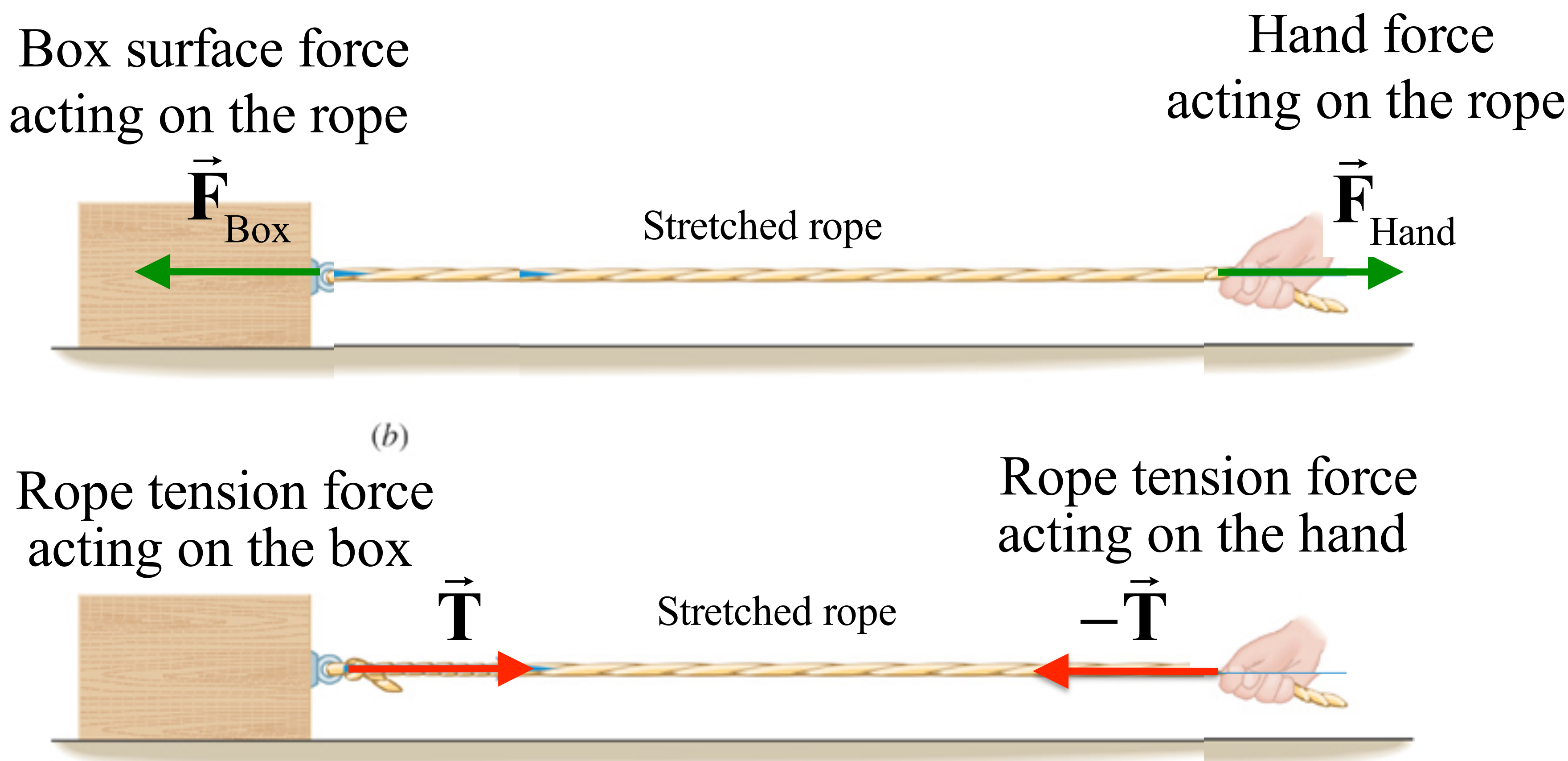
Chapter 4

Forces and Newton's Laws of Motion

Conclusion

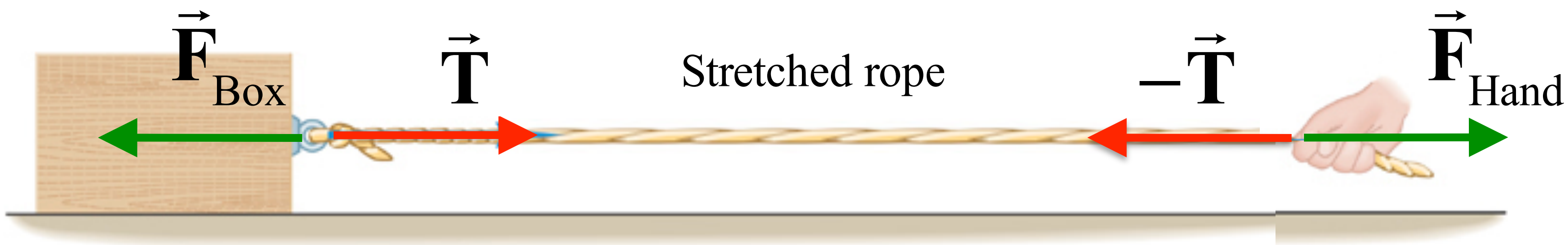
Cables and ropes transmit forces through ***tension***.



$\left(\vec{\mathbf{F}}_{\text{Box}}, \vec{\mathbf{T}}\right)$ These are Newton's 3rd law Action – Reaction pairs $\left(-\vec{\mathbf{T}}, \vec{\mathbf{F}}_{\text{Hand}}\right)$

$$\text{magnitudes: } T = F_{\text{Hand}}$$

Hand force stretches the rope that generates tension forces at the ends of the rope



$\left(\vec{\mathbf{F}}_{\text{Box}}, \vec{\mathbf{T}}\right)$

These are Newton's 3rd law
Action – Reaction pairs

$\left(\vec{\mathbf{F}}_{\text{Hand}}, -\vec{\mathbf{T}}\right)$

Tension pulls on box

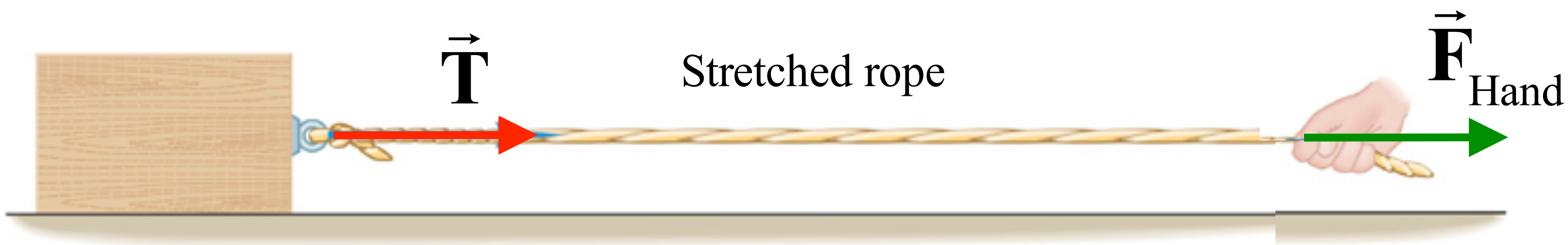
Box pulls on rope

Tension pulls on hand

Hand pulls on rope

Cables and ropes transmit forces through ***tension***.

The stretch of the rope transfers the force of the hand to the box



Hand force causes a tension force on the box

Force magnitudes are the same

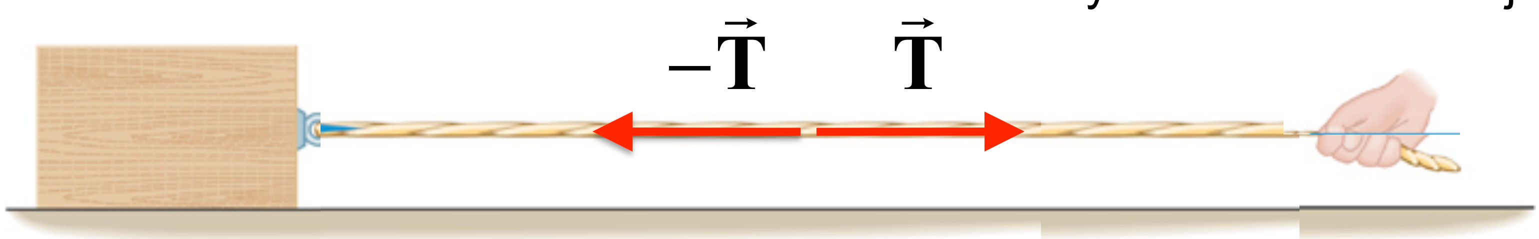
$$T = F_{Hand}$$

4.4 The Tension Force

What tension forces are in action at the center of the rope?

Forces in action at any point on the stretched rope

These cannot be added to get zero they act on different objects.



Tension of left section pulls to the left on the other section

$$\left(-\vec{T}, \vec{T}\right)$$

Tension of right section pulls to the right on the other section

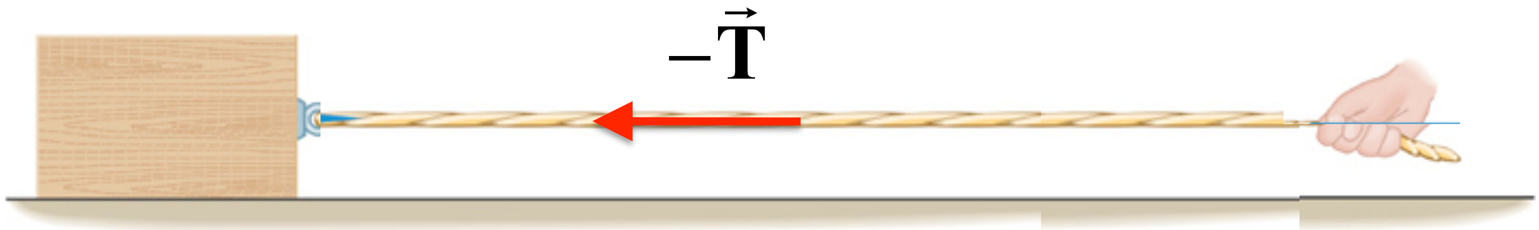
This is a Newton's 3rd law
Action – Reaction pair

The same magnitude of tension acts at any point on the stretched rope

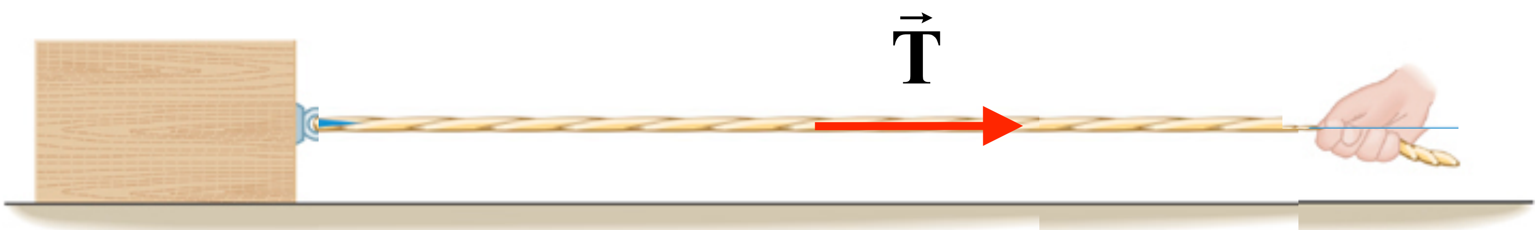
4.4 The Tension Force

Tension forces at any point on the rope are an Action-Reaction pair.

Forces in action at any point on the stretched rope

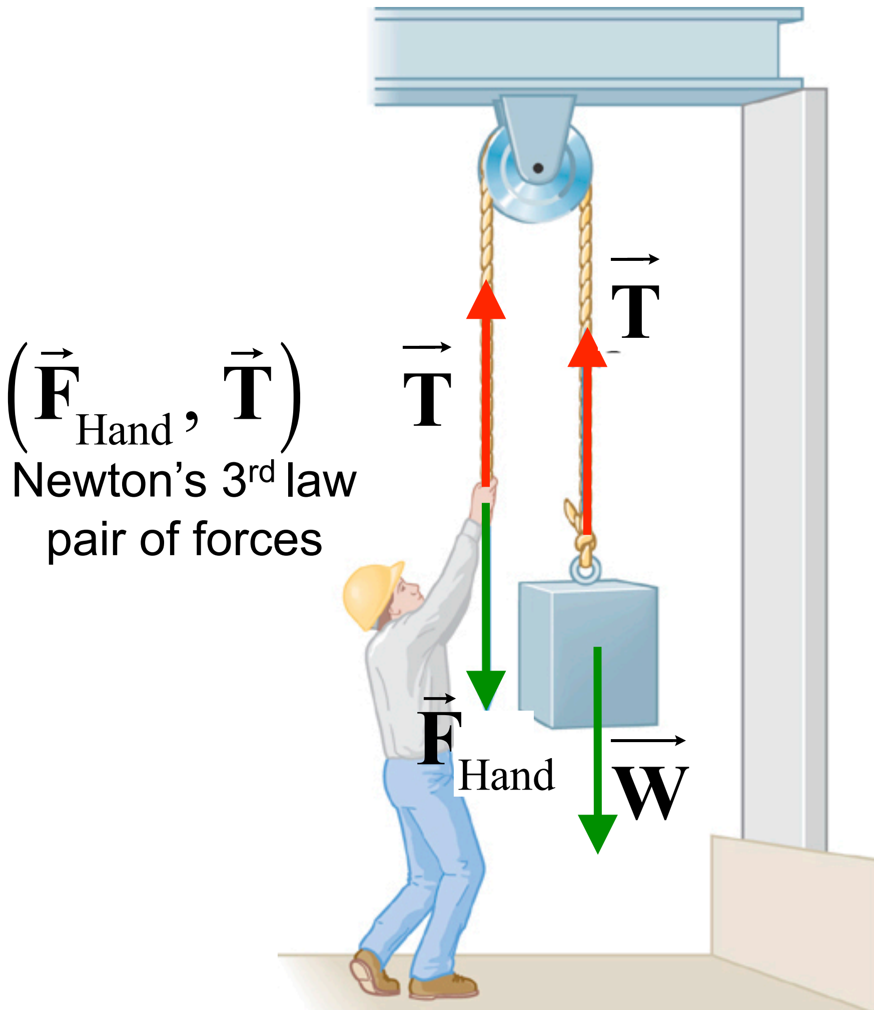


Tension of left section pulls to the left on the other section



Tension of right section pulls to the right on the other section

4.4 *The Tension Force*



A massless rope will transmit tension magnitude undiminished from one end to the other.

A massless, frictionless pulley, transmits the tension undiminished to the other end.

If the mass is at rest or moving with a constant speed & direction the Net Force on the mass is zero!

$$\sum \vec{F} = \vec{W} + \vec{T} = 0 \; (\vec{a} = 0)$$

$$0 = -mg + \vec{T}$$

$$\vec{T} = +mg, \text{ and } \mathbf{F}_{Hand} = -mg$$

Note: the weight of the person must be larger than the weight of the box, or the mass will drop and the tension force will accelerate the person upward.

Definition of Equilibrium

An object is in equilibrium when it has zero acceleration in all directions

$$\sum F_x = 0$$

$$\sum F_y = 0$$

We have been using this concept for the entire Chapter 4

Reasoning Strategy

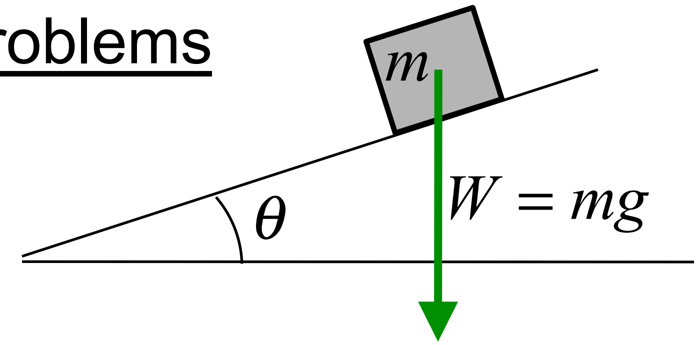
- Select an object(s) to which the equations of equilibrium are to be applied.

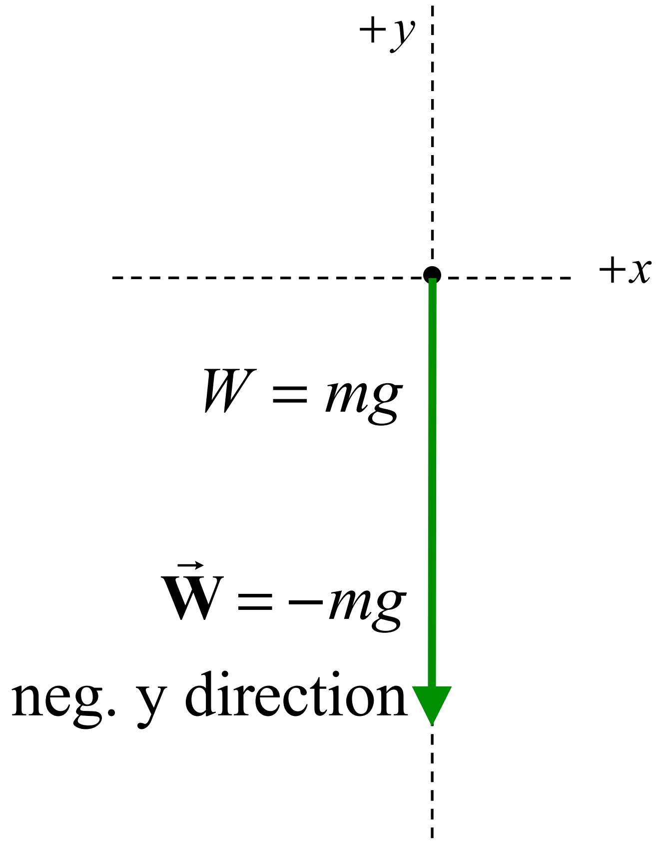
- Draw a free-body diagram for each object chosen above. Include only forces acting on each object, not forces objects exert on its environment.

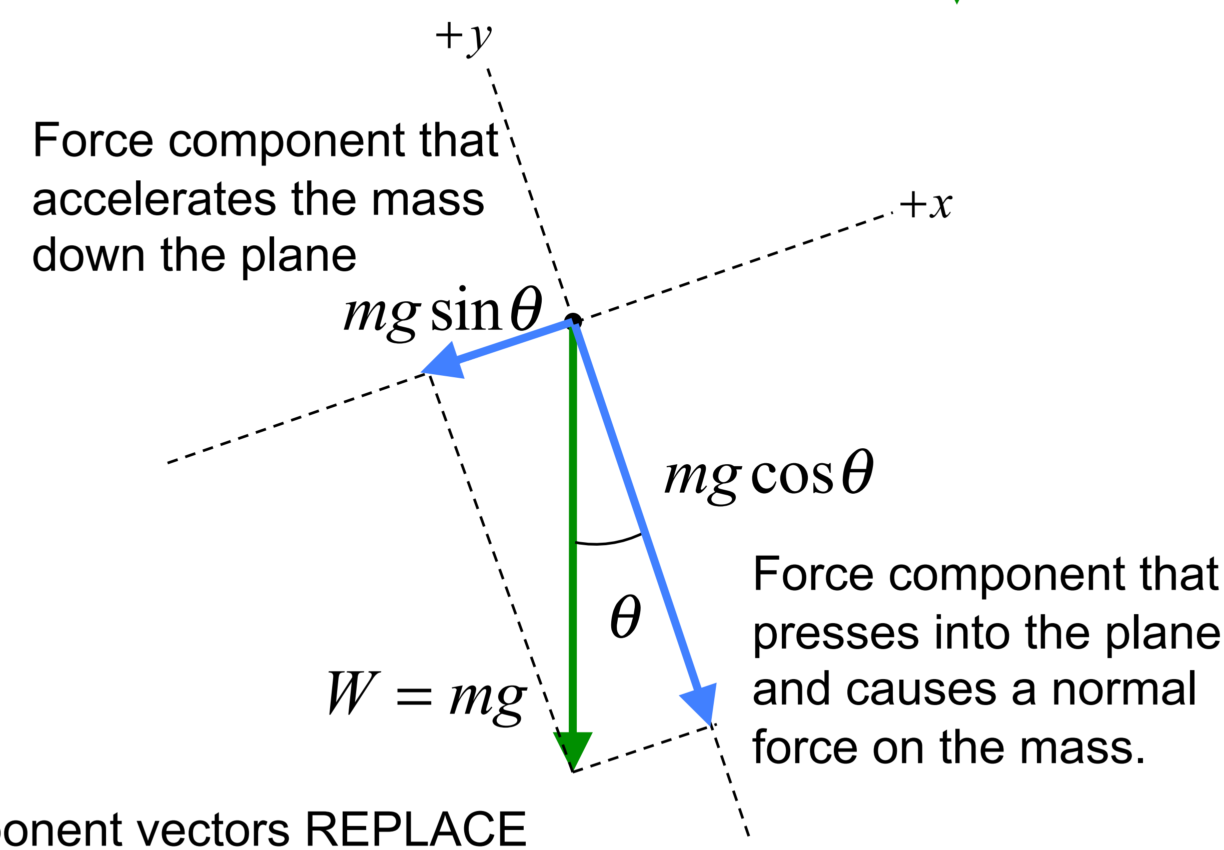
- Choose a set of *x*, *y* axes for each object and resolve all forces in the free-body diagram into components that point along these axes.

- Apply the Equilibrium equations and solve for unknowns.

Inclined plane and similar problems



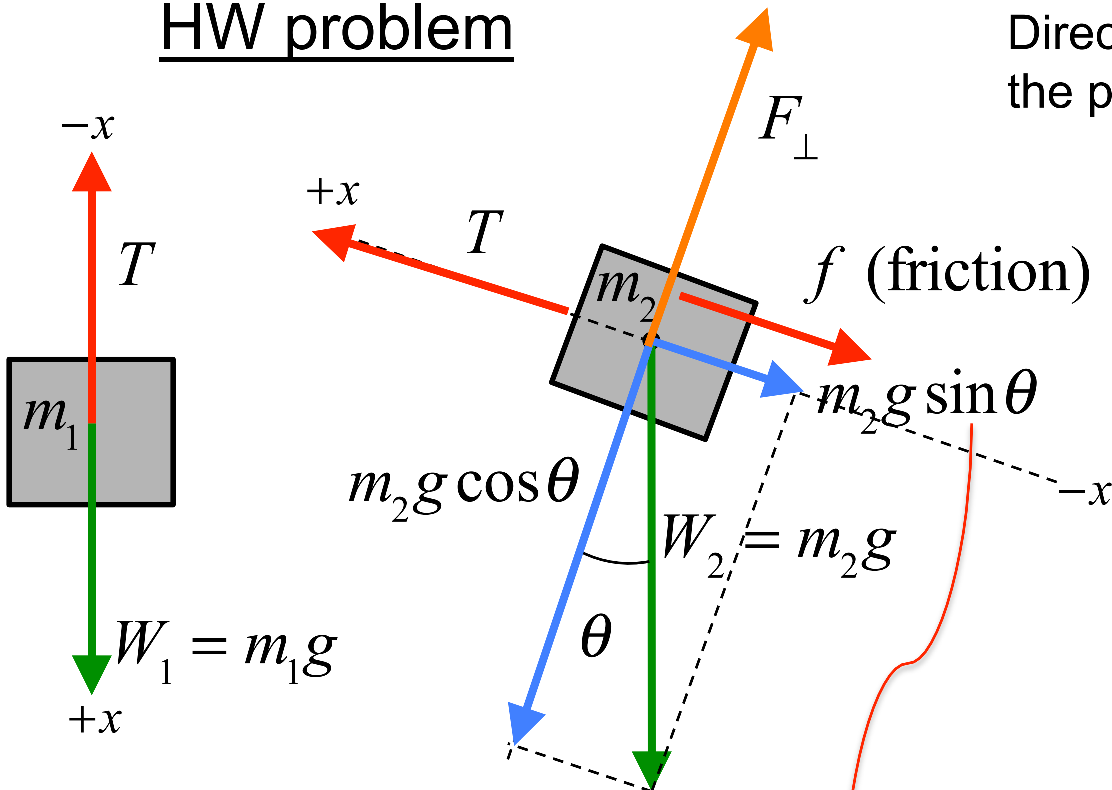




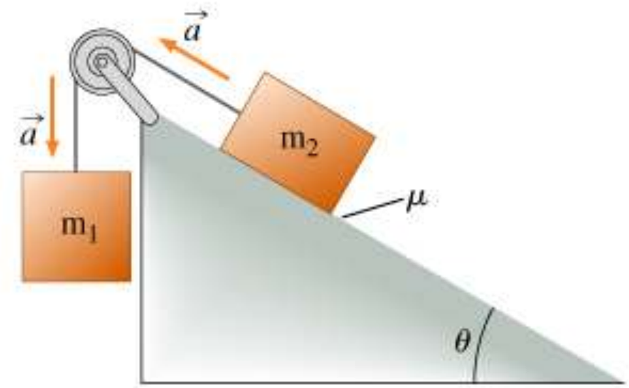
Blue component vectors REPLACE (equivalent to) the Green Weight vector

4.4 HW Application of Newton's Laws of Motion

HW problem



Direction of **a** (shown) determines the positive direction for both masses!



$$f \text{ (friction)} = \mu_k F_\perp = \mu_k m_2 g \cos\theta$$

Newton's 2[nd] Law for each mass

$$\text{(2) Net-Force on } m_2\text{:} \quad T + (-m_2 g \sin\theta) + (-\mu_k m_2 g \cos\theta) = m_2 a$$

$$\text{(1) Net-Force on } m_1\text{:} \quad m_1 g + (-T) = m_1 a$$

$$\text{(1)}: T = m_1(g-a), \text{ replace } T \text{ in (2): } m_1(g-a) = m_2[a + g(\sin\theta + \mu_k \cos\theta)]$$

$$\text{Finally: } \quad m_2 = m_1(g-a) / [a + g(\sin\theta + \mu_k \cos\theta)]$$

Chapter 5

Work and Energy

5.1 Work Done by a Constant Force

The concept of forces acting on a mass (one object) is intimately related to the concept of ENERGY production or storage.

- A mass accelerated to a non-zero speed carries energy (mechanical)
- A mass raised up carries energy (gravitational)
- The atom in a molecule carries energy (chemical)
- The molecule in a hot gas carries energy (thermal)
- The nucleus of an atom carries energy (nuclear)

(The energy carried by radiation will be discussed in PHY232)

The concept of energy relates to the net force acting on a moving mass.

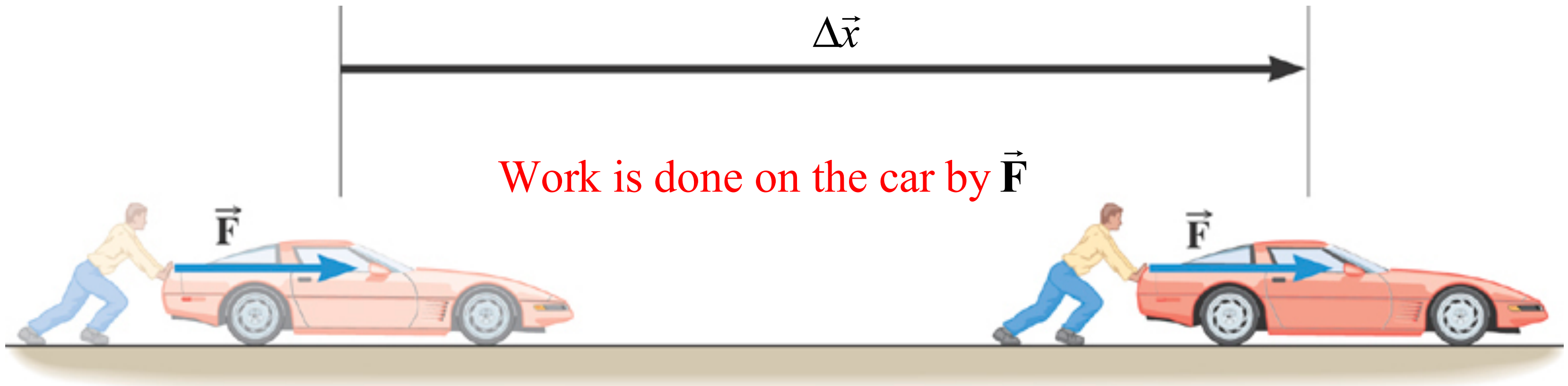
WORK

Sorry, but work is essential to understand the concept of energy.

5.1 Work Done by a Constant Force

Work is *done on* an object (a mass) *by* the force components acting on the object that are parallel to the displacement of the object.

Only acceptable definition.



The case shown is the simplest: the directions of $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{x}}$ are the same.

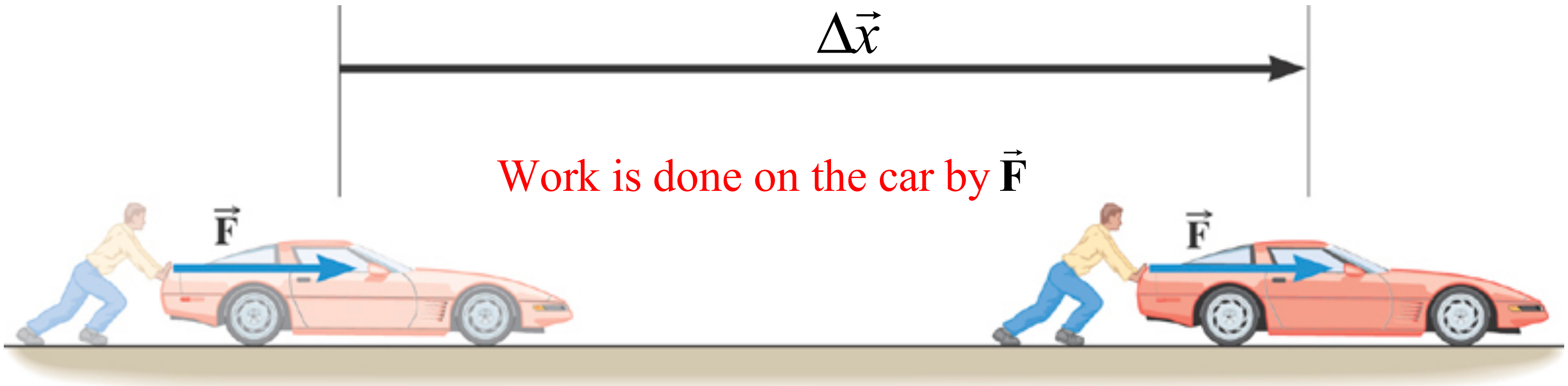
F and Δx are the magnitudes of these vectors.

The case where directions of $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{x}}$ are different is covered later.

5.1 Work Done by a Constant Force

Only acceptable definition.

Work is *done on* a moving object (a mass) *by* a force component acting on the object that is parallel to the displacement of the object.



Sorry about using the symbol W again. Hard to avoid it.

$$W = F\Delta x$$

$$1\,\mathrm{N\cdot m} = 1\ \text{joule}\ (\mathrm{J})$$

Work is a scalar (no direction - but it can have a sign)

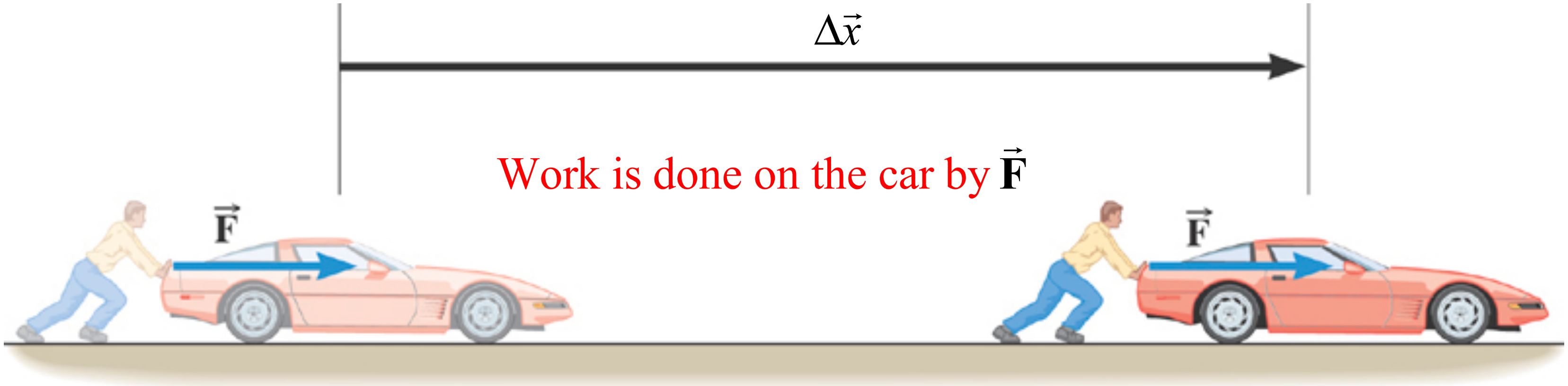
The work is positive if $\vec{\mathbf{F}}$ and $\Delta\vec{x}$ point in the same direction.

The work is negative if $\vec{\mathbf{F}}$ and $\Delta\vec{x}$ point in opposite directions.

Don't focus on the guy pushing the car!

It is the FORCE acting on the car that does the work.

5.1 Work Done by a Constant Force



With only one force acting on the car (m_{Car}), the car must accelerate, and over the displacement $\Delta\vec{x}$, the speed of the car will increase.

Starting with velocity v_0, find the final speed.

Newton's 2nd law: acceleration of the car, $a = F/m_{Car}$

$$v^2 = v_0^2 + 2a\Delta x$$

$$v = \sqrt{v_0^2 + 2a\Delta x}$$

The work done on the car by the force:

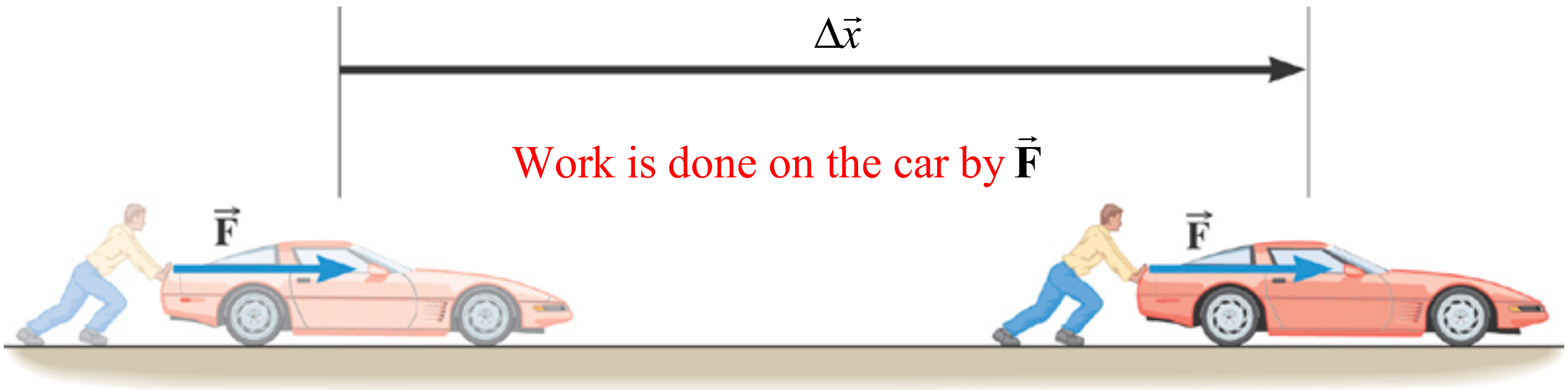
$$W = F\Delta x$$

$$1\ \text{N}\cdot\text{m} = 1\ \text{joule (J)}$$

has increased the speed of the car.

5.1 Work Done by a Constant Force

Other forces may be doing work on the object at the same time.



Example:

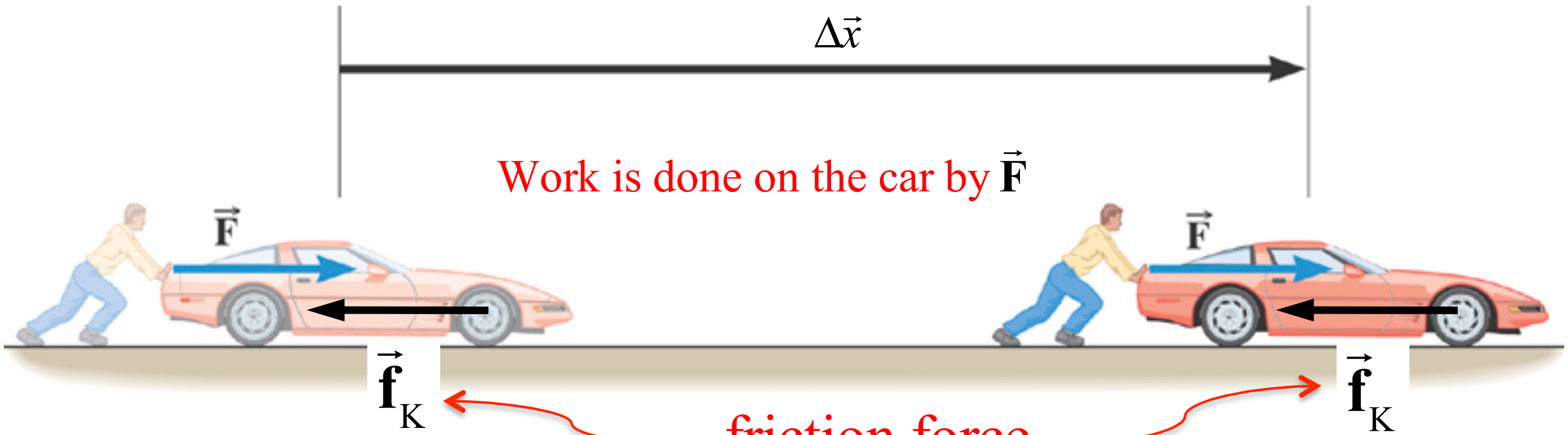
This time the car is not accelerating, but maintaining a constant speed, v_0.

Constant speed and direction: net force $\sum \mathbf{F} = 0$.

There must be at least one other force acting on the car !

5.1 Work Done by a Constant Force

The car is not accelerating, instead it maintains a constant speed, v_0.
The other force acting on the car is friction ($\vec{\mathbf{f}}$) .



NOTE :

$\vec{\mathbf{f}}_K$ and $\Delta\vec{\mathbf{x}}$ point in opposite directions, work is negative!

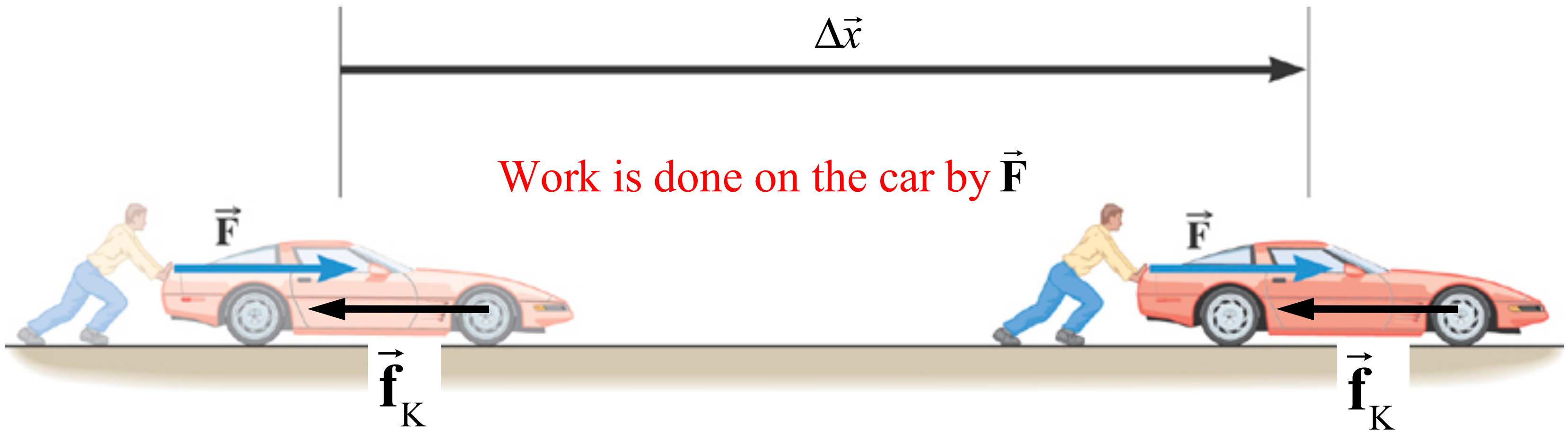
Acting on the car is a kinetic friction force, $\vec{\mathbf{f}}_K = -\vec{\mathbf{F}}$.

Because the car does not accelerate

the Net force on car must be ZERO !

5.1 Work Done by a Constant Force

$\vec{\mathbf{f}}_K$ and $\Delta\vec{\mathbf{x}}$ point in opposite directions, work is negative!



Also acting on the car is a kinetic friction force, $\vec{\mathbf{f}}_K = -\vec{\mathbf{F}}$.

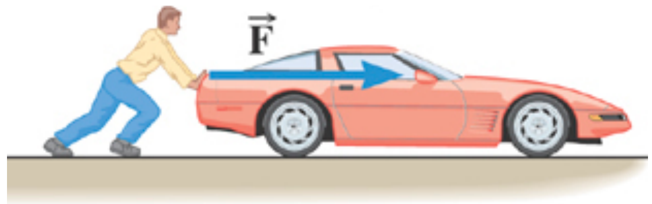
Net force on car must be ZERO, because the car does not accelerate !

$$W = F\Delta x$$

$$W_f = -f_K \Delta x = -F\Delta x$$

The work done on the car by $\vec{\mathbf{F}}$ was countered by the work done by the kinetic friction force, $\vec{\mathbf{f}}_K$.

5.1 Work Done by a Constant Force



Car's emergency brake was not released. What happens?

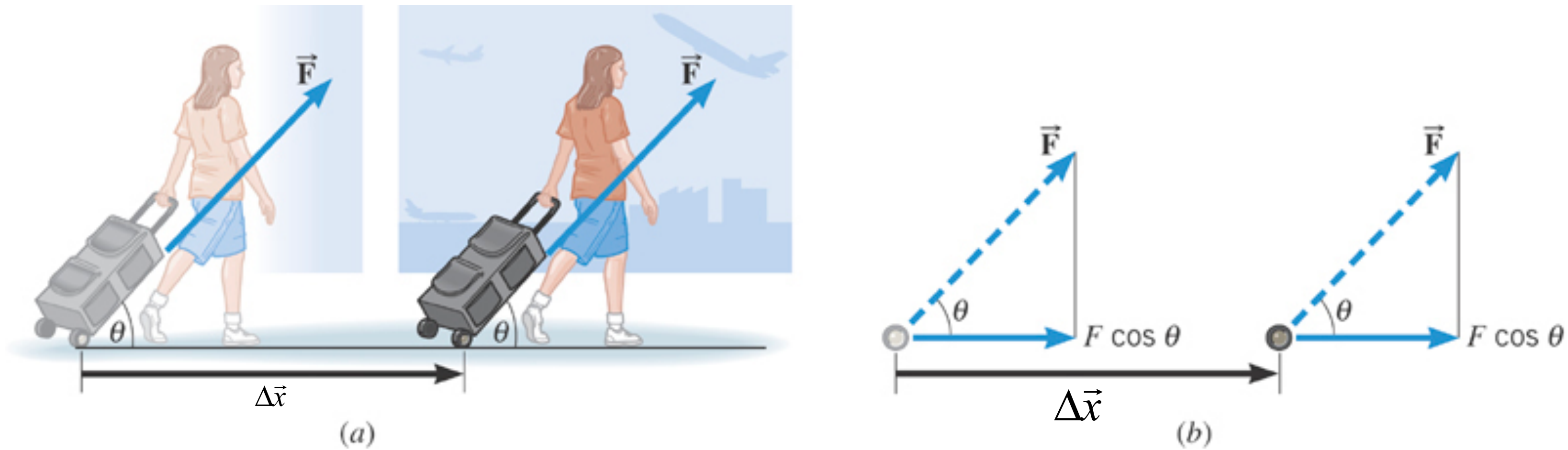
The car does not move. No work done on the car.

Work by force $\vec{\mathbf{F}}$ is zero. What about the poor person?

The person's muscles are pumping away but the attempt to do work on the car, has failed. What happens to the person does not affect the work done.

What must concern us here is: if the car does not move the work done on the car by the force $\vec{\mathbf{F}}$ is ZERO.

5.1 Work Done by a Constant Force



If the force and the displacement are not in the same direction, work is done by only the component of the force parallel to the displacement.

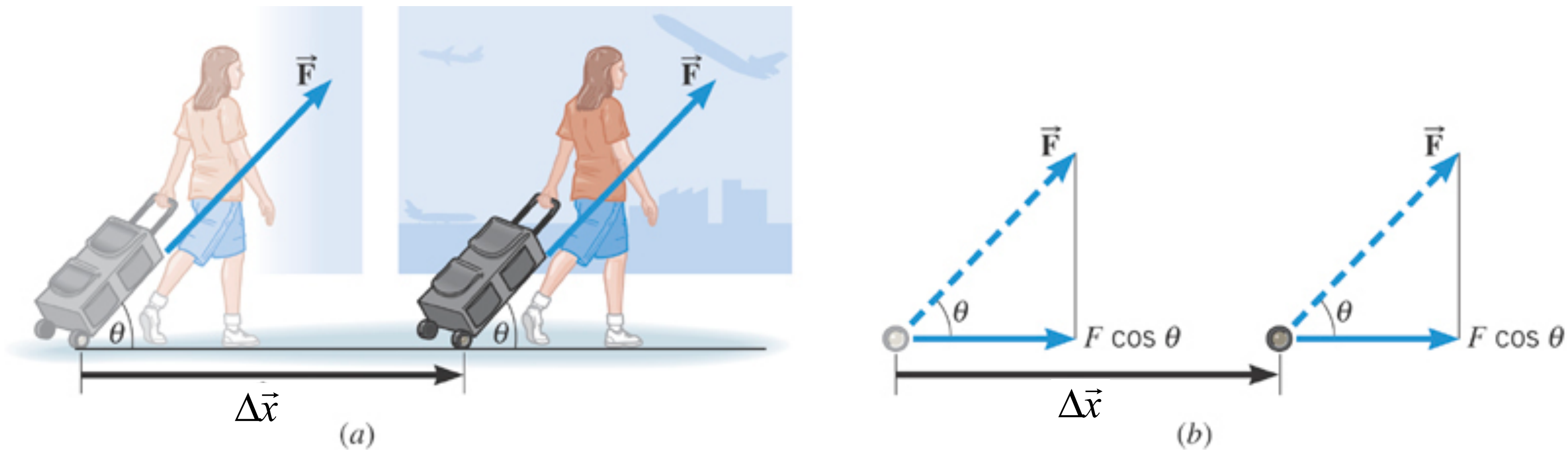
$$W = \left(F\cos\theta\right)\Delta x$$

F and Δx are magnitudes

$\cos 0^\circ = 1$ — $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{x}}$ in the same direction. $W = F\Delta x$

$\cos 90^\circ = 0$ — $\vec{\mathbf{F}}$ perpendicular to $\Delta\vec{\mathbf{x}}$. $W = 0$

$\cos 180^\circ = -1$ — $\vec{\mathbf{F}}$ in the opposite direction to $\Delta\vec{\mathbf{x}}$. $W = -F\Delta x$



Example: Pulling a Suitcase-on-Wheels

Find the work done if the force is 45.0-N, the angle is 50.0 degrees, and the displacement is 75.0 m.

$$W = (F\cos\theta)\Delta x = [(45.0\ \text{N})\cos 50.0^\circ](75.0\ \text{m})$$

$$= 2170\ \text{J}$$

5.1 Work Done by a Constant Force

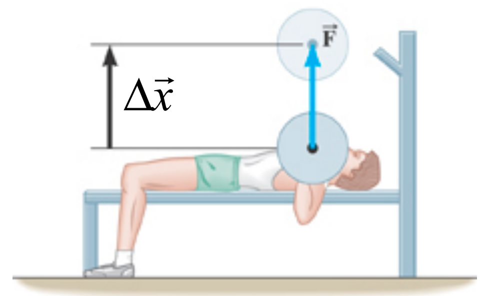
The bar bell (mass m) is moved slowly at a constant speed $\Rightarrow\ F = mg$.
The work done by the gravitational force will be discussed later.


(a)

Raising the bar bell, the displacement is up, and the force is up.

$$W = \left(F\cos 0°\right)\Delta x = F\Delta x$$

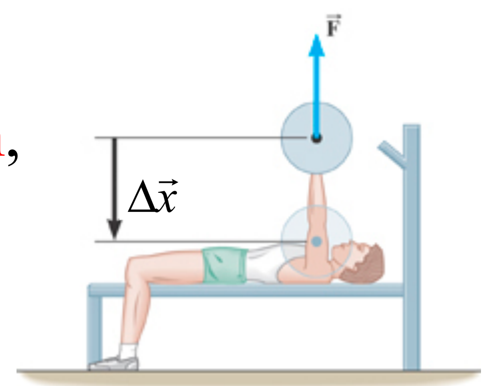
these are magnitudes!



(b)

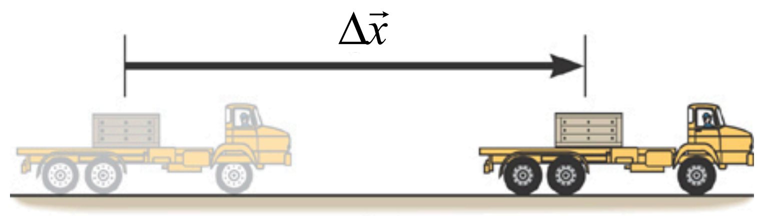
Lowering the bar bell, the displacement is down, and the force is (STILL) up.

$$W = \left(F\cos 180°\right)\Delta x = -F\Delta x$$



(c)

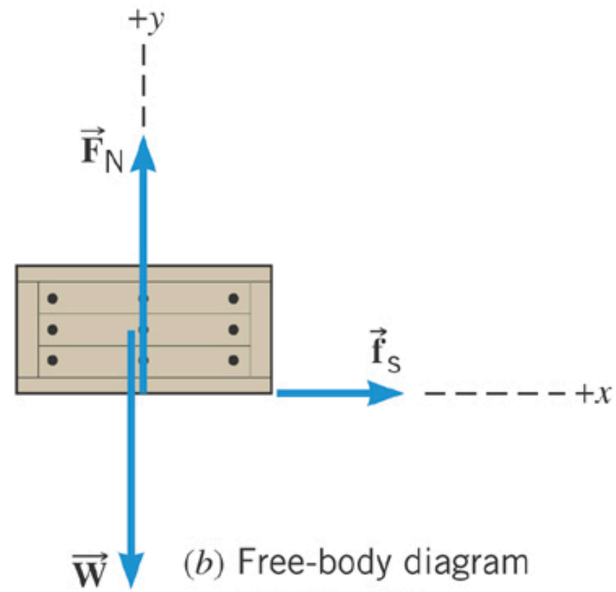
Example: Accelerating a Crate



(a)

The truck is accelerating at a rate of +1.50 m/s². The mass of the crate is 120-kg and it does not slip. The magnitude of the displacement is 65 m.

What is the total work done on the crate by all of the forces acting on it?



(b) Free-body diagram for the crate

$$(\text{normal force})\ W = \left(F_N \cos 90°\right)\Delta x = 0$$

$$(\text{gravity force})\ W = \left(F_G \cos 90°\right)\Delta x = 0$$

$$(\text{friction force})\ W = \left(f_S \cos 0°\right)\Delta x = f_S \Delta x$$

$$= (180\text{ N})(65\text{ m}) = 12\text{ kJ}$$

$$f_S = ma = (120\text{ kg})(1.50\text{ m/s}^2)$$
$$= 180\,\text{N}$$

$$1\text{ N}\cdot\text{m} = 1\text{ joule (J)}$$

5.2 Work on a Spring & Work by a Spring

HOOKE'S LAW

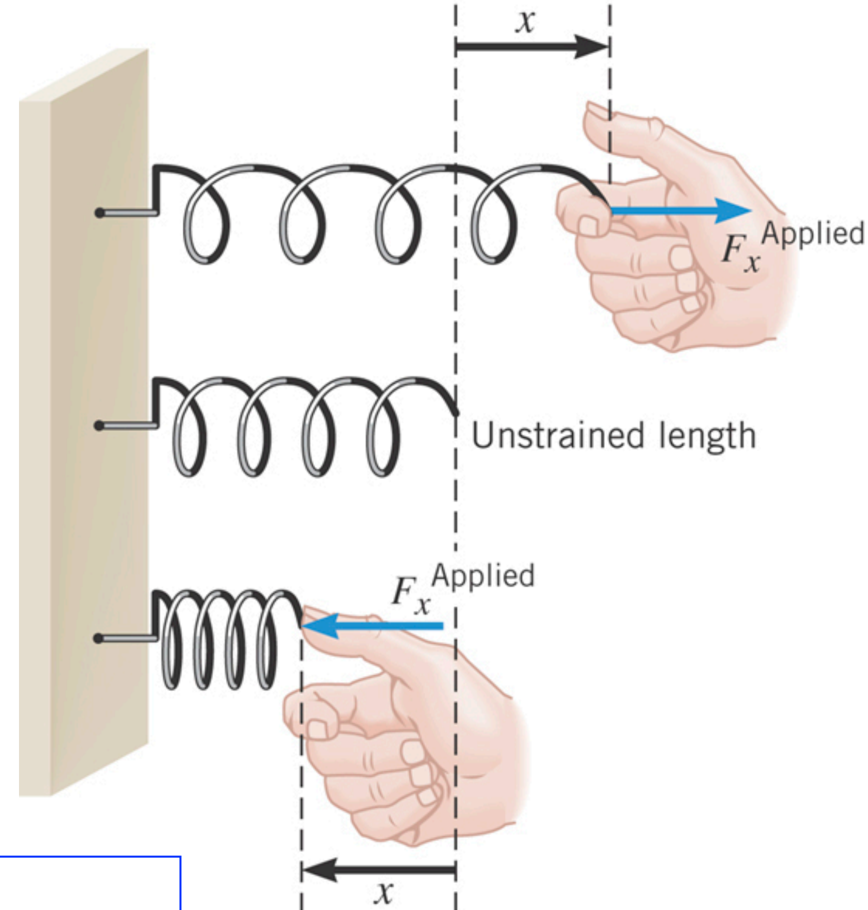
Force Required to Distort an Ideal Spring

The force applied to an ideal spring is proportional to the displacement of its end.

$$F_x^{\text{Applied}} = kx$$

spring constant

Units: N/m



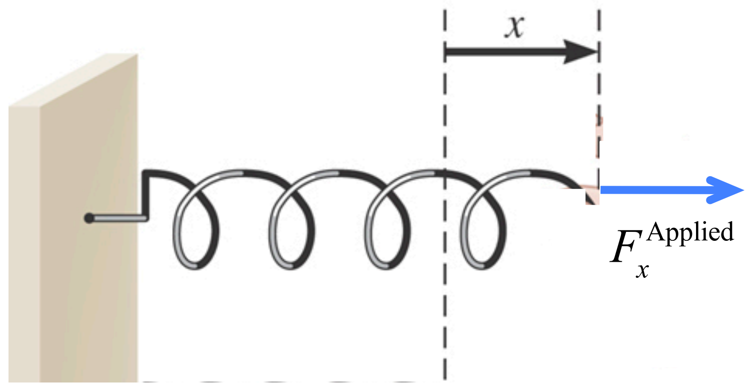
This is a scalar equation

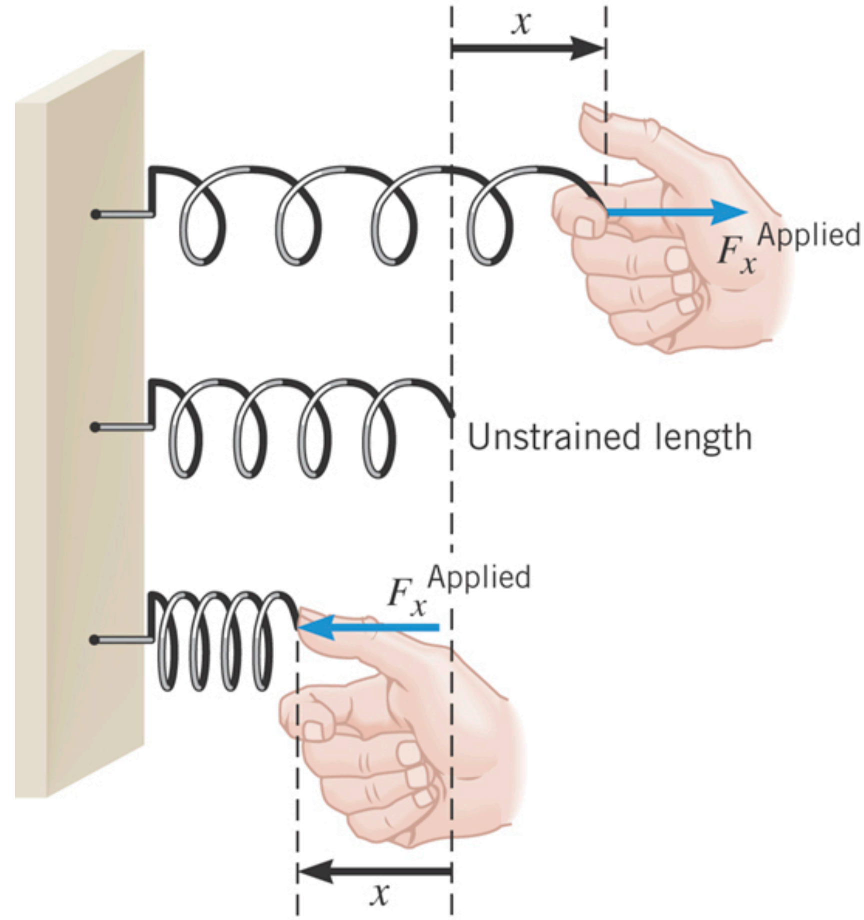
F_x^{Applied} is magnitude of applied force.

x is the magnitude of the spring displacement

k is the spring constant (strength of the spring)

5.2 Work on a Spring & Work by a Spring





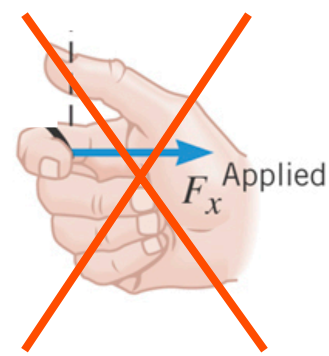
F_x^{Applied} is applied to the spring.

This force can come from anywhere.

The wall generates a force on the spring.

F_x^{Applied} acts ON the SPRING

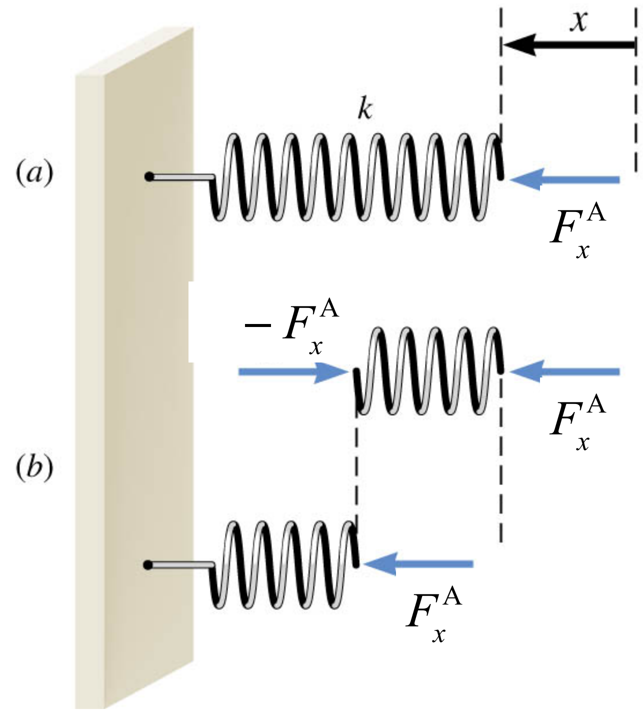
NOT on the HAND



5.2 Work on a Spring & Work by a Spring

Conceptual Example: Is ½ a spring stronger or weaker? A 10-coil spring has a spring constant *k*. The spring is cut in half, so there are two 5-coil springs. What is the spring constant of each of the smaller springs?

Original Spring: $F_x^{\mathrm{A}} = kx; \quad k = \dfrac{F_x^{\mathrm{A}}}{x}$



Compression of each piece $x' = x/2$.

Apply the same force as before!

Spring constant of each piece

$$k' = \frac{F_x^{\mathrm{A}}}{x'} = \frac{F_x^{\mathrm{A}}}{x/2}$$

$$= 2\left(\frac{F_x^{\mathrm{A}}}{x}\right) = 2k \text{ (twice as strong)}$$

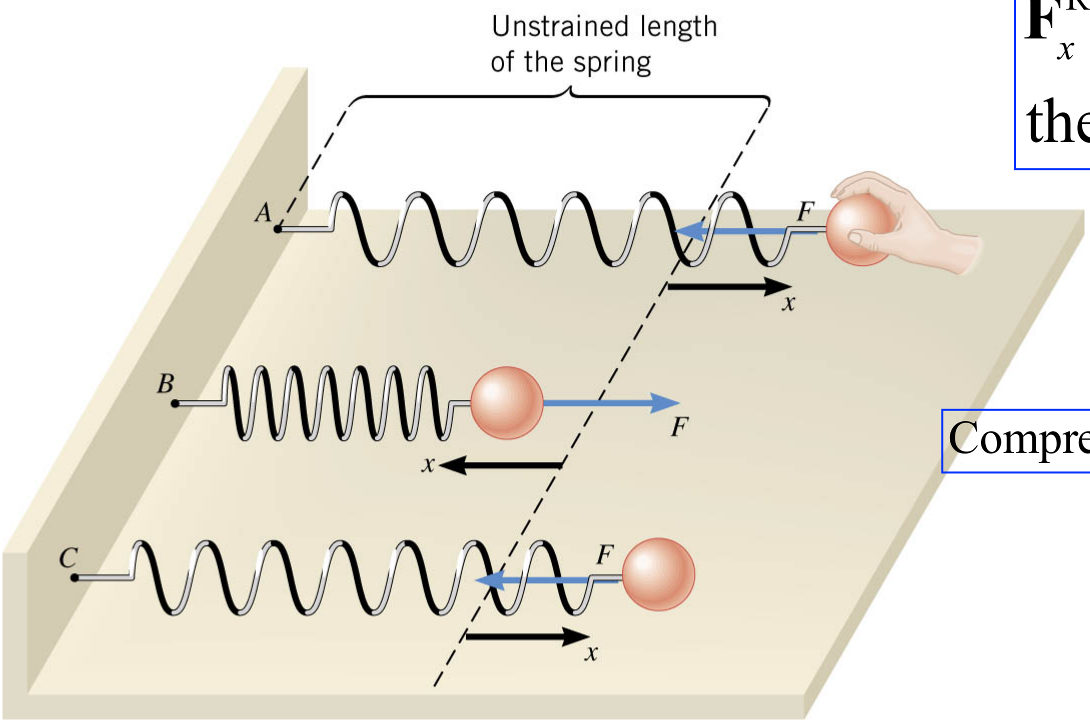
5.2 Work on a Spring & Work by a Spring

HOOKE'S LAW
Restoring Force Generated by a Distorted Ideal Spring

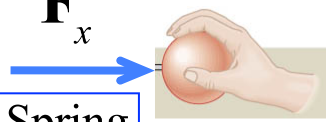
The restoring force generated by an ideal spring is proportional to the displacement of its end:

$$F_x^R = -kx$$

F_x^R : restoring force generated by the stretched or compressed spring.

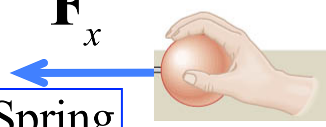


F_x^R



Compressed Spring

F_x^R



Stretched Spring

Restoring forces act on ball/hand.

Conceptual Example 2 Are Shorter Springs Stiffer?

A 10-coil spring has a spring constant *k*. If the spring is cut in half, so there are two 5-coil springs, what is the spring constant of each of the smaller springs?

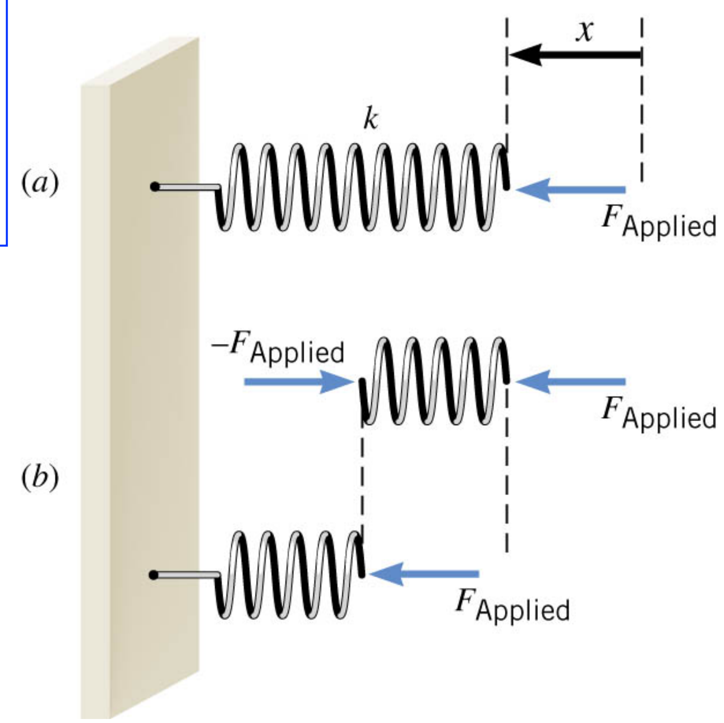
$$F_A = kx; \quad k = \frac{F_A}{x}$$

Each piece $x' = x/2$. Same force applied.

New spring constant of each piece

$$k' = \frac{F_A}{x'} = \frac{F_A}{x/2}$$

$$= 2\left(\frac{F_A}{x}\right) = 2k \text{ (twice as strong)}$$



5.2 Work on a Spring & Work by a Spring

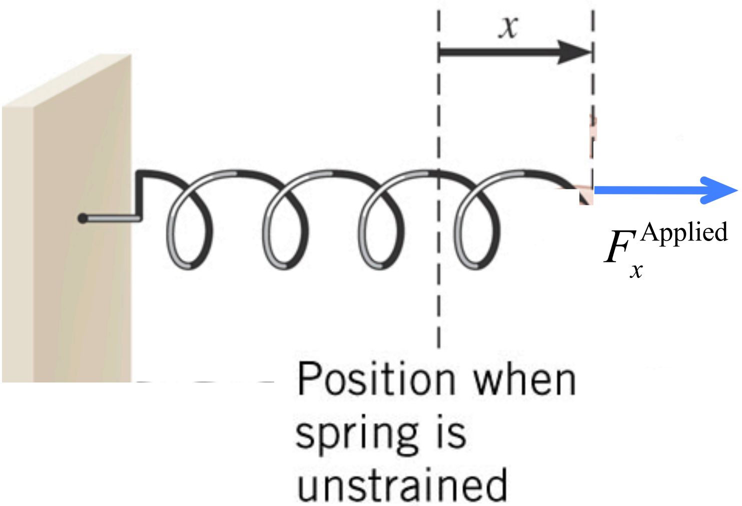
Work done by applied force stretching (or compressing) a spring.
Force is changing while stretching – so use the average force.

$\bar{F}$ is the magnitude of the <u>average force</u> while stretching, $\frac{1}{2}(kx+0)$

Δx is the magnitude of the displacement, (x)

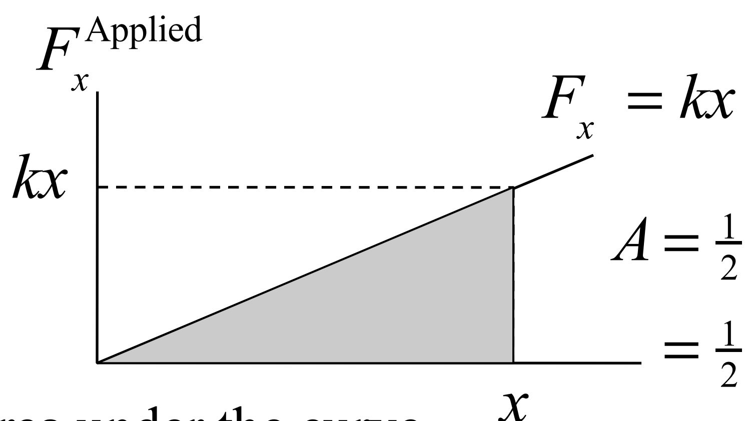
θ is the angle between the force and displacement vectors, $(0°)$

W is the work done on the spring by the applied force



$$W = \left(\bar{F}\cos\theta\right)\Delta x$$

$$= \tfrac{1}{2}\left(kx\right)\cos(0^\circ)(x) = \tfrac{1}{2}kx^2 \quad \text{(positive)}$$



$$A = \tfrac{1}{2}bh$$

$$= \tfrac{1}{2}(x)(kx) = \tfrac{1}{2}kx^2$$

work is the area under the curve

5.2 Work on a Spring & Work by a Spring

Restoring force of a stretched spring can do work on a mass.

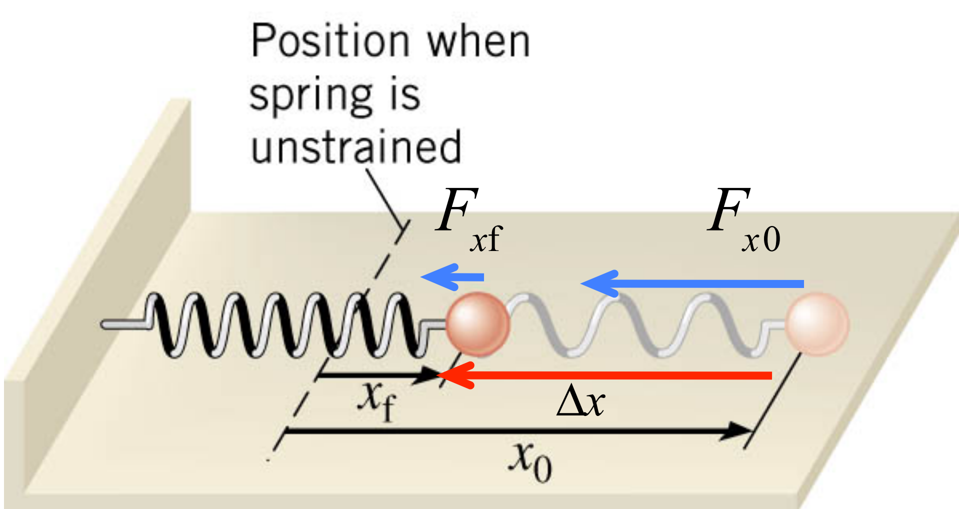
$\bar{F}$ is the magnitude of the average force, $\frac{1}{2}(kx_0 + kx_f)$

Δx is the magnitude of the displacement, $|\Delta\vec{x}| = (x_0 - x_f)$, $x_0 > x_f$

θ is the angle between the force and displacement vectors, (0°)

$$W_{\text{elastic}} = \left(\bar{F}\cos\theta\right)\Delta x$$

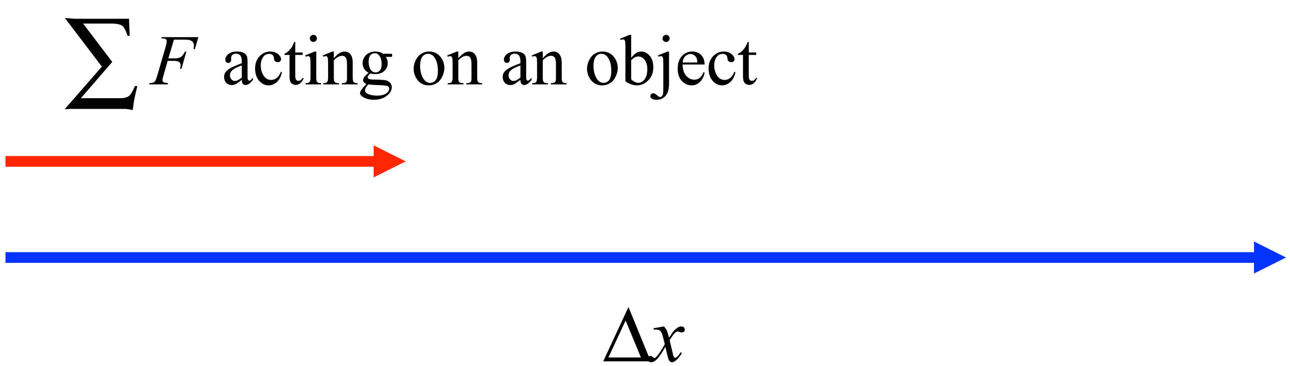
$$= \tfrac{1}{2}\left(kx_f + kx_0\right)\cos(0^\circ)(x_0 - x_f) = \tfrac{1}{2}kx_0^2 - \tfrac{1}{2}kx_f^2 \quad \text{(positive)}$$



5.3 The Work-Energy Theorem and Kinetic Energy

Consider a constant net external force acting on an object.

The object is displaced a distance Δx, in the same direction as the net force.



The work is simply $W = \left(\sum F\right)\Delta x = \left(ma\right)\Delta x$

5.3 The Work-Energy Theorem and Kinetic Energy

We have often used this 1D motion equation using v_x for final velocity:

$$v_x^2 = v_{0x}^2 + 2a\Delta x$$

Multiply equation by $\frac{1}{2}m$ (why?)

$$\frac{1}{2}mv_x^2 = \frac{1}{2}mv_{0x}^2 + ma\Delta x \qquad \text{but } F_{\text{Net}} = ma$$

$$\frac{1}{2}mv_x^2 = \frac{1}{2}mv_{0x}^2 + F_{\text{Net}}\Delta x \qquad \text{but net work, } W_{\text{Net}} = F_{\text{Net}}\Delta x$$

DEFINE KINETIC ENERGY of an object with mass m speed v:

$$K = \frac{1}{2}mv^2$$

Now it says, Kinetic Energy of a mass changes due to Work:

$$K = K_0 + W_{\text{Net}}$$

or

$$K - K_0 = W_{\text{Net}}$$

Work–Energy Theorem