbf{C}}$

magnitude $C = \sqrt{C_x^2 + C_y^2}$; $\quad \theta = \tan^{-1}(C_y / C_x)$

Clicker Question 3.3

$\vec{\mathbf{F}}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$, and $\vec{\mathbf{F}}_2 = c\hat{\mathbf{i}} + d\hat{\mathbf{j}}$.



What is the vector $\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$?

A) $(a+b+c+d)(\hat{\mathbf{i}}+\hat{\mathbf{j}})$

B) $(a+c)\hat{\mathbf{i}}+(b+d)\hat{\mathbf{j}}$

C) $(a+b)\hat{\mathbf{i}}+(c+d)\hat{\mathbf{j}}$

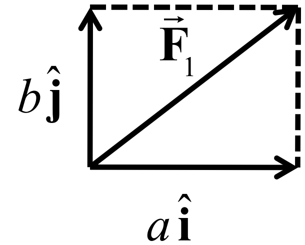
D) $(a-b)\hat{\mathbf{i}}+(c-d)\hat{\mathbf{j}}$

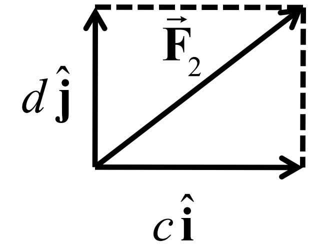
E) $(a+b)\hat{\mathbf{j}}+(c+d)\hat{\mathbf{i}}$

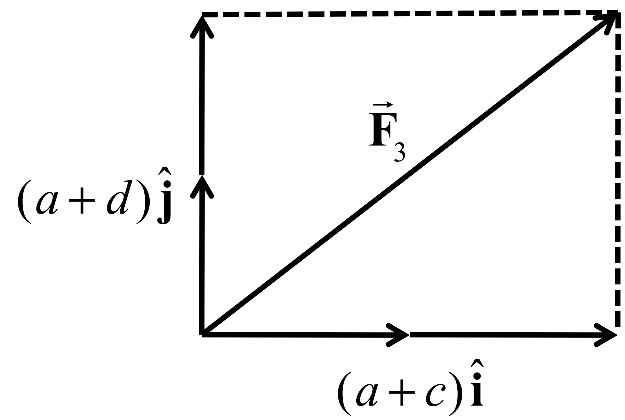
Clicker Question 3.3

$\vec{\mathbf{F}}_1 = a\,\hat{\mathbf{i}} + b\,\hat{\mathbf{j}}$, and $\vec{\mathbf{F}}_2 = c\,\hat{\mathbf{i}} + d\,\hat{\mathbf{j}}$.

What is the vector $\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$?







A) $(a+b+c+d)(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

B) $(a+c)\hat{\mathbf{i}} + (b+d)\hat{\mathbf{j}}$

C) $(a+b)\hat{\mathbf{i}} + (c+d)\hat{\mathbf{j}}$

D) $(a-b)\hat{\mathbf{i}} + (c-d)\hat{\mathbf{j}}$

E) $(a+b)\hat{\mathbf{j}} + (c+d)\hat{\mathbf{i}}$

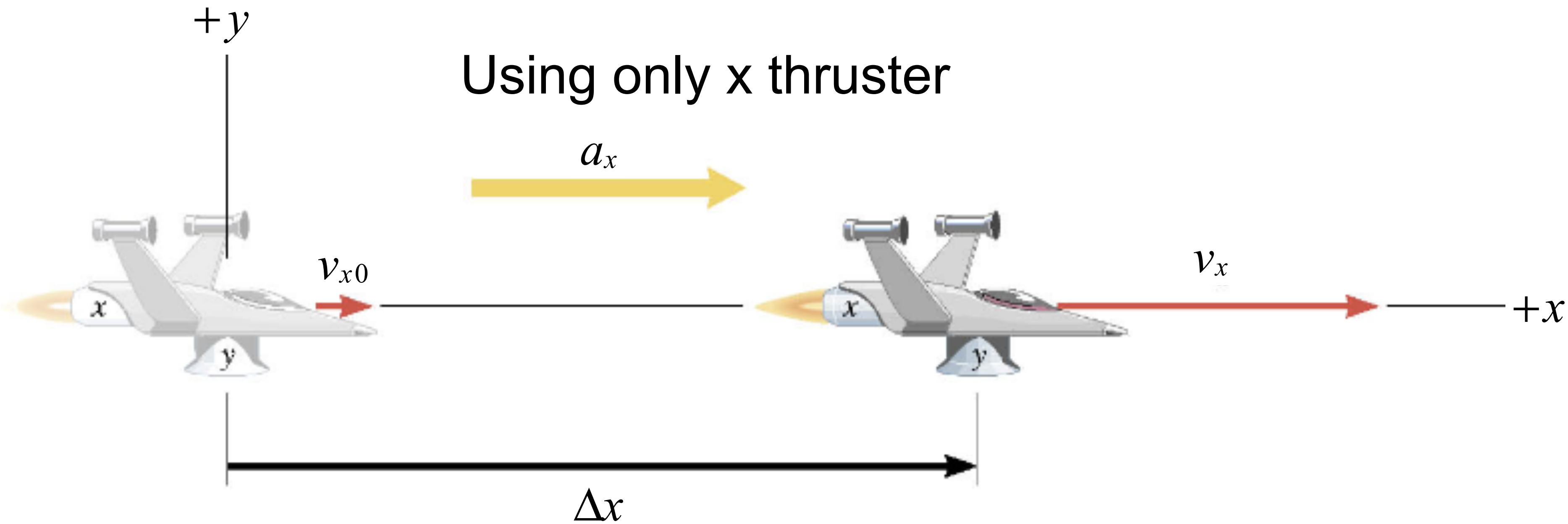
$$\begin{array}{rcccc} \vec{\mathbf{F}}_1 & = & a\,\hat{\mathbf{i}} & + & b\,\hat{\mathbf{j}} \\ +\vec{\mathbf{F}}_2 & = & c\,\hat{\mathbf{i}} & + & d\,\hat{\mathbf{j}} \\ \hline \end{array}$$

$$\vec{\mathbf{F}}_3 = (a+c)\,\hat{\mathbf{i}} + (b+d)\,\hat{\mathbf{j}}$$

Homework: What is the vector $\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_1 - \vec{\mathbf{F}}_2$?

3.2 Equations of Kinematics in Two Dimensions

Except for time, motion in x and y directions are INDEPENDENT.



Motion in x direction with constant acceleration.

$$v_x = v_{x0} + a_x t$$

$$\Delta x = \tfrac{1}{2}\left(v_{x0} + v_x\right)t$$

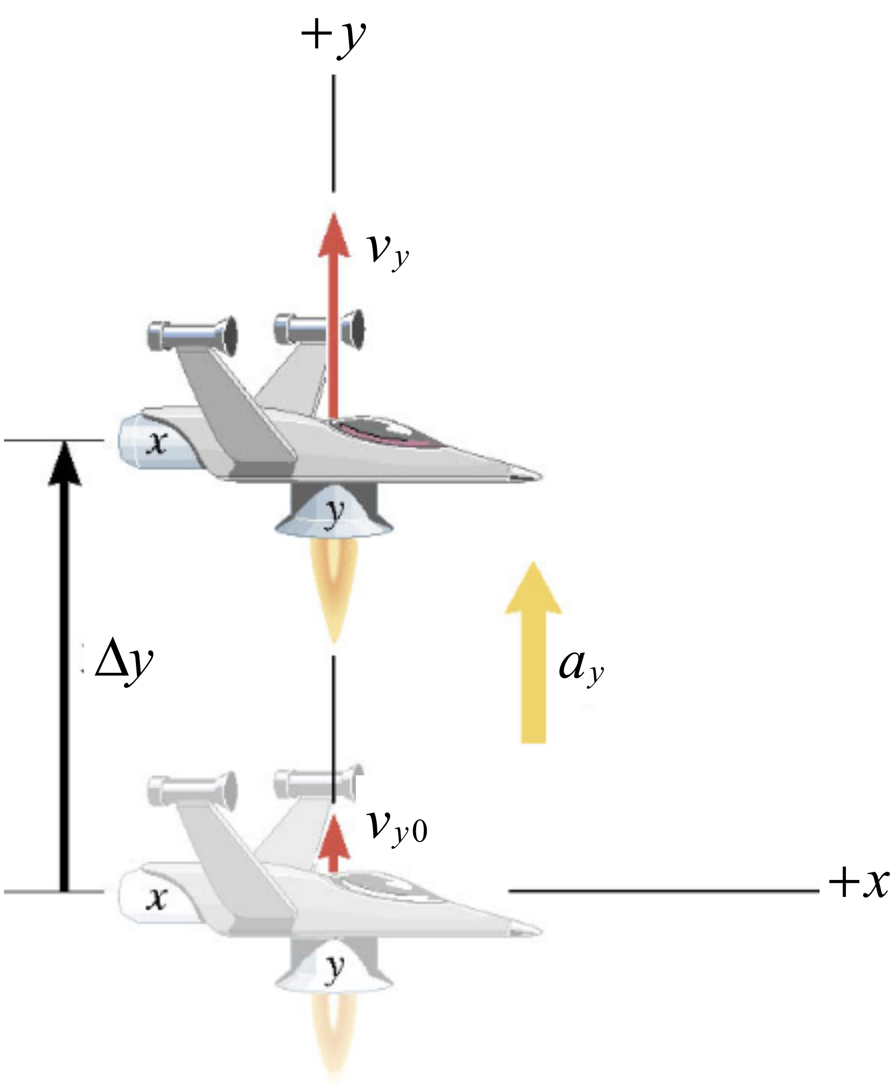
$$\Delta x = v_{x0}t + \tfrac{1}{2}a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x \Delta x$$

3.2 Equations of Kinematics in Two Dimensions

Except for time, motion in x and y directions are INDEPENDENT.

Using only y thruster



Constant acceleration motion in y direction.

$$v_y = v_{y0} + a_y t$$

$$\Delta y = v_{y0} t + \tfrac{1}{2} a_y t^2$$

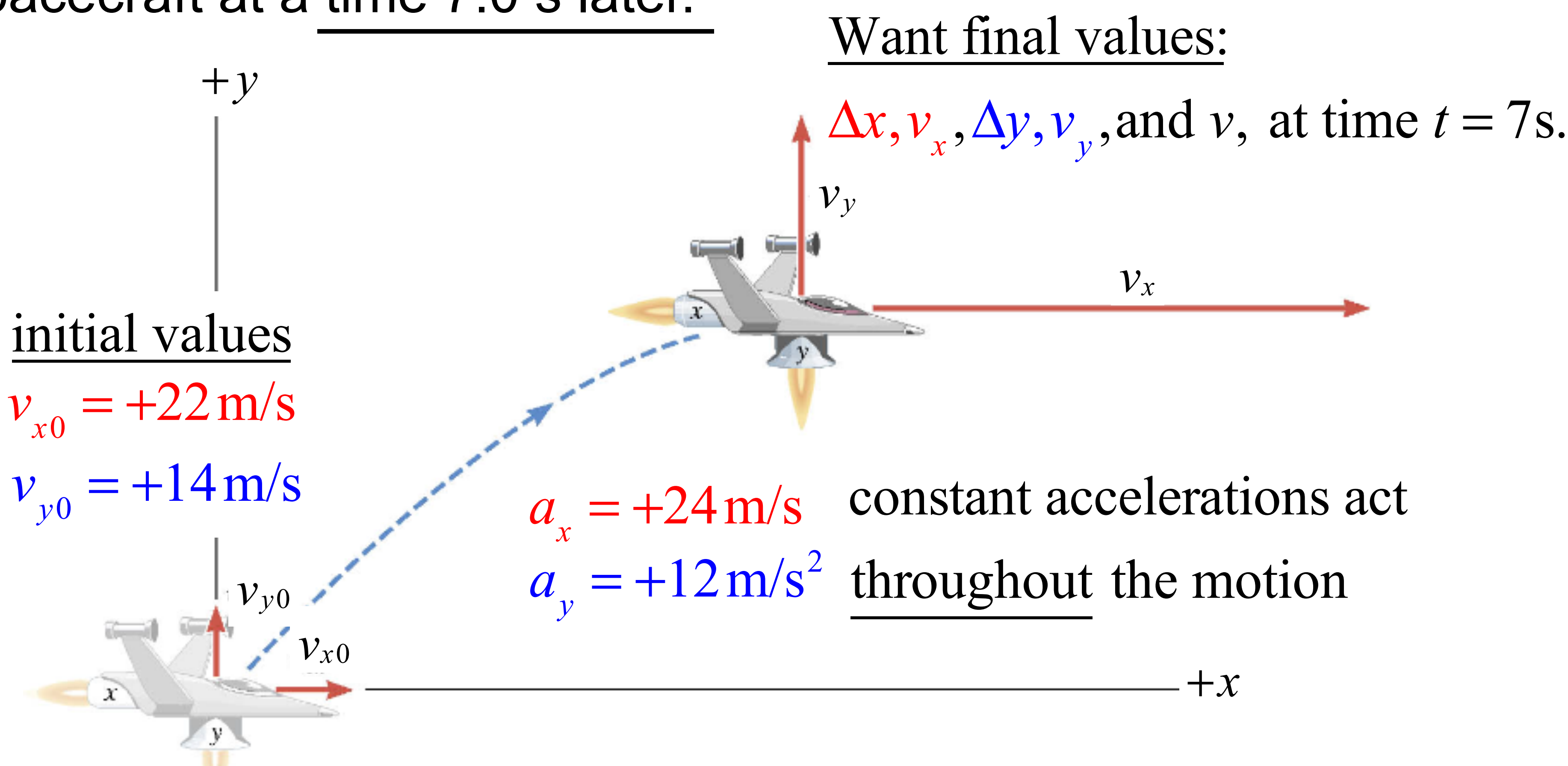
$$\Delta y = \tfrac{1}{2}\left(v_{y0} + v_y\right) t$$

$$v_y^2 = v_{y0}^2 + 2 a_y \Delta y$$

3.2 *Equations of Kinematics in Two Dimensions*

Example: A Moving Spacecraft

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s². In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s². Find (a) Δx and v_x, (b) Δy and v_y, and (c) the final velocity of the spacecraft at a time 7.0 s later.



3.2 Equations of Kinematics in Two Dimensions

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (-).

3. Write down the values that are given for any of the five kinematic variables associated with each direction.

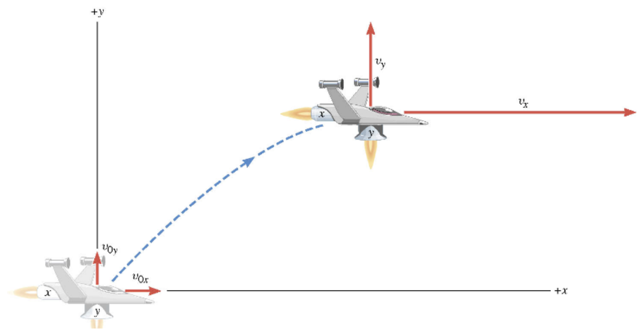
4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y*. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.

6. Keep in mind that there may be two possible answers to a kinematics problem.

3.2 *Equations of Kinematics in Two Dimensions*

Example: A Moving Spacecraft:



x direction motion

Δx	a_x	v_x	v_{x0}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$\Delta x = v_{x0}t + \tfrac{1}{2}a_x t^2$$

x displacement

$$= (22\,\text{m/s})(7.0\ \text{s}) + \tfrac{1}{2}(24\,\text{m/s}^2)(7.0\ \text{s})^2 = +740\ \text{m}$$

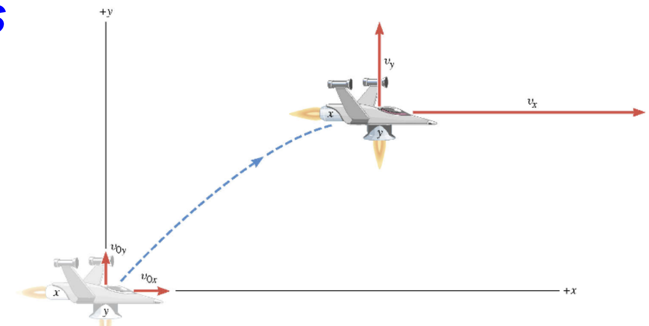
$$v_x = v_{x0} + a_x t$$

x component of velocity

$$= (22\,\text{m/s}) + (24\,\text{m/s}^2)(7.0\ \text{s}) = +190\,\text{m/s}$$

3.2 *Equations of Kinematics in Two Dimensions*

Example: A Moving Spacecraft:



y direction motion

Δy	a_y	v_y	v_{y0}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

$$\Delta y = v_{y0} t + \tfrac{1}{2} a_y t^2$$

y displacement

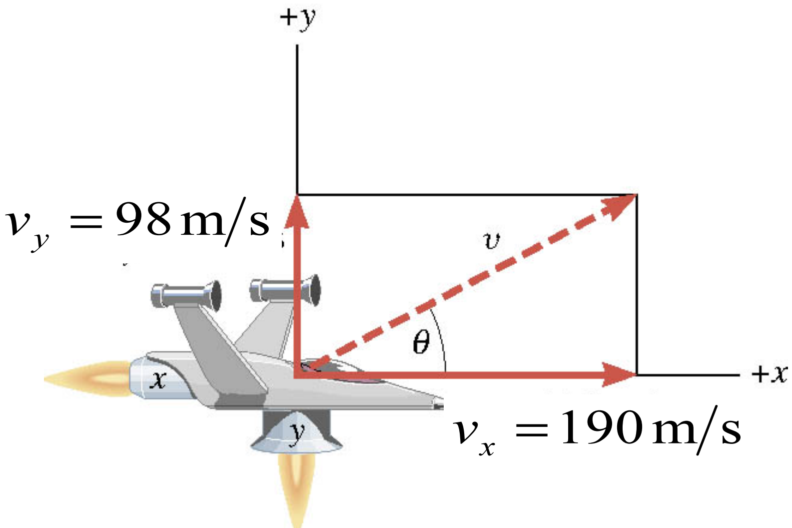
$$= (14\,\text{m/s})(7.0\ \text{s}) + \tfrac{1}{2}(12\,\text{m/s}^2)(7.0\ \text{s})^2 = +390\ \text{m}$$

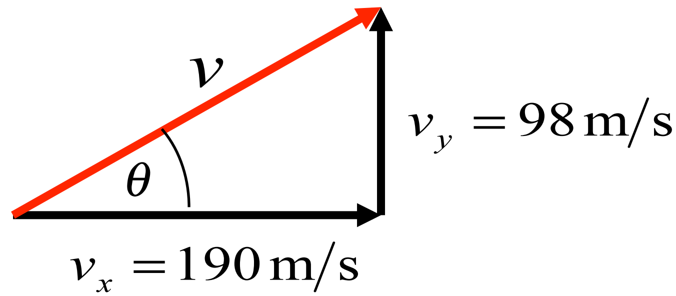
$$v_y = v_{y0} + a_y t$$

y component of velocity

$$= (14\,\text{m/s}) + (12\,\text{m/s}^2)(7.0\ \text{s}) = +98\,\text{m/s}$$

Can also find final speed and direction (angle) at $t = 7\text{s}$.





$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(190\,\text{m/s})^2 + (98\,\text{m/s})^2} = 210\,\text{m/s}$$

$$\theta = \tan^{-1}(98/190) = 27^\circ$$

3.3 Projectile Motion

Under the influence of gravity alone, an object near the surface of the Earth will accelerate downwards at 9.81m/s^2.

Great simplification for projectiles !

<u>*y* direction</u>

up is positive so

$$a_y = -9.81\,\mathrm{m/s^2}$$

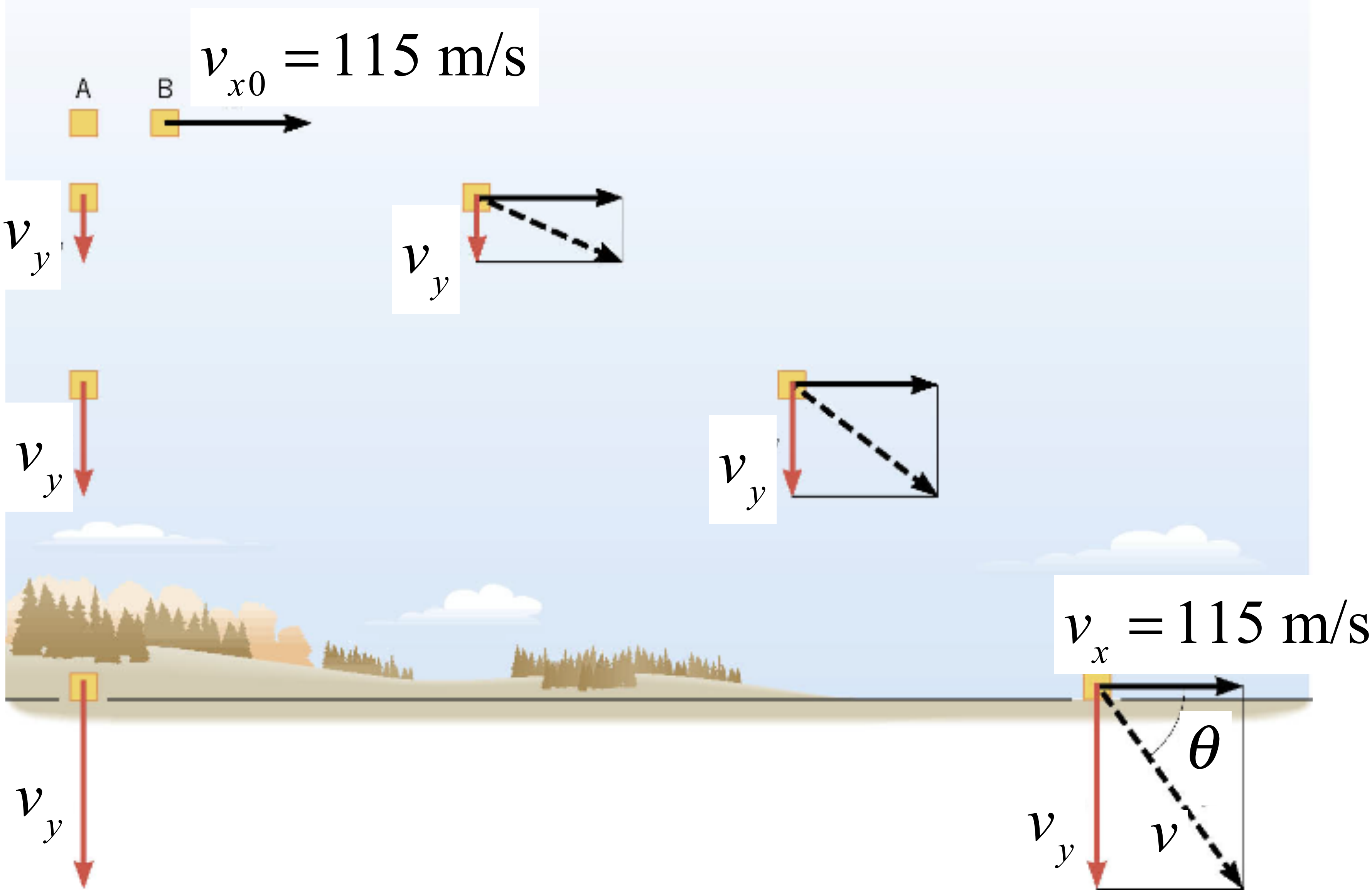
<u>*x* direction</u>

$$a_x = 0$$

↓

$$v_x = v_{x0} = \text{ constant}$$

Example: A Falling Care Package

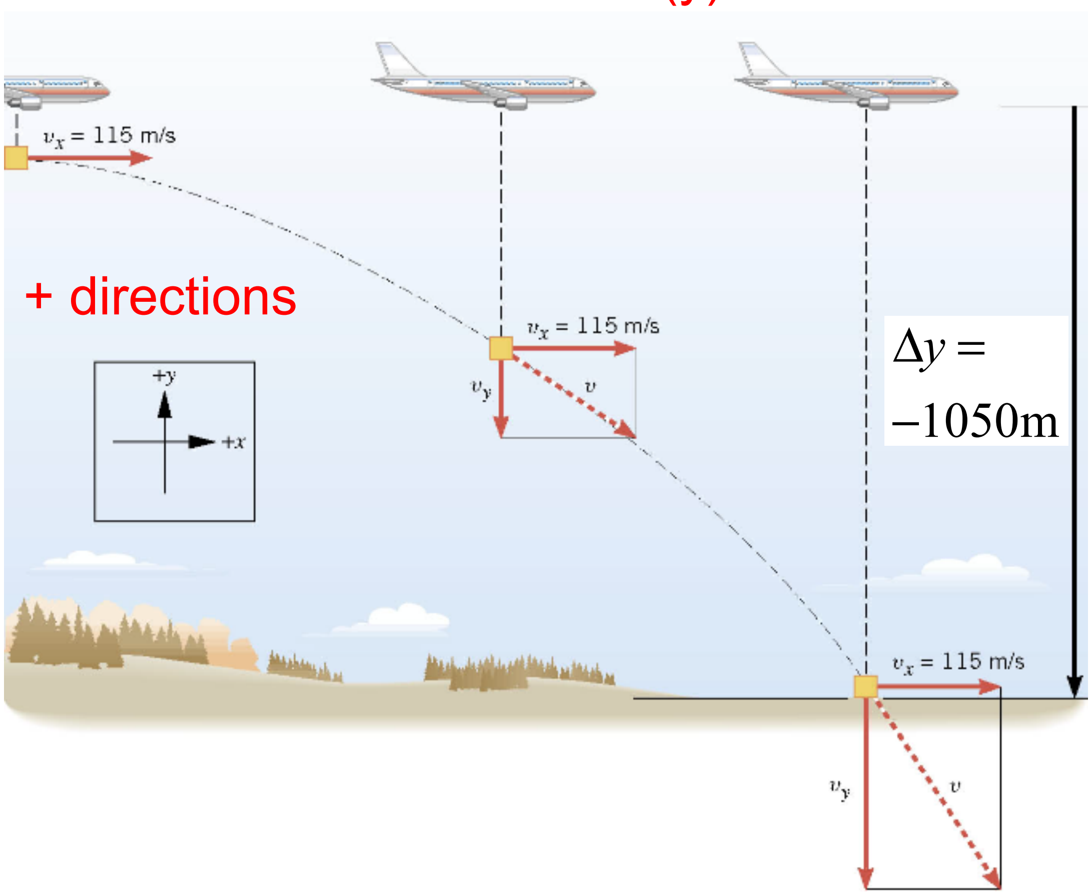


3.3 Projectile Motion

Example: A Falling Care Package

The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.

Time to hit the ground depends ONLY on vertical (y) motion



$$v_{y0} = 0$$

$$a_y = -9.81 \text{ m/s}^2$$

$$\Delta y = -1050 \text{ m}$$

Displacement in y is in the negative direction

3.3 Projectile Motion

Δy	a_y	v_y	v_{y0}	t
–1050 m	–9.81 m/s^2		0 m/s	?

$$\Delta y = v_{y0}t + \tfrac{1}{2}a_y t^2 \quad \Rightarrow \quad \Delta y = \tfrac{1}{2}a_y t^2$$

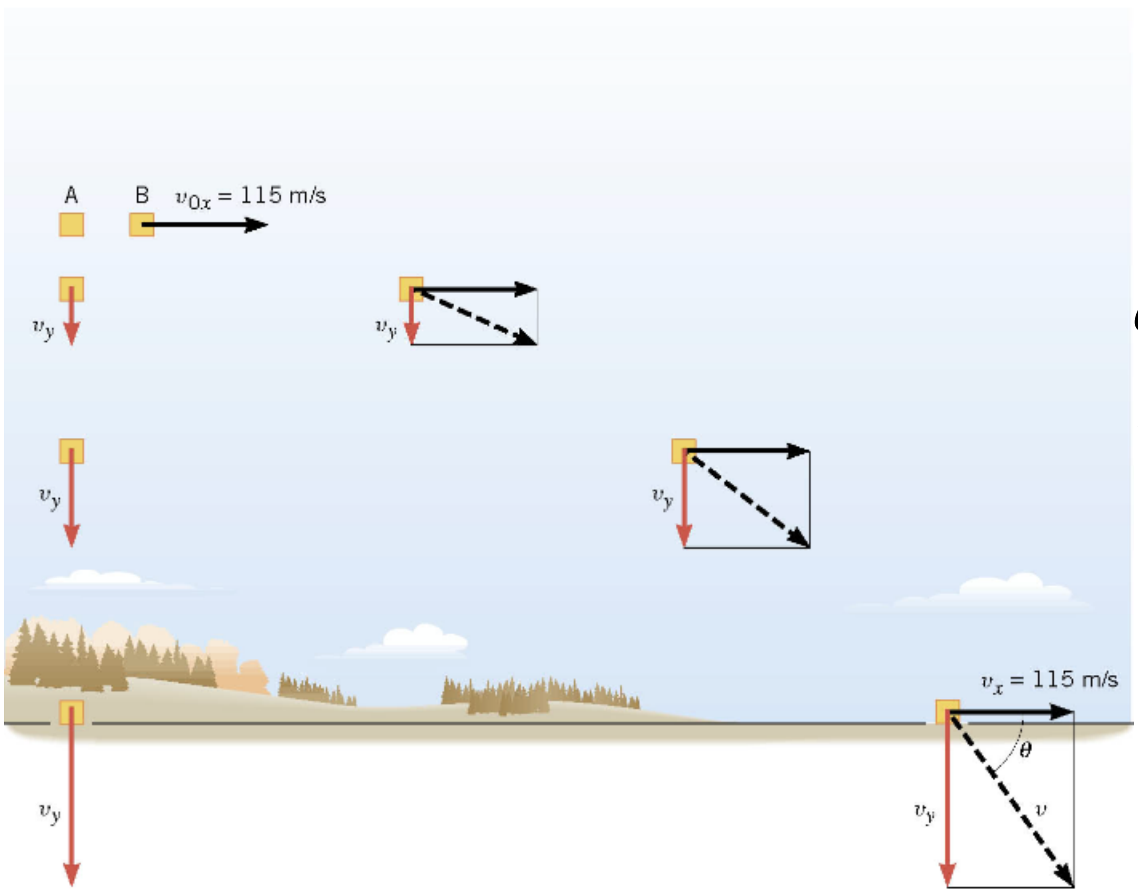
$$t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-1050\text{ m})}{-9.81\text{m}/\text{s}^2}}$$

$$= 14.6\text{ s}$$

3.3 Projectile Motion

Example: The Velocity of the Care Package

What are the magnitude and direction of the final velocity of the care package?



$$t = 14.6 \text{ s}$$

BECAUSE x-component of acceleration is zero

$$a_x = 0; \; v_{x0} = +115 \text{ m/s}$$

$$v_x = v_{x0} + a_x t$$

$$= +115 \text{ m/s}$$

x-component of velocity does not change

3.3 Projectile Motion

Δy	a_y	v_y	v_{y0}	t
–1050 m	–9.81 m/s^2	?	0 m/s	14.6 s

$$v_y = v_{y0} + a_y t = 0 + \left(-9.81\,\text{m/s}^2\right)\left(14.6\text{ s}\right)$$

$$= -143\,\text{m/s}$$ y-component of final velocity.

Now ready to get final speed and direction

$$v_x = v_{x0} = +115\,\text{m/s}$$

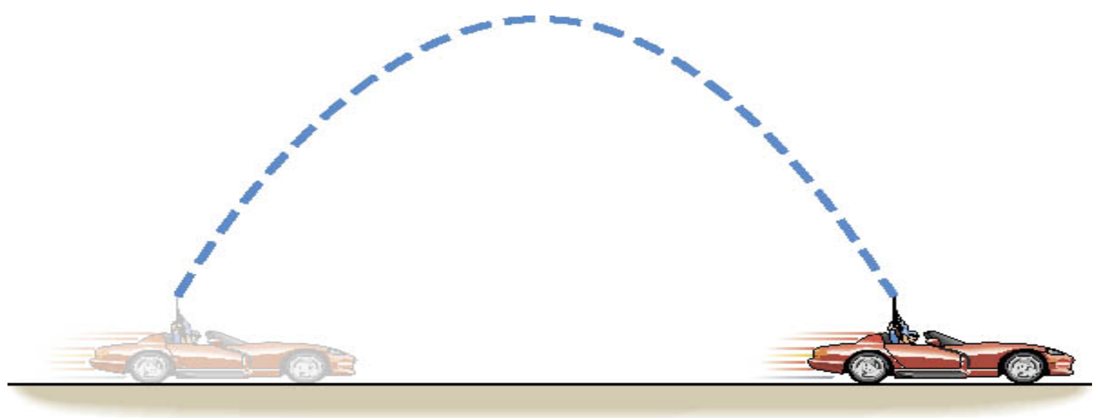
$$v = \sqrt{v_x^2 + v_y^2} = 184\text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-143}{+115}\right) = -51^\circ$$

3.3 Projectile Motion

Conceptual Example: I Shot a Bullet into the Air...

Suppose you are driving a convertible with the top down. The car is moving to the right at constant velocity. You point a rifle straight up into the air and fire it. In the absence of air resistance, where would the bullet land – behind you, ahead of you, or in the barrel of the rifle?



Ballistic Cart Demonstration

Clicker Question 3.4

A cannon is on a flat-car train moving at constant velocity. Which direction should the cannon be AIMED so that the cannon-ball lands right on the cannon? Ignore air friction.

A) You have to “lead” the train
B) If the train is moving fast, it can’t be done
C) Far ahead if the the train is moving really fast
D) At exactly 45 degrees for all train speeds
E) Straight upward

Clicker Question 3.4

A cannon is on a flat-car train moving at constant velocity. Which direction should the cannon be AIMED so that the cannon-ball lands right on the cannon? Ignore air friction.

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E) Straight upward

Example: The Height of a Kickoff

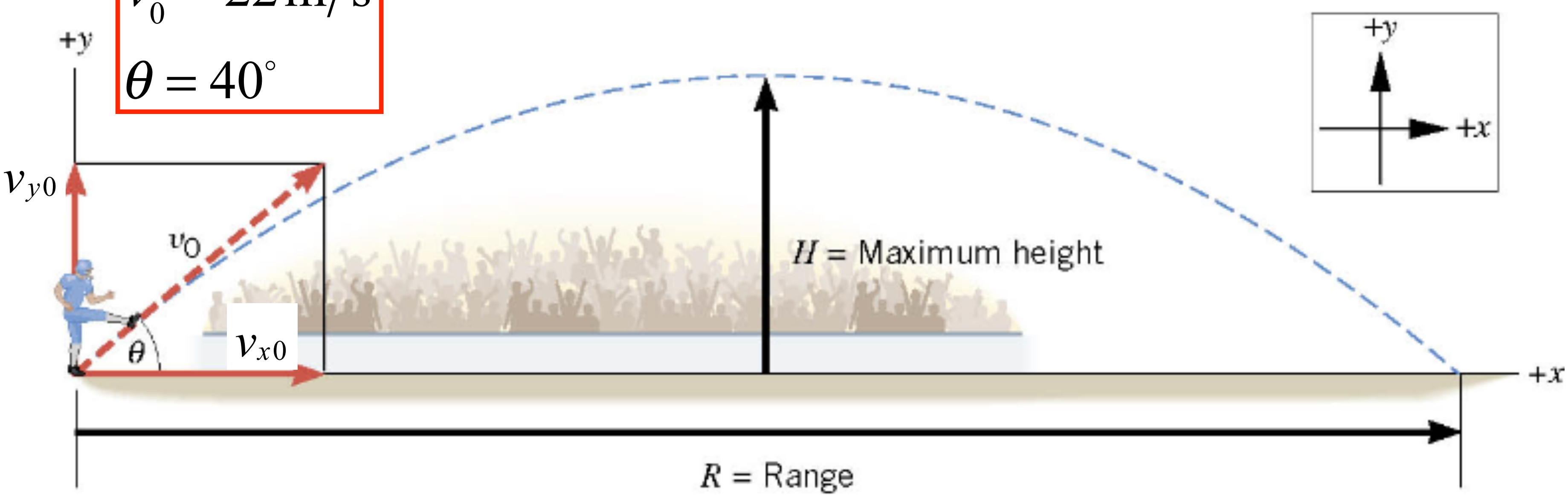
A placekicker kicks a football at and angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.

initial velocity

$$v_0 = 22\,\mathrm{m/s}$$

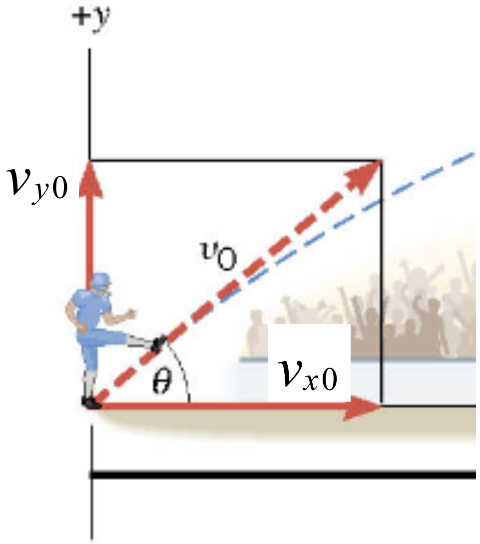
$$\theta = 40^\circ$$

maximum height and "hang time" depend only on the y-component of initial velocity



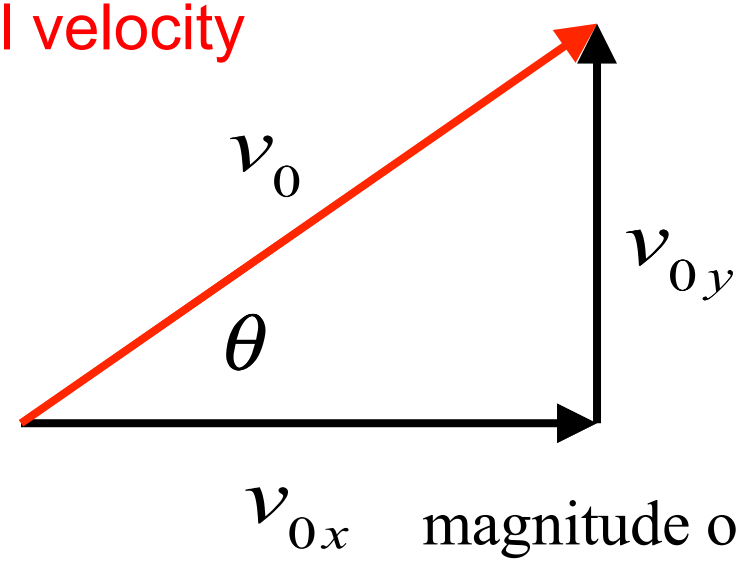
3.3 Projectile Motion

Find *x* and *y* components of initial velocity



$$v_0 = 22\,\text{m/s}$$

$$\theta = 40^\circ$$

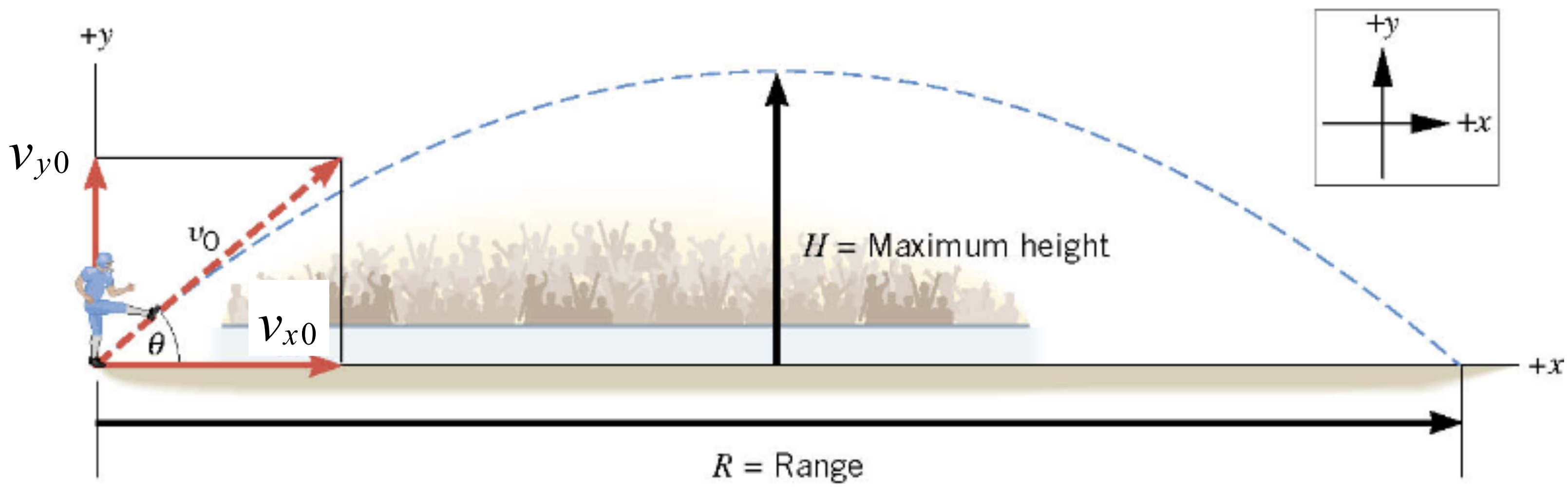


magnitude of the component vectors

$$v_{y0} = v_0 \sin\theta = (22\,\text{m/s})\sin 40^\circ = 14\,\text{m/s}$$

$$v_{x0} = v_0 \cos\theta = (22\,\text{m/s})\cos 40^\circ = 17\,\text{m/s}$$

3.3 Projectile Motion



Δy	a_y	v_y	v_{y0}	t
?	–9.80 m/s^2	0	14 m/s	

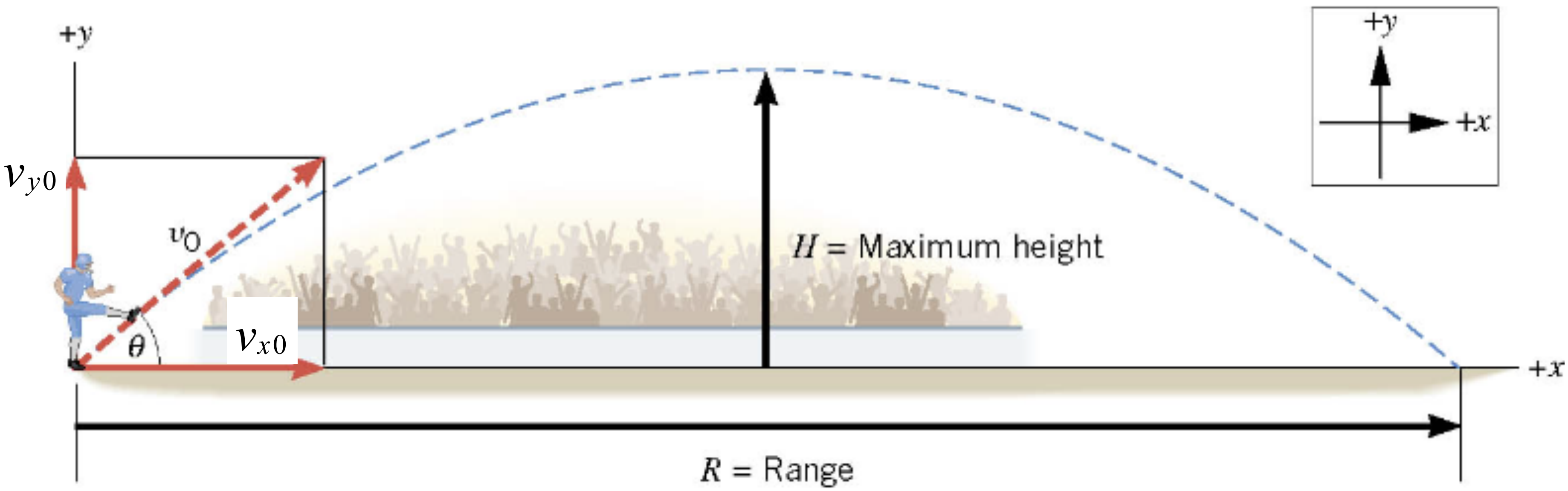
$$v_y^2 = v_{0y}^2 + 2a_y \Delta y \quad \Rightarrow \quad \Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y}$$

maximum height

$$H = \Delta y = \frac{0 - (14\,\text{m/s})^2}{2(-9.8\,\text{m/s}^2)} = +10\ \text{m}$$

Example: The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?



Δy	a_y	v_y	v_{y0}	t
0	$-9.80\ m/s^2$		14 m/s	?

3.3 Projectile Motion

Δy	a_y	v_y	v_{y0}	t
0	–9.81 m/s²		14 m/s	?

$$\Delta y = v_{y0}t + \tfrac{1}{2}a_y t^2$$

$$0 = \left(14\,\mathrm{m/s}\right)t + \tfrac{1}{2}\left(-9.81\,\mathrm{m/s^2}\right)t^2$$

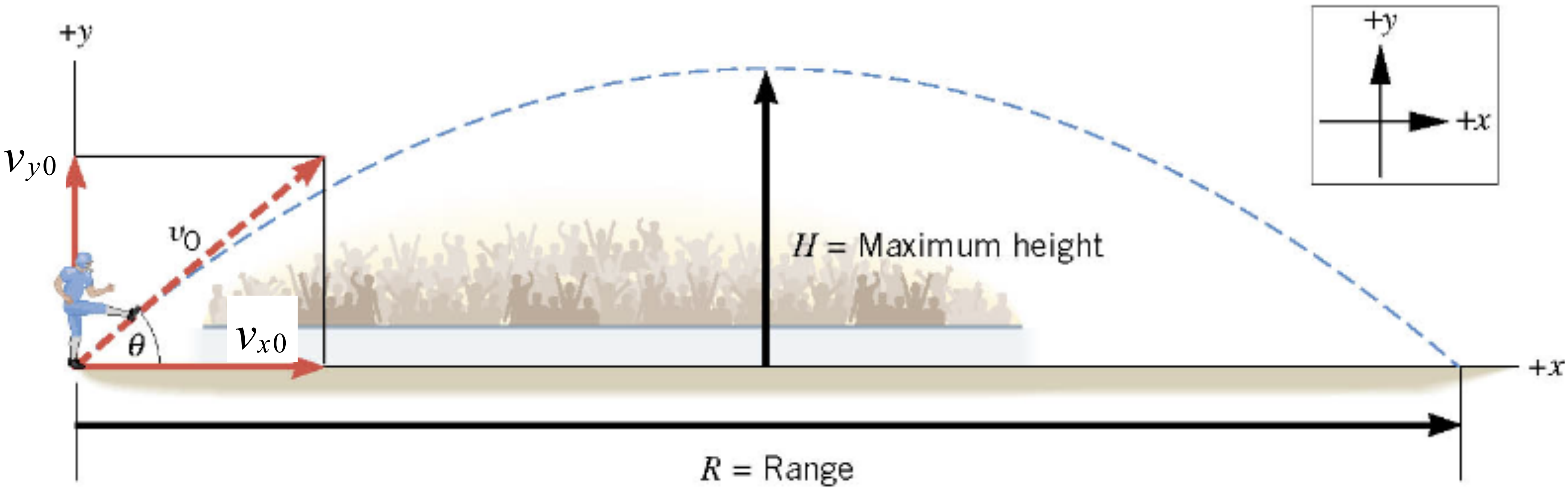
$$0 = 2\left(14\,\mathrm{m/s}\right) + \left(-9.81\,\mathrm{m/s^2}\right)t$$

$$t = 2.9\,\mathrm{s}$$

3.3 Projectile Motion

Example: The Range of a Kickoff
Calculate the range R of the projectile.

Range depends on the hang time (2.9 s) and x-component of initial velocity



x-direction

$a_x = 0\,!$

$$\Delta x = v_{x0}t + \tfrac{1}{2}a_x t^2 = v_{x0}t$$

$$= \left(17\,\text{m/s}\right)\left(2.9\ \text{s}\right) = +49\ \text{m}$$

3.3 Projectile Motion

Conceptual Example: Two Ways to Throw a Stone

From the top of a cliff, a person throws two stones. The stones have identical initial speeds, but stone 1 is thrown downward at some angle above the horizontal and stone 2 is thrown at the same angle below the horizontal. Neglecting air resistance, which stone, if either, strikes the water with greater velocity?