Chapter 5

Work and Energy

continued

5.2 Work on a Spring & Work by a Spring

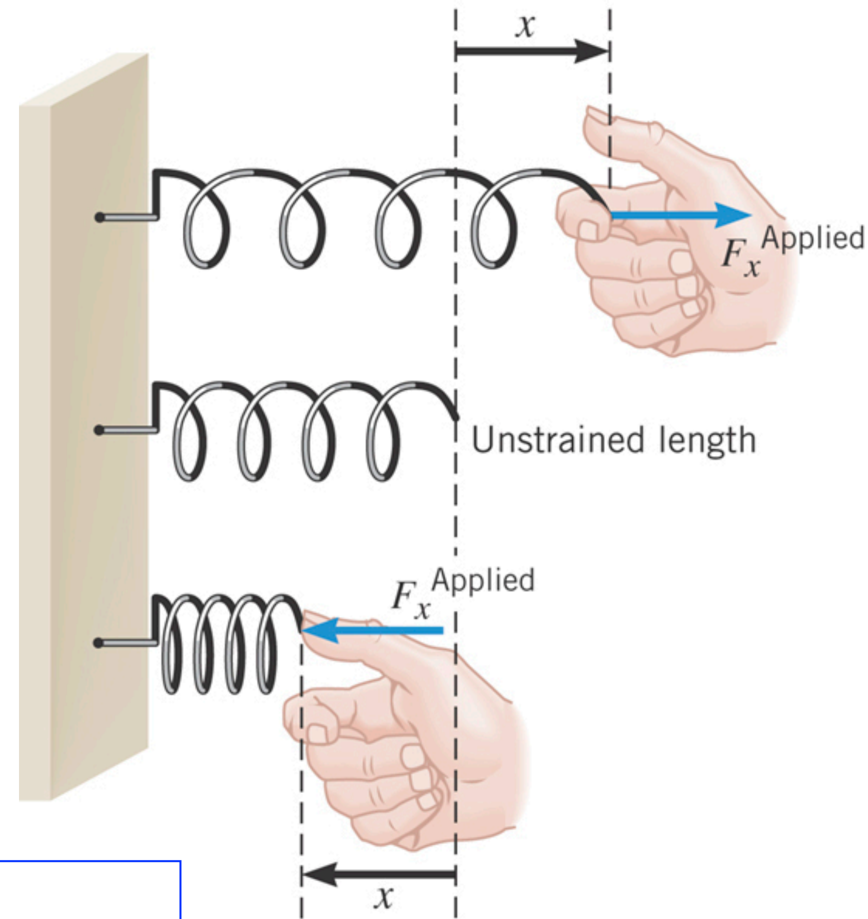
HOOKE'S LAW

Force Required to Distort an Ideal Spring

The force applied to an ideal spring is proportional to the displacement of its end.

$$F_x^{\text{Applied}} = kx$$

spring constant
Units: N/m



This is a scalar equation

F_x^{Applied} is magnitude of applied force.

x is the magnitude of the spring displacement

k is the spring constant (strength of the spring)

5.2 Work on a Spring & Work by a Spring

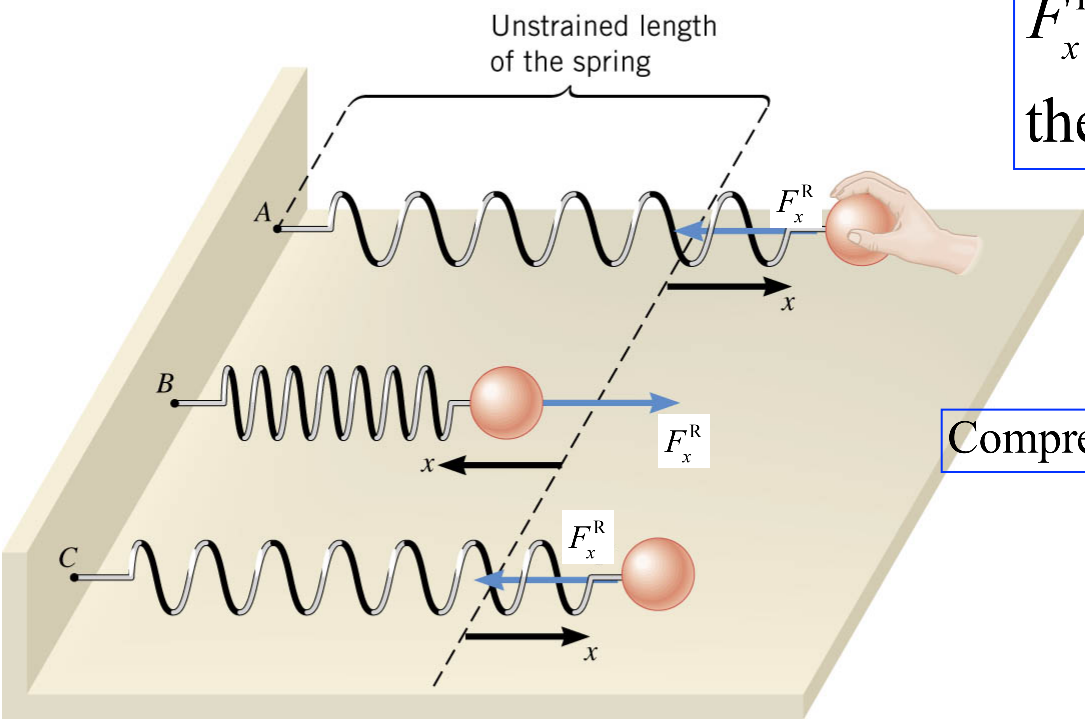
HOOKE'S LAW

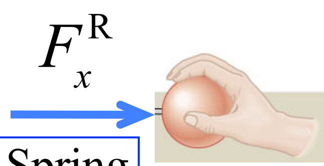
Restoring Force Generated by a Distorted Ideal Spring

The restoring force generated by an ideal spring is proportional to the displacement of its end:

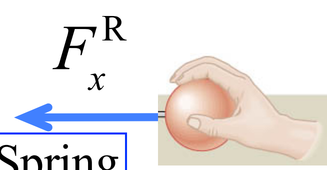
$$F_x^R = -kx$$

F_x^R : restoring force generated by the stretched or compressed spring.





Compressed Spring



Stretched Spring

Restoring forces act on ball/hand.

5.2 Work on a Spring & Work by a Spring

Work done by the restoring force of a stretched spring

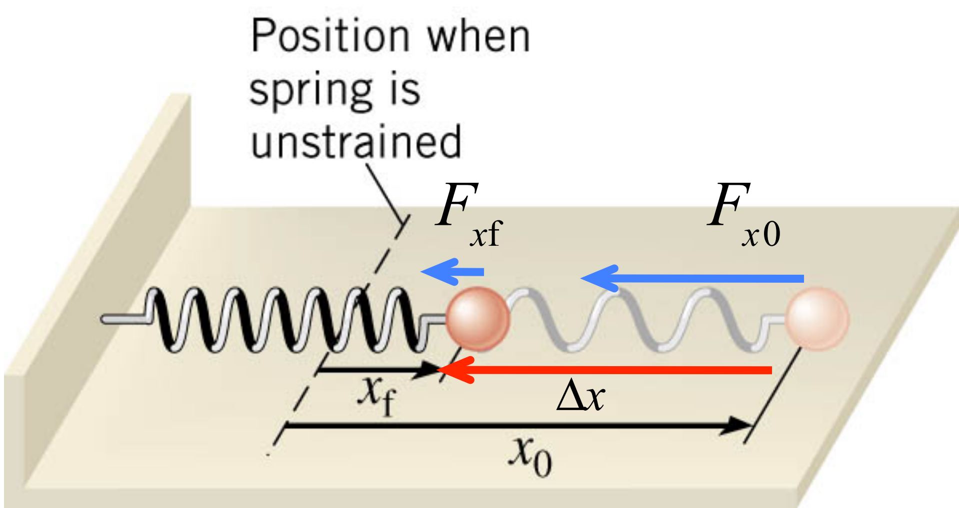
$\bar{F}$ is the magnitude of the average restoring force, $\frac{1}{2}(kx_0 + kx_f)$

Δx is the magnitude of the displacement, $|\Delta \vec{\mathbf{x}}| = (x_0 - x_f)$, $x_0 > x_f$

θ is the angle between the force and displacement vectors, (0°)

$$W_{\text{elastic}} = (\bar{F}\cos\theta)\Delta x$$

$$= \tfrac{1}{2}\left(kx_f + kx_0\right)\cos(0^\circ)(x_0 - x_f) = \tfrac{1}{2}kx_0^2 - \tfrac{1}{2}kx_f^2 \quad > 0!$$



5.3 The Work-Energy Theorem and Kinetic Energy

We have often used this 1D motion equation with

v_x for final velocity for a constant acceleration:

$$v_x^2 = v_{0x}^2 + 2a\Delta x$$

Multiply equation by $\frac{1}{2}m$ (you will see why)

$$\tfrac{1}{2}mv_x^2 = \tfrac{1}{2}mv_{0x}^2 + ma\Delta x \quad \text{but } F_{\text{Net}} = ma$$

$$\tfrac{1}{2}mv_x^2 = \tfrac{1}{2}mv_{0x}^2 + F_{\text{Net}}\Delta x \quad \text{and } W = F_{\text{Net}}\Delta x$$

DEFINE KINETIC ENERGY of object with mass m speed v:

$$K = \tfrac{1}{2}mv^2$$

Now equation says, Kinetic Energy changes due to Work on object:

$$K = K_0 + W$$

or

$$K - K_0 = W$$

Work–Energy Theorem

5.3 The Work-Energy Theorem and Kinetic Energy

Work and Energy

Work: the effect of a force acting on an object making a displacement.

$$W = (F_{\text{Net}} \cos\theta)\Delta x,$$

where W is the work done, $F_{\text{Net}}, \Delta x$ are the magnitudes of the force and displacement, and θ is the angle between $\vec{\mathbf{F}}_{\text{Net}}$ and $\Delta\vec{\mathbf{x}}$.

The origin of the force does not affect the calculation of the work done.

Work can be done by: gravity, elastic, friction, explosion, or human forces.

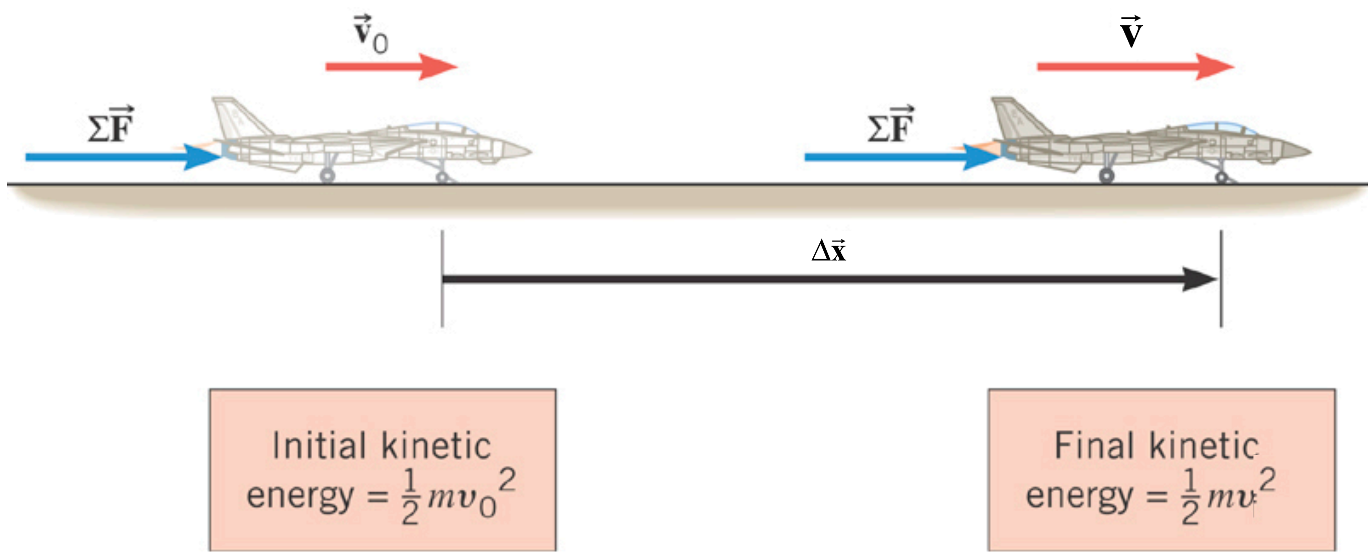
Kinetic energy: property of a mass (m) and the square of its speed (v).

$$\mathrm{K} = \tfrac{1}{2} m v^2$$

Work-Energy Theorem: Work changes the Kinetic Energy of an object.

$$\mathrm{K} = \mathrm{K}_0 + W \qquad \text{or} \qquad \mathrm{K} - \mathrm{K}_0 = W$$

5.3 The Work-Energy Theorem and Kinetic Energy



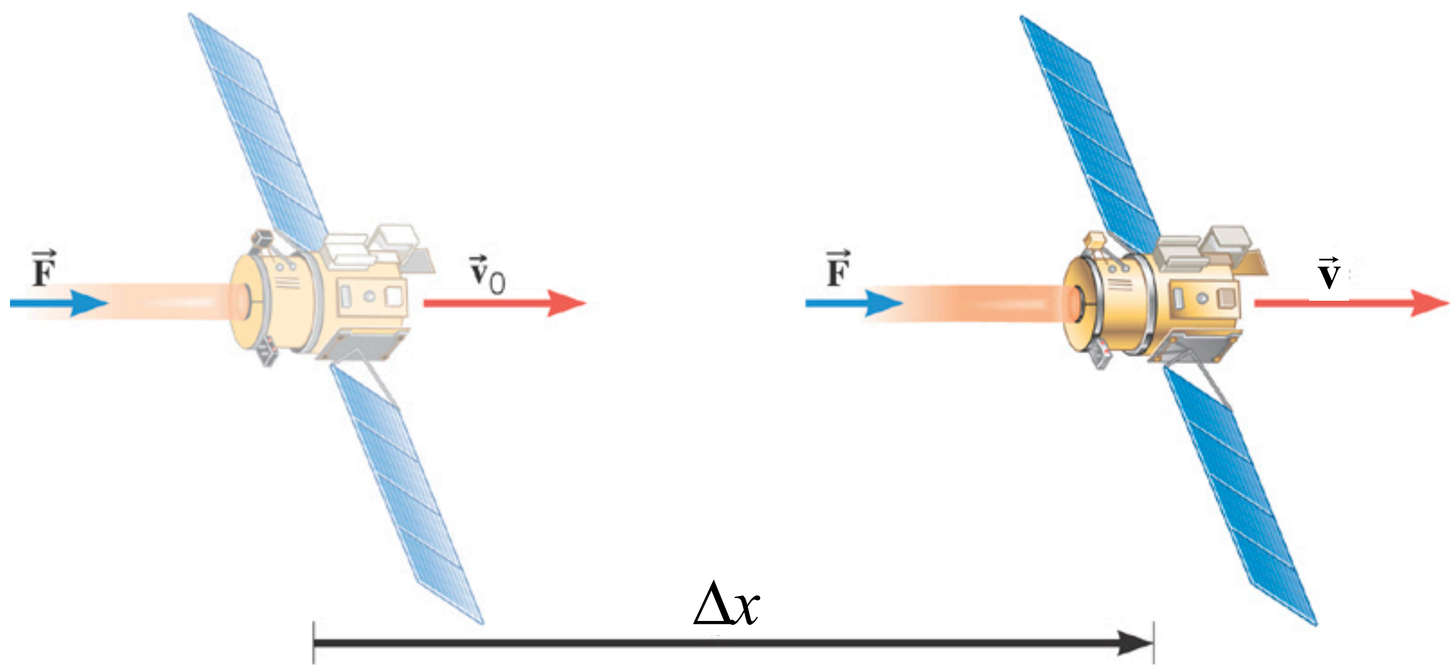
THE WORK-ENERGY THEOREM

When a net external force does work on an object, the kinetic energy of the object changes according to

$$W = K - K_0 = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2$$

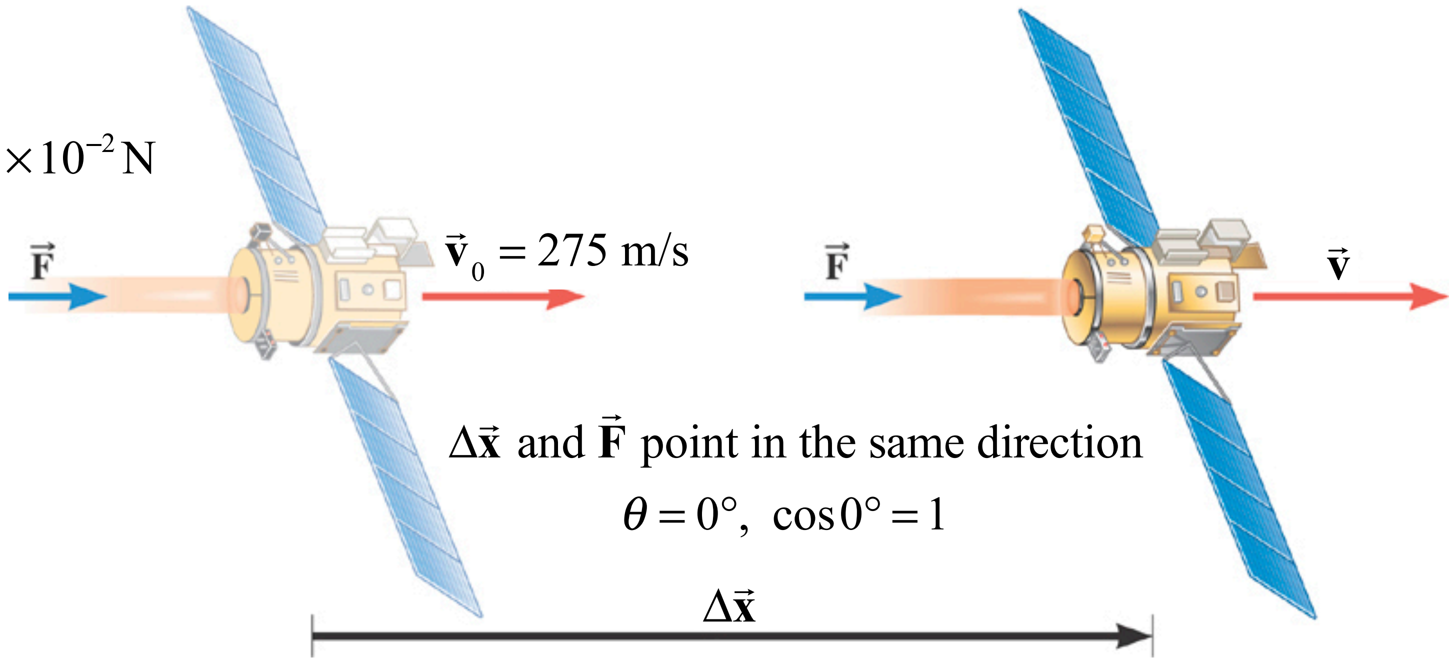
Example:

The mass of the space probe is 474-kg and its initial velocity is 275 m/s. If the 56.0-mN force acts on the probe through a displacement of 2.42×10^9m, what is its final speed?



5.3 The Work-Energy Theorem and Kinetic Energy

$\vec{\mathbf{F}} = 5.60 \times 10^{-2}\,\text{N}$



$$W = \left[F\cos\theta\right]\Delta x = \left(5.60\times10^{-2}\,\text{N}\right)\left(2.42\times10^{9}\,\text{m}\right) = 1.36\times10^{8}\,\text{J}$$

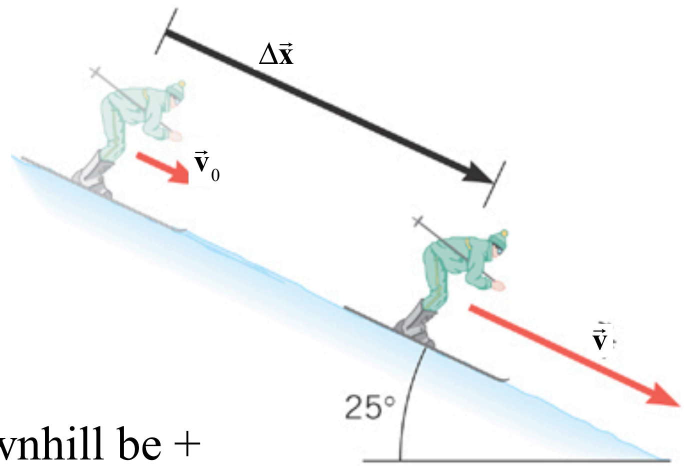
$$W = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2$$ Solve for final velocity v

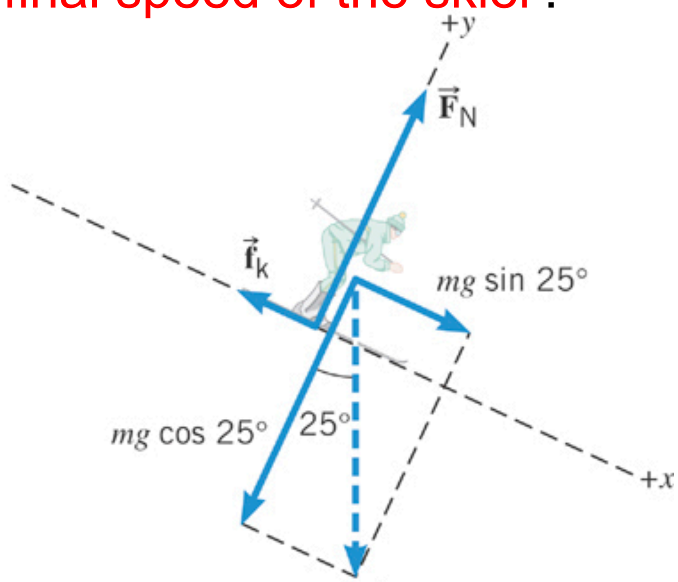
$$v^2 = \frac{2W}{m} + v_0^2 = \frac{2.72\times10^{8}\,\text{J}}{474\ \text{kg}} + (275\,\text{m/s})^2$$

$$v = 806\ \text{m/s}$$

5.3 The Work-Energy Theorem and Kinetic Energy

Example - A 58.0 kg skier experiences a kinetic frictional force of 71.0N while traveling down a 25° hill for a distance of 57.0 m. If the skier's initial speed was 3.60 m/s, what is the final speed of the skier?





Let downhill be +

The net force is $F_{Net} = mg\sin 25^\circ - f_k = 170\text{N}$

Decomposition of the downward gravitational force, *mg*.

Work-Energy Theorem:

$$K - K_0 = W \quad \Rightarrow \quad K = K_0 + W$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}mv_0^2 + F_{Net}(\cos 0^\circ)\Delta x$$

$$v^2 = v_0^2 + \frac{2F_{Net}\Delta x}{m} \Rightarrow v = \sqrt{v_0^2 + \frac{2F_{Net}\Delta x}{m}} = 18.6 \text{ m/s}$$

5.4 Gravitational Potential Energy

Magnitude of $\Delta\vec{\mathbf{y}}$ written as Δy

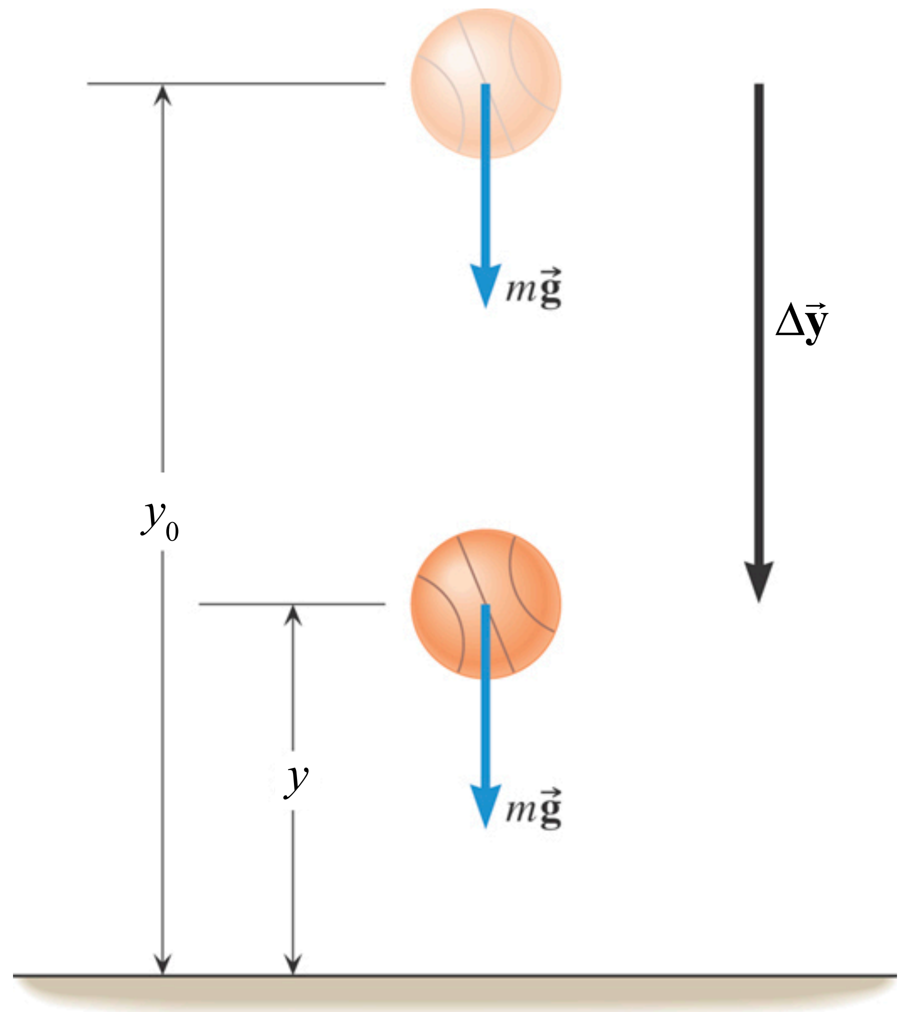
$\Delta y = \text{distance of fall} = (y_0 - y)$

This θ is the angle between $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{y}}$.

$$W = \left(F\cos\theta\right)\Delta y$$

$$= mg\Delta y$$

$$W_G = mg\left(y_0 - y\right)$$



5.4 Conservative Versus Nonconservative Forces

DEFINITION OF A CONSERVATIVE FORCE

Version 1 A force is conservative when the work it does on a moving object is independent of the path between the object's initial and final positions.

Version 2 A force is conservative when it does no net work on an object moving around a closed path, starting and finishing at the same point.

Also:
Version 2' A force is conservative when it can remove energy from a mass over its displacement and then reverse the displacement and return the energy to the mass without loss.

5.4 Conservative Versus Nonconservative Forces

Some Conservative and Nonconservative Forces

Conservative Forces Conservation of energy OK

- Gravitational force
- Elastic spring force
- Electric force

Nonconservative Forces Add or remove energy

- Static and kinetic frictional forces (remove energy)
- Air resistance (removes energy)
- Muscular forces (add or remove energy)
- Explosions (add energy)
- Jet or rocket forces (add or remove energy)

5.4 *Gravitational Potential Energy*

Because gravity is a conservative force, when a mass moves upward against the gravitational force, the kinetic energy of the mass decreases, but when the mass falls to its initial height that kinetic energy returns completely to the mass.

When the kinetic energy decreases, where does it go?

DEFINITION OF GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy U is the energy that an object (mass m) has by virtue of its position relative to the surface of the earth. That position is measured by the height y of the object relative to an arbitrary zero level:

$$\boxed{U = mgy} \qquad 1\,\mathrm{N \cdot m} = 1\ \mathrm{joule}\,(\mathrm{J})$$

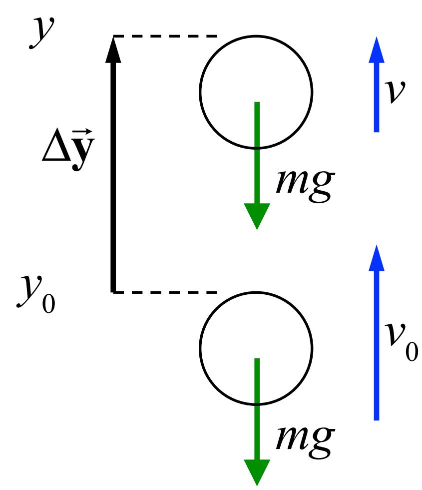
(y can be + or –)

5.4 Gravitational Potential Energy

Thrown upward

Gravitational work is negative with $\Delta\vec{\mathbf{y}}$ positive.

$$W_G = (F\cos 180°)\Delta y$$
$$= -mg(y - y_0)$$



Gravitational Potential Energy (*U*) increases.

$$U - U_0 = mgy - mgy_0 = mg(y - y_0)$$
$$= -W_G$$

With just Gravity acting, Work-Energy Theorem becomes:

$$K - K_0 = W_G$$
$$= -(U - U_0)$$

Final values to the left side

Initial values to the right side

$$K + U = K_0 + U_0 \quad \text{Conservation of Energy}$$

5.4 Gravitational & Spring Potential Energy

GRAVITATIONAL POTENTIAL ENERGY

Energy of mass m due to its position relative to the surface of the earth.
Position measured by the height y of mass relative to an arbitrary zero level:

$$U = mgy$$

U replaces Work by gravity in the Work-Energy Theorem

Work-Energy Theorem becomes Mechanical Energy Conservation:

$$K + U = K_0 + U_0$$
$$E = E_0$$

Initial total energy, $\mathrm{E}_0 = K_0 + U_0$ doesn't change.
It is the same as final total energy, $E = K + U$.

IDEAL SPRING POTENTIAL ENERGY

Potential energy will be stored by a spring stretched or compressed from its natural length.

$$U = \tfrac{1}{2}kx^2$$

k is the spring constant from Hooke's law $F_x^R = -kx$
x is the distortion of the spring from its natural length

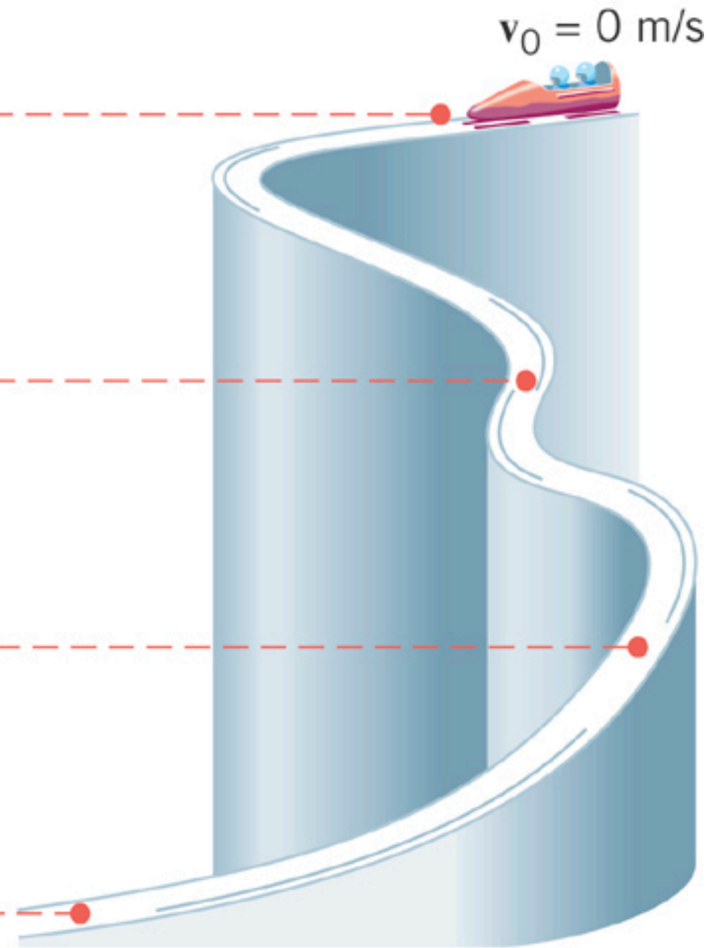
Use in the Mechanical Energy Conservation equation above.

5.5 *The Conservation of Mechanical Energy*

Sliding without friction: only gravity does work.

Normal force of ice is always perpendicular to displacements.

K	U	$E = K + U$
0 J	600 000 J	600 000 J
200 000 J	400 000 J	600 000 J
400 000 J	200 000 J	600 000 J
600 000 J	0 J	600 000 J



5.5 Conservative Versus Nonconservative Forces

In many situations both conservative and non-conservative forces act simultaneously on an object, so the work done by the net external force can be written as

$$W_{\mathrm{Net}} = W_{\mathrm{C}} + W_{\mathrm{NC}}$$

W_{C} = work by conservative force such as work by gravity W_{G}

But replacing W_{C} with $-(U - U_0)$

Work-Energy Theorem becomes:

$$K + U = K_0 + U_0 + W_{\mathrm{NC}}$$
$$\mathrm{E}_{\mathrm{f}} = \mathrm{E}_0 + W_{\mathrm{NC}}$$

work by non-conservative forces will add or remove energy from the mass

$$E = K + U \neq E_0 = K_0 + U_0$$

Another (equivalent) way to think about it:

$$\left(K - K_0\right) + \left(U - U_0\right) = W_{\mathrm{NC}}$$
$$\Delta K + \Delta U = W_{\mathrm{NC}}$$

if non-conservative forces do work on the mass, energy changes will not sum to zero

5.5 The Conservation of Mechanical Energy

Just remember and use this:

$$K + U = K_0 + U_0 + W_{NC}$$

non-conservative forces add or remove energy

$$\text{If } W_{NC} \neq 0, \text{ then } E \neq E_0$$

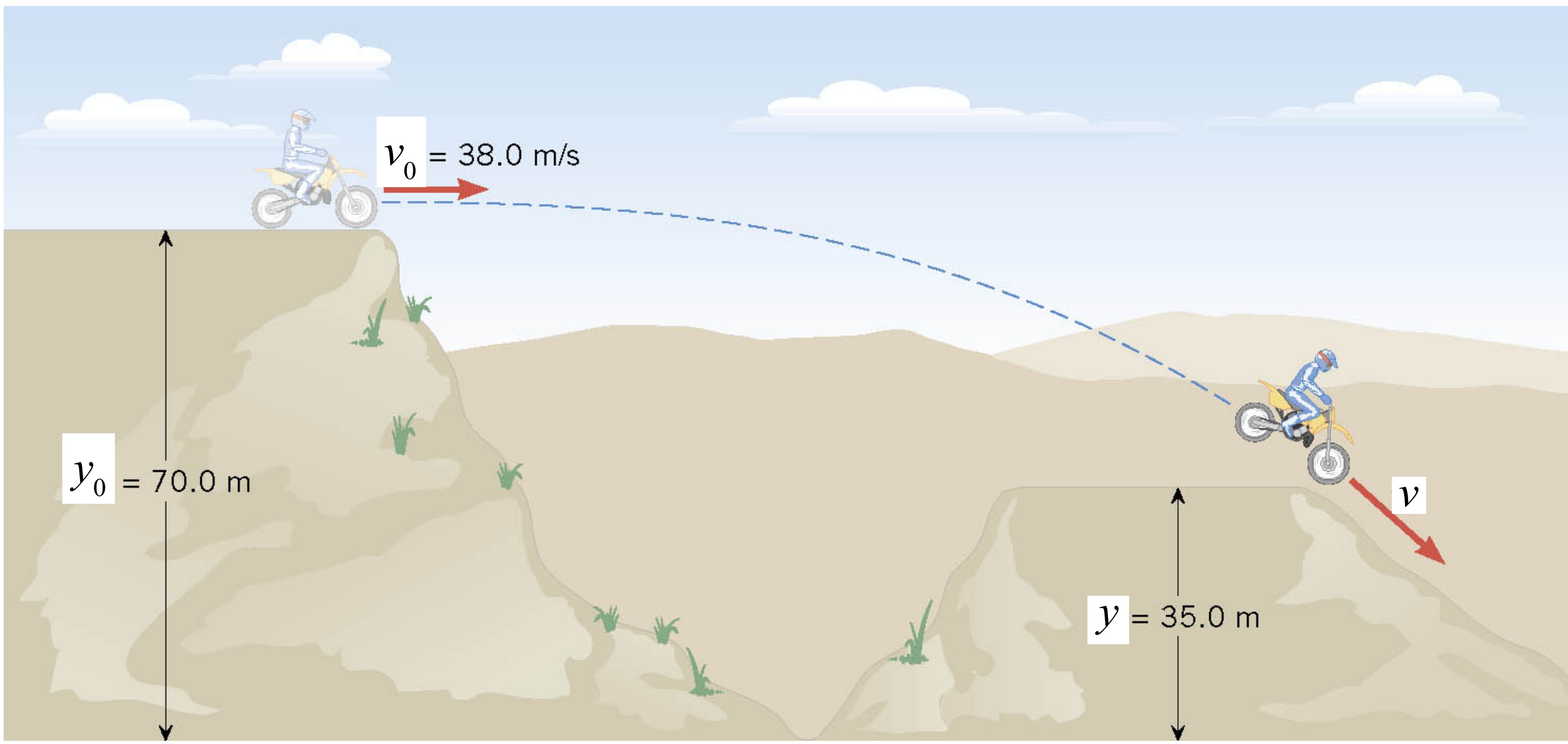
If the net work on a mass by non-conservative forces is zero, then its total energy does not change:

$$\text{If } W_{NC} = 0, \text{ then } E = E_0$$

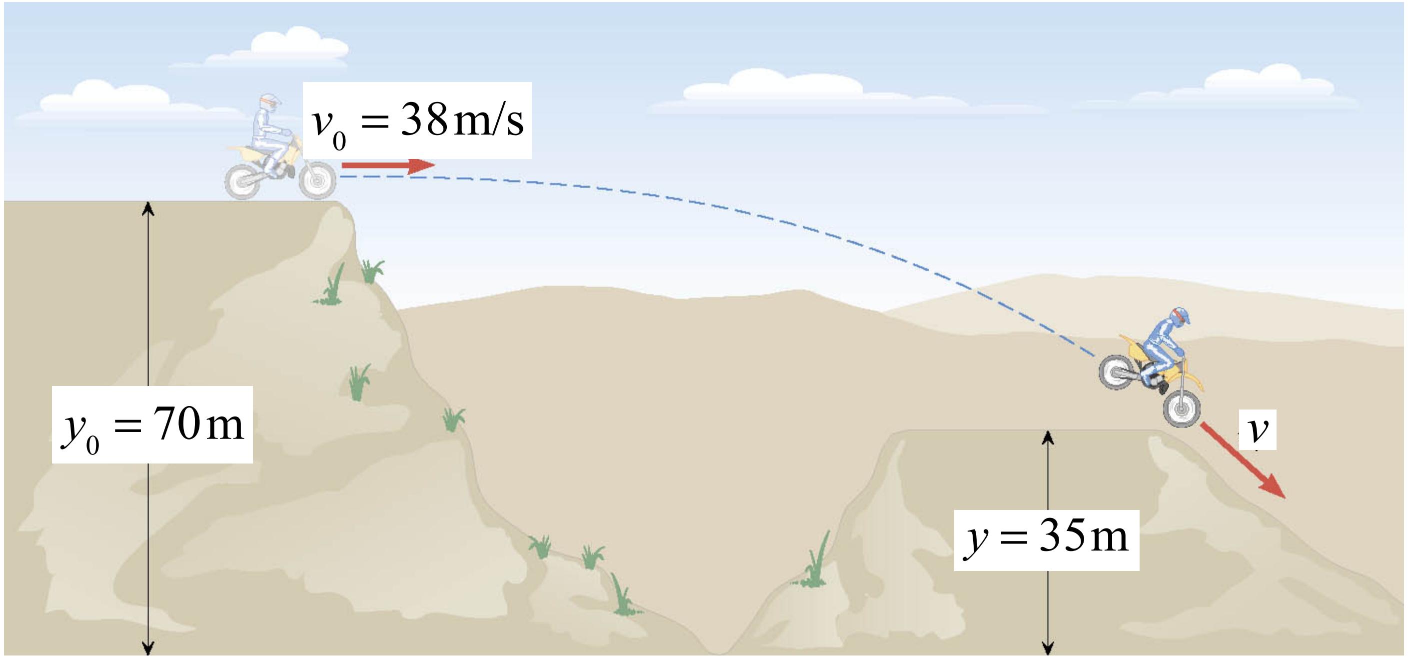
$$K + U = K_0 + U_0$$

Example: A Daredevil Motorcyclist

A motorcyclist is trying to leap across the canyon by driving horizontally off a cliff 38.0 m/s. Ignoring air resistance, find the speed with which the cycle strikes the ground on the other side.



5.5 The Conservation of Mechanical Energy



$$E = E_0$$

$$mgy + \tfrac{1}{2}mv^2 = mgy_0 + \tfrac{1}{2}mv_0^2$$

(only gravity - a conservative force)

$$gy + \tfrac{1}{2}v^2 = gy_0 + \tfrac{1}{2}v_0^2$$

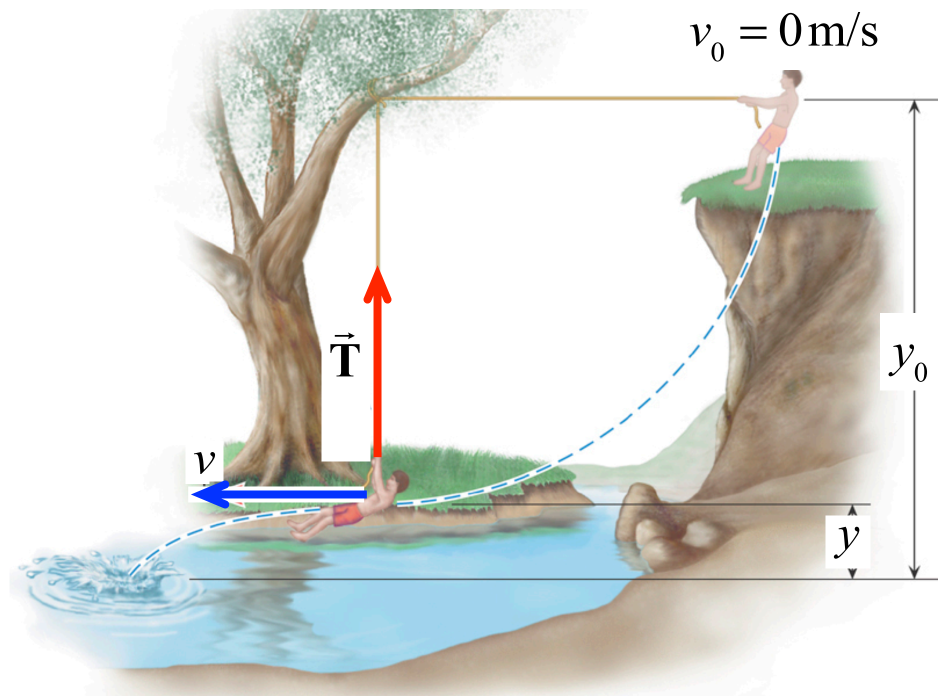
(common factor of m)

$$v = \sqrt{2g\left(y_0 - y\right) + v_0^2}$$

$$v = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(35.0\,\text{m}\right) + \left(38.0\,\text{m/s}\right)^2} = 46.2\,\text{m/s}$$

Conceptual Example: The Favorite Swimming Hole

The person starts from rest, with the rope held in the horizontal position, swings downward, and then lets go of the rope, with no air resistance. Two forces act on him: gravity and the tension in the rope.

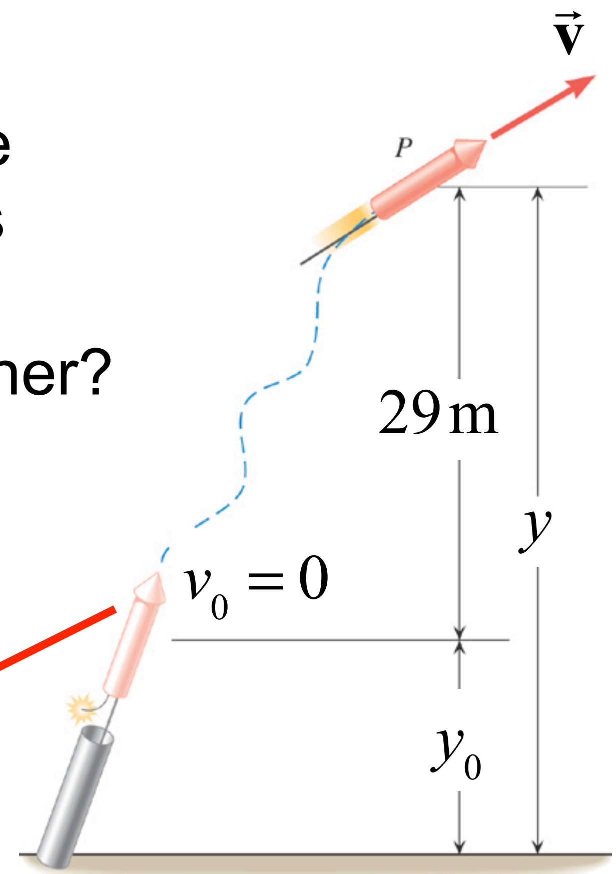


Note: tension in rope is always perpendicular to displacement, and so, does no work on the mass.

The principle of conservation of energy can be used to calculate his final speed.

Example: Fireworks

Assuming that the nonconservative force generated by the burning propellant does 425 J of work, what is the final speed of the rocket (*m* = 0.20kg) when 29m higher? Ignore air resistance.



$$E = E_0 + W_{NC} \qquad W_{NC} = 425\text{J}$$

$$\rightarrow W_{NC} = E - E_0$$

$$W_{\text{NC}} = \left(mgy + \tfrac{1}{2}mv^2\right) - \left(mgy_0 + \tfrac{1}{2}mv_0^2\right)$$

$$= mg\left(y - y_0\right) + \tfrac{1}{2}mv^2$$

$$v^2 = 2W_{\text{NC}}/m - 2g\left(y - y_0\right)$$

$$= 2\left(425\text{ J}\right)/\left(0.20\text{ kg}\right) - 2\left(9.81\text{m/s}^2\right)\left(29.0\text{ m}\right)$$

$$v = 60.7\text{ m/s}$$

5.6 *Power*

DEFINITION OF AVERAGE POWER

Average power is the rate at which work is done, and it is obtained by dividing the work by the time required to perform the work.

$$\overline{P} = \frac{\text{Work}}{\text{Time}} = \frac{W}{t}$$

$$\text{joule/s} = \text{watt (W)}$$

Note: 1 horsepower = 745.7 watts

$$\overline{P} = \frac{\text{Change in energy}}{\text{Time}}$$

$$\overline{P} = \frac{W}{t} = \frac{F_x \Delta x}{t} = F_x\left(\frac{\Delta x}{t}\right) = F_x \overline{v}_x$$

Table of **Human Metabolic Rates**[a]

Activity	Rate (watts)
Running (15 km/h)	1340 W
Skiing	1050 W
Biking	530 W
Walking (5 km/h)	280 W
Sleeping	77 W

[a]For a young 70-kg male.

THE PRINCIPLE OF CONSERVATION OF ENERGY

Energy can neither be created not destroyed, but can only be converted from one form to another.

Heat energy is the kinetic or vibrational energy of molecules. The result of a non-conservative force is often to remove mechanical energy and transform it into heat.

Examples of heat generation: sliding friction, muscle forces.