Chapter 6

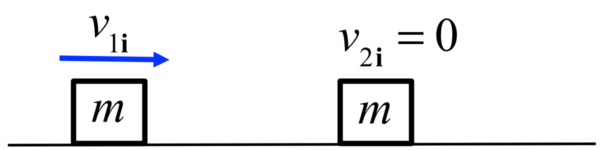
Impulse and Momentum

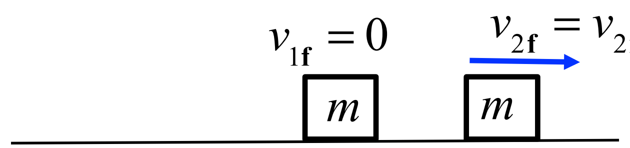
Continued

6.3 Collisions in One Dimension

Elastic collisions $(x - \text{components of the velocities})$

Initial — Equal masses — Final





$(m_1 = m_2 = m)$ masses cancel in both equations

Momentum conservation: $v_{1i} = v_{1f} + v_{2f}$

Energy conservation: $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$

$$v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f} = v_{1f}^2 + v_{2f}^2$$

$$2v_{1f}v_{2f} = 0$$

$$v_{1f} = 0; \quad v_{2f} = v_{1i}$$

Incoming mass stops, target mass gets initial momentum

6.3 Collisions in One Dimension

Elastic collisions $(x-\text{components of the velocities})$

Initial — **Equal masses** — Final

$(m_1 = m_2 = m)$ masses cancel in both equations

Momentum conservation: $v_{1i} = v_{1f} + v_{2f}$

Energy conservation: $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$

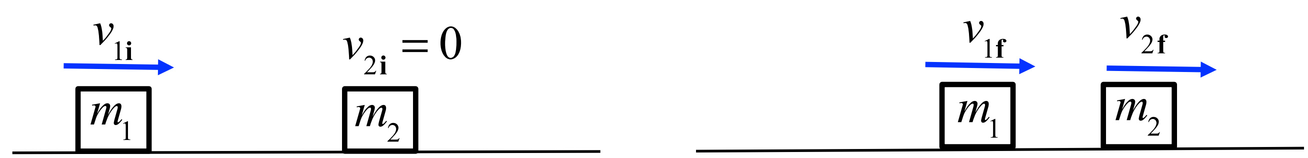
$$v_{1f}^2 + v_{2f}^2 + 2v_{1f}v_{2f} = v_{1f}^2 + v_{2f}^2$$

$$2v_{1f}v_{2f} = 0$$

$$v_{1f} = 0; \quad v_{2f} = v_{1i}$$

Incoming mass stops, target mass gets initial momentum

Initial — **Unequal masses** — Final



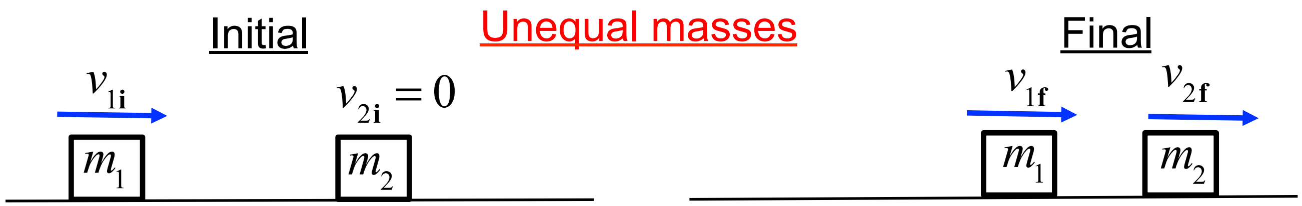
Momentum conservation: $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \Rightarrow \quad v_{1i} - v_{1f} = \frac{m_2}{m_1} v_{2f}$

Energy conservation: $\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \Rightarrow v_{1i}^2 - v_{1f}^2 = \frac{m_2}{m_1} v_{2f}^2$

Solve for v_{1f}, v_{2f}

6.3 Collisions in One Dimension

Elastic collisions with arbitrary masses (solve for final velocities)



Momentum conservation: $$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \Rightarrow \quad v_{1i} - v_{1f} = \frac{m_2}{m_1} v_{2f} \qquad (1)$$

Energy conservation: $$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \quad \Rightarrow \quad v_{1i}^2 - v_{1f}^2 = \frac{m_2}{m_1} v_{2f}^2 \qquad (2)$$

rewrite (2) as $$\left(v_{1i} - v_{1f}\right)\left(v_{1i} + v_{1f}\right) = \frac{m_2}{m_1} v_{2f}^2$$

insert (1) $$\frac{m_2}{m_1} v_{2f}\left(v_{1i} + v_{1f}\right) = \frac{m_2}{m_1} v_{2f}^2 \quad \Rightarrow \quad v_{1i} + v_{1f} = v_{2f} \qquad (3)$$

(1) with (3) $$v_{1i} - v_{1f} = \frac{m_2}{m_1}\left(v_{1i} + v_{1f}\right) \quad \Rightarrow \quad \boxed{v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i}} \qquad (4)$$

(1) with (4) $$v_{1i} - \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} = \frac{m_2}{m_1} v_{2f} \quad \Rightarrow \quad \boxed{v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i}}$$

6.3 Collisions in One Dimension

Elastic collisions with arbitrary masses

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \quad \text{and} \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

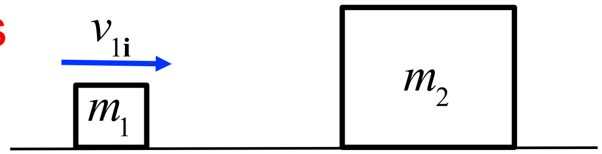
Equal mass solution is here too

Little mass hits big mass

$$m_2 >> m_1$$

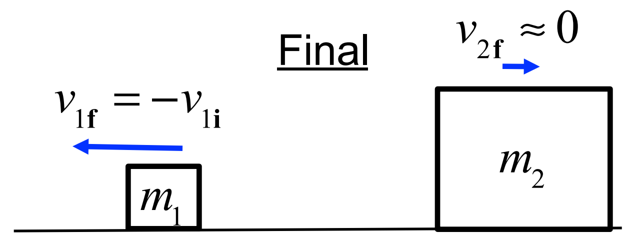
Let $m_1 \to 0$

Initial



Bounces back

Final



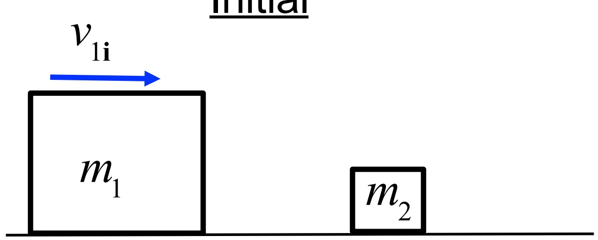
$$v_{1f} = -v_{1i} \quad \text{and} \quad v_{2f} \approx 0$$

Big mass hits little mass

$$m_1 >> m_2$$

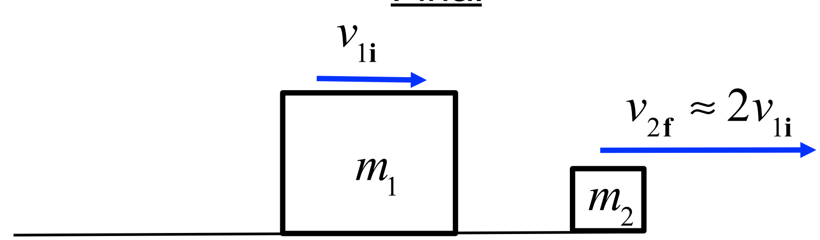
Let $m_1 \to \infty$

Initial



Small mass ejected at twice the speed

Final

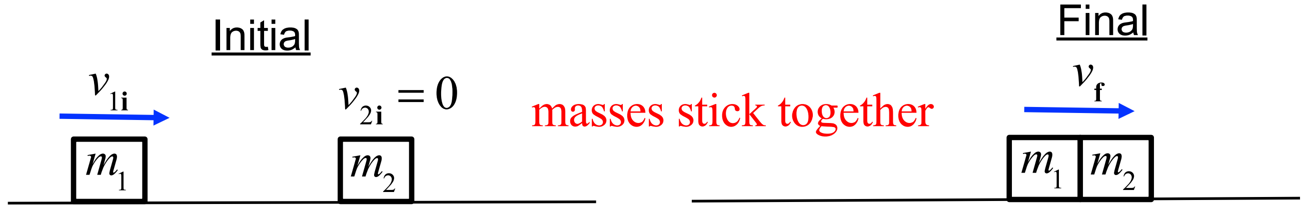


$$v_{1f} \approx v_{1i} \quad \text{and} \quad v_{2f} \approx 2v_{1i}$$

6.3 Collisions in One Dimension

Inelastic collisions (with only internal forces affecting the motion)

Equal masses:



$(x-\text{components of the velocities})$

Momentum conservation: $m_1 v_{1\mathrm{i}} = (m_1 + m_2) v_{\mathrm{f}}$

Energy NOT conserved:

$$v_{\mathrm{f}} = \frac{m_1}{m_1 + m_2} v_{1\mathrm{i}}$$

How much kinetic energy was converted to heat in the collision.

$$K_{\mathrm{i}} = \tfrac{1}{2} m_1 v_{1\mathrm{i}}^2$$

$$K_{\mathrm{f}} = \tfrac{1}{2}(m_1 + m_2) v_{\mathrm{f}}^2 = \tfrac{1}{2}(m_1 + m_2)\left[\frac{m_1}{m_1 + m_2} v_{1\mathrm{i}}\right]^2$$

$$= \frac{m_1^2}{2(m_1 + m_2)} v_{1\mathrm{i}}^2 = \frac{m_1}{m_1 + m_2}\left[\tfrac{1}{2} m_1 v_{1\mathrm{i}}^2\right]$$

$$= \frac{m_1}{m_1 + m_2} K_{\mathrm{i}}$$

$$\Delta K = K_{\mathrm{f}} - K_{\mathrm{i}} = \frac{m_1}{m_1 + m_2} K_{\mathrm{i}} - K_i$$

$$= \left(\frac{m_2}{m_1 + m_2}\right) K_i \quad \text{(converted to heat)}$$

If $m_2 >> m_1, \quad \Delta K \approx K_i$

(all kinetic energy converted to heat)

Clicker Question 6.5

A 9-kg object is at rest. Suddenly, it explodes and breaks into two pieces. The mass of one piece is 6 kg and the other is a 3-kg piece. Which one of the following statements concerning these two pieces is correct?

a) The speed of the 6-kg piece will be one eighth that of the 3-kg piece.

b) The speed of the 3-kg piece will be one fourth that of the 6-kg piece.

c) The speed of the 6-kg piece will be one forth that of the 3-kg piece.

d) The speed of the 3-kg piece will be one half that of the 6-kg piece.

e) The speed of the 6-kg piece will be one half that of the 3-kg piece.

Clicker Question 6.5

A 9-kg object is at rest. Suddenly, it explodes and breaks into two pieces. The mass of one piece is 6 kg and the other is a 3-kg piece. Which one of the following statements concerning these two pieces is correct?

a) The speed of the 6-kg piece will be one eighth that of the 3-kg piece.

b) The speed of the 3-kg piece will be one fourth that of the 6-kg piece.

c) The speed of the 6-kg piece will be one fourth that of the 3-kg piece.

d) The speed of the 3-kg piece will be one half that of the 6-kg piece.

e) The speed of the 6-kg piece will be one half that of the 3-kg piece.

$$m_1 = 6\,\text{kg}, \quad m_2 = 3\,\text{kg}$$

$$m_1 \vec{\mathbf{v}}_{1\mathbf{f}} + m_2 \vec{\mathbf{v}}_{2\mathbf{f}} = 0$$

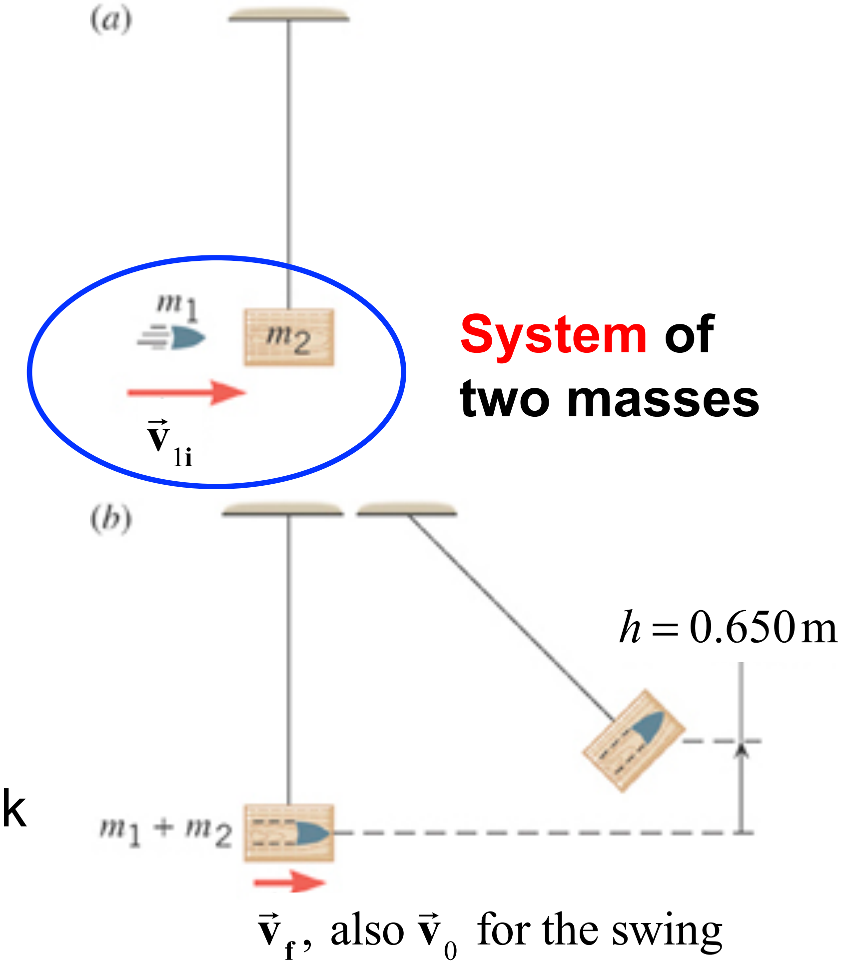
$$\vec{\mathbf{v}}_1 = -\frac{m_2}{m_1}\vec{\mathbf{v}}_2 \quad \text{speeds: } v_1 = \frac{m_2}{m_1} v_2 = \frac{3\,\text{kg}}{6\,\text{kg}} v_2 \Rightarrow v_1 = \tfrac{1}{2} v_2$$

$$m_1 = 6\,\text{kg}$$
$$m_2 = 3\,\text{kg}$$

Example: A Ballistic Pendulum

The mass of the block of wood is 2.50-kg and the mass of the bullet is 0.0100-kg. The block swings to a maximum height of 0.650 m above the initial position.

Find the initial speed of the bullet.



Strategy **–** 1) After the bullet hits the block the swing will conserve energy. From the height determine the starting velocity.

2) Use this velocity as the final velocity of the collision. Then momentum conservation determines the bullet's initial velocity.

6.3 Collisions in One Dimension

Apply conservation of energy to the swinging motion:

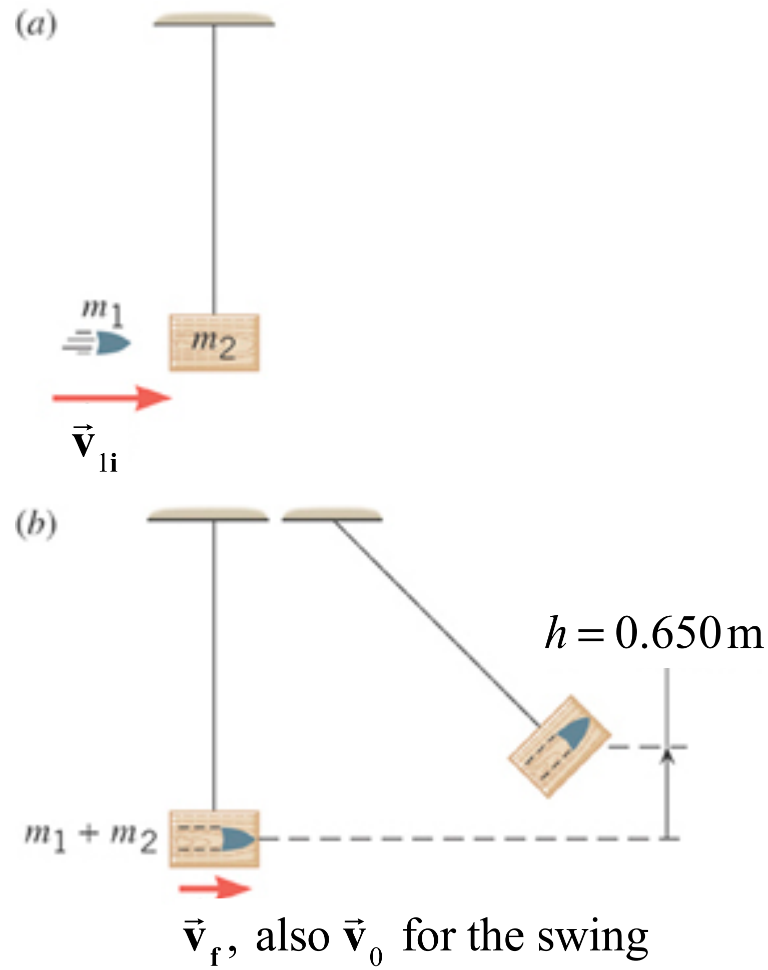
$$K + U = K_0 + U_0$$
$$0 + mgh = \tfrac{1}{2} m v_0^2 + 0$$
$$v_0 = \sqrt{2gh}$$

Apply conservation of momentum in the collision:

$$\vec{\mathbf{p}}_{Total,\mathbf{i}} = \vec{\mathbf{p}}_{bullet,\mathbf{i}} + \vec{\mathbf{p}}_{block,\mathbf{i}} = m_1 v_{1\mathbf{i}} + 0$$
$$\vec{\mathbf{p}}_{Total,\mathbf{f}} = (m_1 + m_2) v_{\mathbf{f}}$$

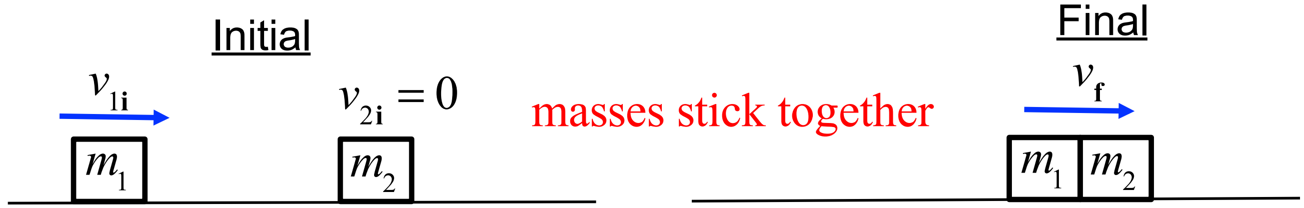
Momentum conservation: $\vec{\mathbf{p}}_{Total,\mathbf{f}} = \vec{\mathbf{p}}_{Total,\mathbf{i}}$

$$v_{1\mathbf{i}} = \frac{(m_1 + m_2)}{m_1} v_{\mathbf{f}} = \frac{(m_1 + m_2)}{m_1} \sqrt{2gh}$$
$$= \frac{2.50 + .01}{.01} \sqrt{2(9.81)(0.65)} \text{ m/s} = 896 \text{ m/s}$$



6.3 *Collisions in One Dimension*

Inelastic collisions (with only internal forces affecting the motion)



($x-$components of the velocities)

Momentum conservation: $m_1 v_{1\mathbf{i}} = (m_1 + m_2) v_{\mathbf{f}}$

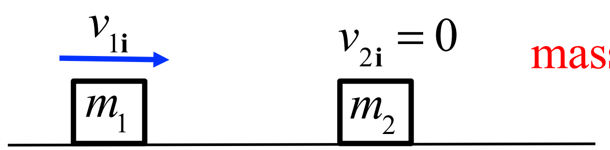
$$v_{\mathbf{f}} = \frac{m_1}{m_1 + m_2} v_{1\mathbf{i}}$$

Energy NOT conserved:

6.3 Collisions in One Dimension

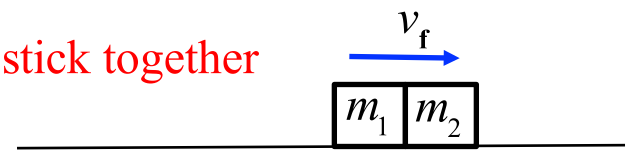
Inelastic collisions (with only internal forces affecting the motion)

Initial



masses stick together

Final



(x – components of the velocities)

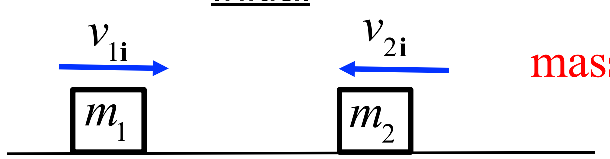
Momentum conservation: $m_1 v_{1i} = (m_1 + m_2) v_f$

Energy NOT conserved:

$$v_f = \frac{m_1}{m_1 + m_2} v_{1i}$$

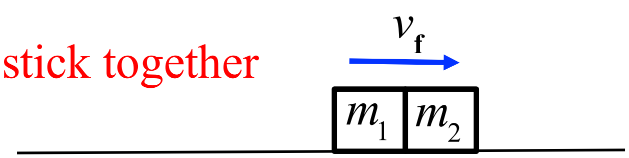
Masses initially moving toward each other

Initial



masses stick together

Final



(x – components of the velocities)

Momentum conservation: $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$

Energy NOT conserved:

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

For example: $v_{1i} = +5.0$ m/s, $v_{2i} = -10.0$ m/s, $m_1 = m_2 = m$ (same mass)

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{m}{2m}(v_{1i} + v_{2i}) = 0.5(+5 - 10)\,\text{m/s} = -2.5\ \text{m/s}$$

Clicker Question 6.6

A mass with a momentum of $+10.0\ \mathrm{kg \cdot m/s}$, collides with a mass twice as big with a momentum of $-6.0\ \mathrm{kg \cdot m/s}$, and they stick together. What is the momentum of the combined system after the collision?

a) $-2.0\ \mathrm{kg \cdot m/s}$

b) $+2.0\ \mathrm{kg \cdot m/s}$

c) $+4.0\ \mathrm{kg \cdot m/s}$

d) $+6.0\ \mathrm{kg \cdot m/s}$

e) $+16.0\ \mathrm{kg \cdot m/s}$

Clicker Question 6.6

A mass with a momentum of $+10.0\ \mathbf{kg \cdot m / s}$, collides with a mass twice as big with a momentum of $-6.0\ \mathbf{kg \cdot m / s}$, and they stick together. What is the momentum of the combined system after the collision?

a) $-2.0\ \mathrm{kg \cdot m/s}$

b) $+2.0\ \mathrm{kg \cdot m/s}$

c) $+4.0\ \mathrm{kg \cdot m/s}$

d) $+6.0\ \mathrm{kg \cdot m/s}$

e) $+16.0\ \mathrm{kg \cdot m/s}$

<u>Momentum is conserved</u>

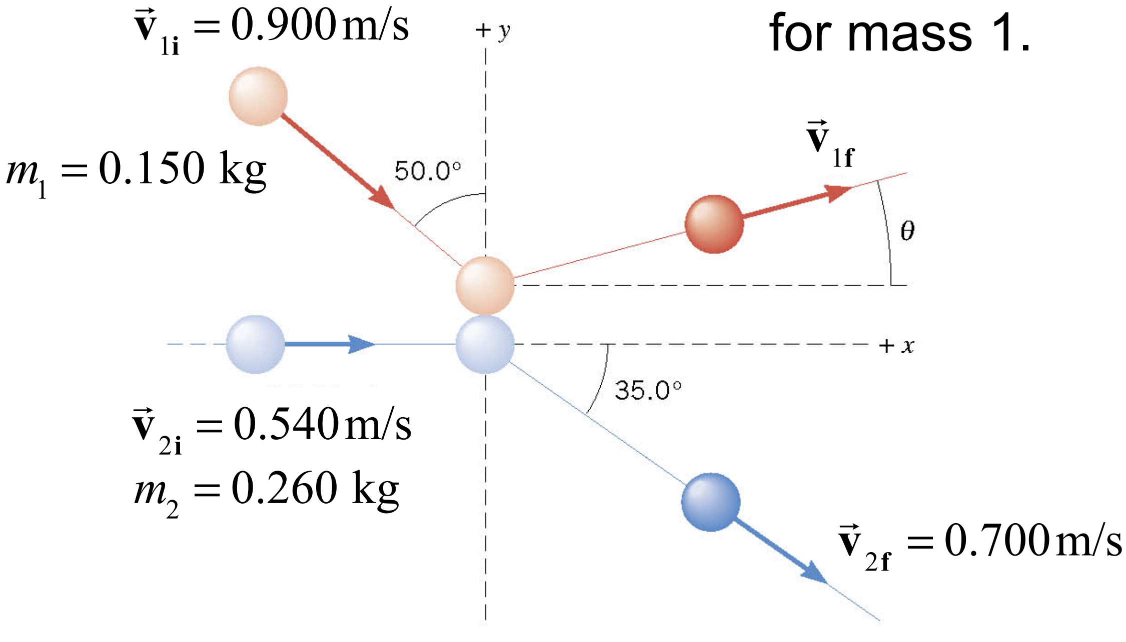
$$\vec{\mathbf{p}}_{(1+2)\mathbf{f}} = \vec{\mathbf{p}}_{1\mathbf{i}} + \vec{\mathbf{p}}_{2\mathbf{i}}$$

$$= \left(+10.0\ \mathrm{kg \cdot m/s}\right) + \left(-6.0\ \mathrm{kg \cdot m/s}\right)$$

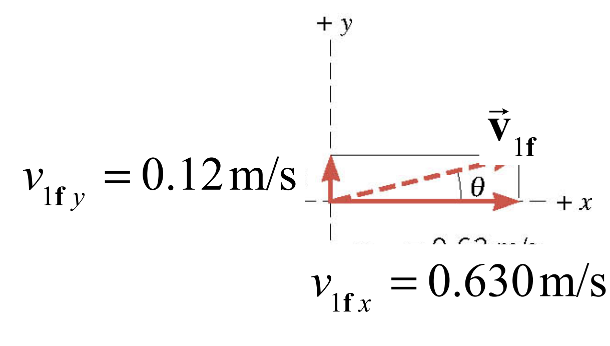
$$= +4.0\ \mathrm{kg \cdot m/s}$$

6.4 Collisions in Two Dimensions

Determine the final momentum vector for mass 1.



(a)



(b)

6.4 Collisions in Two Dimensions

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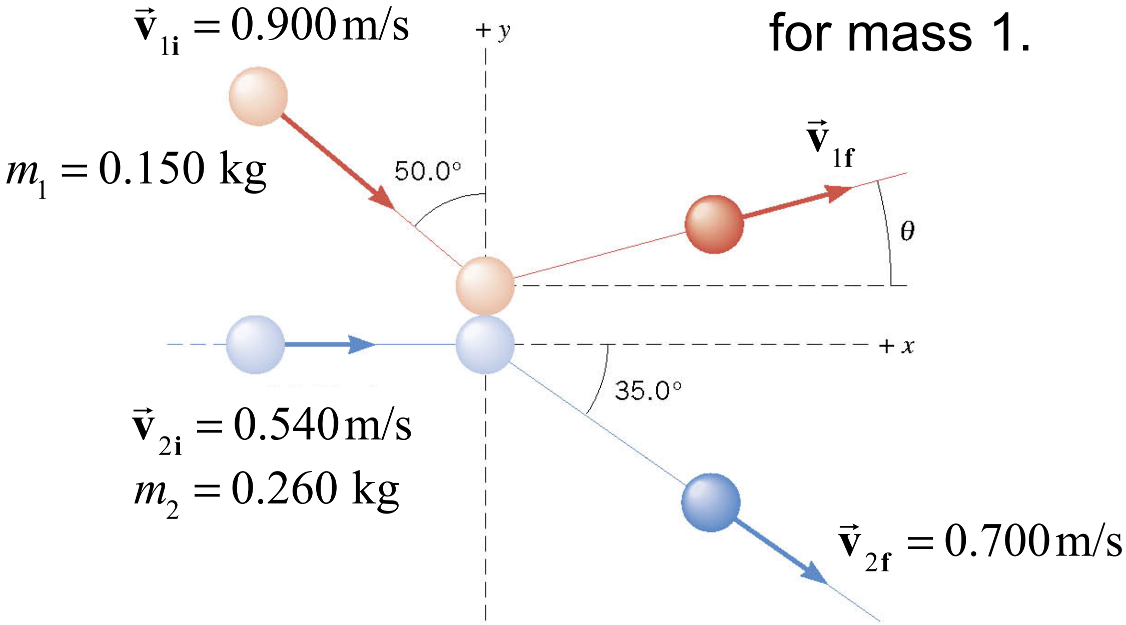
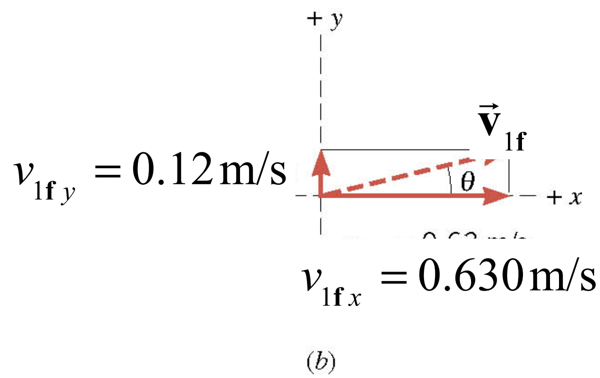


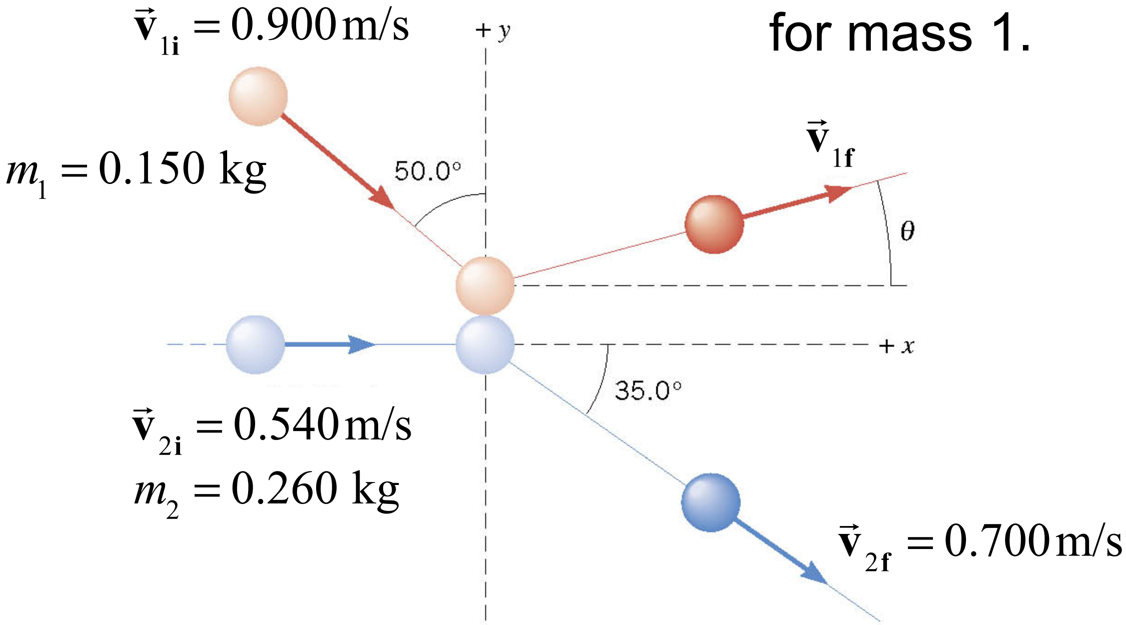


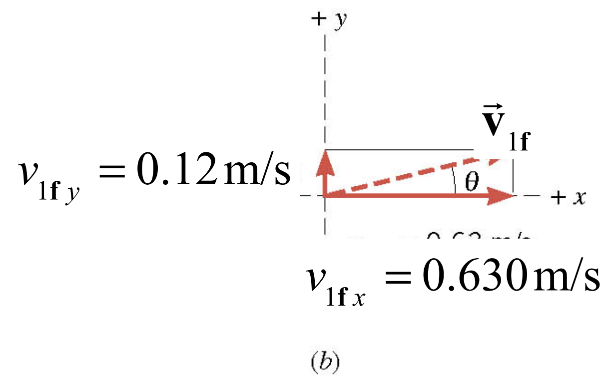
x-components: $m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$

$v_{1i} = +0.900 \sin 50°\ \text{m/s}$, $v_{2i} = +0.540\ \text{m/s}$, $v_{2f} = +0.700 \cos 35°\ \text{m/s}$

6.4 Collisions in Two Dimensions

Determine the final momentum vector for mass 1.





x-components: $m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$

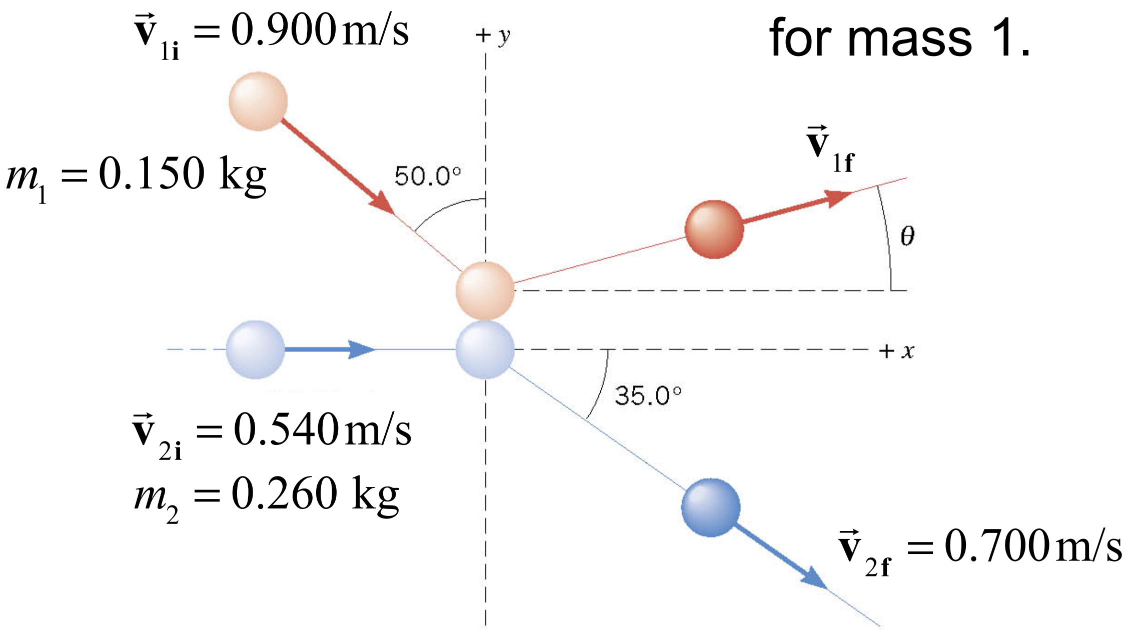
$v_{1i} = +0.900 \sin 50°$ m/s , $v_{2i} = +0.540$ m/s, $v_{2f} = +0.700 \cos 35°$ m/s

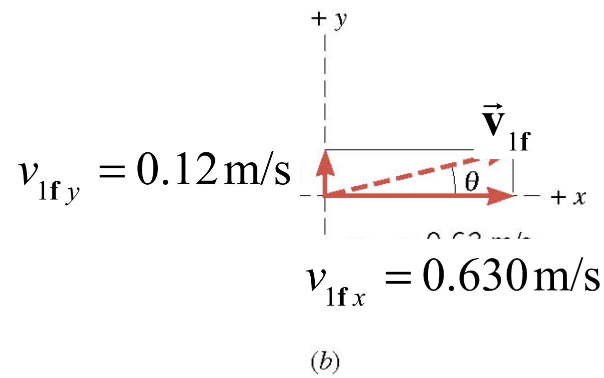
y-components: $m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$

$v_{1i} = -0.900 \cos 50°$ m/s $v_{2i} = 0$, $v_{2f} = -0.700 \sin 35°$ m/s

6.4 Collisions in Two Dimensions

Determine the final momentum vector for mass 1.





x-components: $m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$

$v_{1i} = +0.900 \sin 50°$ m/s , $v_{2i} = +0.540$ m/s, $v_{2f} = +0.700 \cos 35°$ m/s

y-components: $m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$

$v_{1i} = -0.900 \cos 50°$ m/s $v_{2i} = 0$, $v_{2f} = -0.700 \sin 35°$ m/s

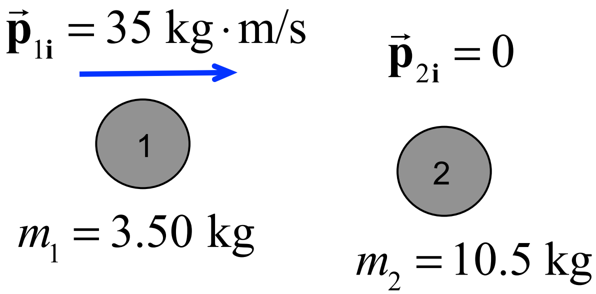
final x : $v_{1x} = +0.63$ m/s final y : $v_{1y} = +0.12$ m/s

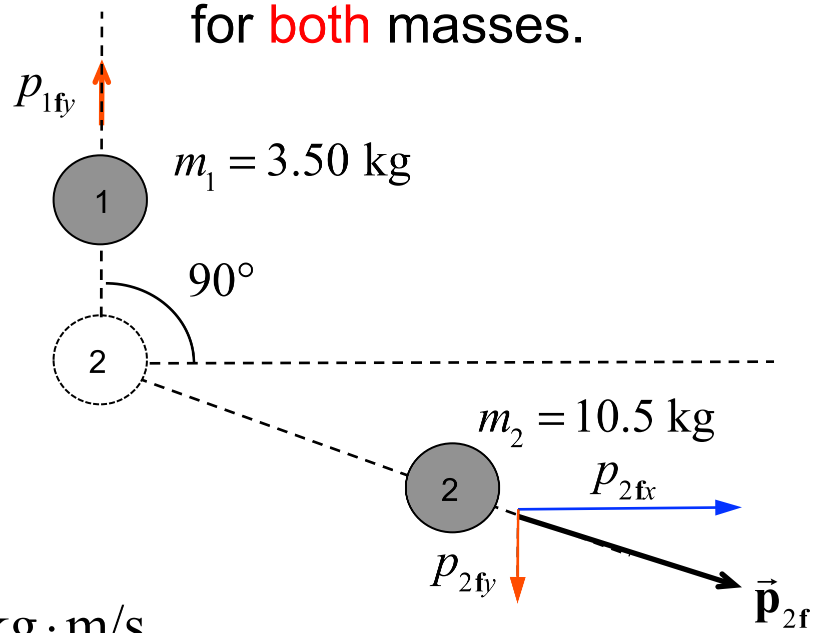
$$v_1 = \sqrt{v_{1x}^2 + v_{1y}^2} = +0.64 \text{ m/s}; \quad \theta_1 = \tan^{-1}\left(v_{1y} / v_{1x}\right) = 11°$$

6.4 Collisions in Two Dimensions

In the elastic collision, m_1 is deflected upward at 90°.

Determine the final momentum vector for both masses.





Momentum conservation

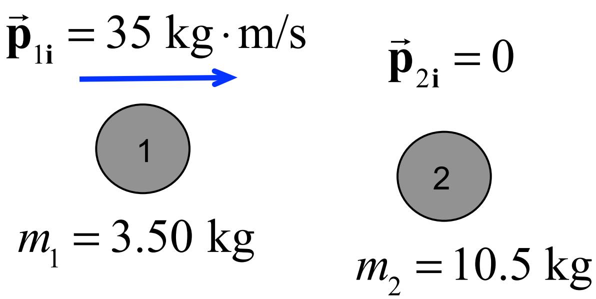
x-components: $p_{1i} = \underline{p_{2fx} = 35\ \text{kg}\cdot\text{m/s}}$

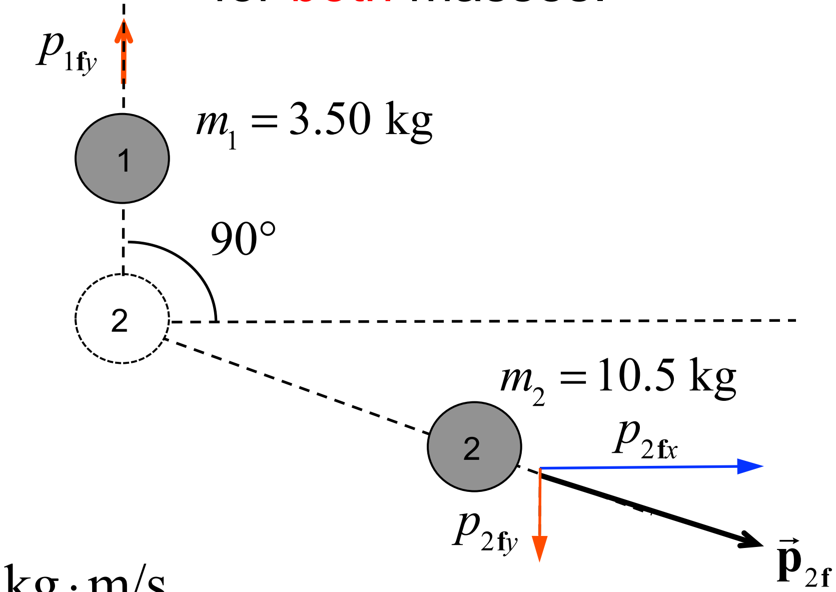
y-components: $p_{1fy} = \underline{-p_{2fy}}$ (need this)

6.4 Collisions in Two Dimensions

Determine the final momentum vector for both masses.

In the elastic collision, m_1 is deflected upward at 90°.





Momentum conservation

x-components: $p_{1\mathbf{i}} = \underline{p_{2\mathbf{f}x} = 35\ \mathrm{kg\cdot m/s}}$

y-components: $p_{1\mathbf{f}y} = \underline{-p_{2\mathbf{f}y}}$ (need this)

Kinetic Energies

$$K_\mathrm{i} = \frac{p_{1\mathbf{i}}^2}{2m_1}$$

$$K_\mathrm{f} = \frac{p_{1\mathbf{f}y}^2}{2m_1} + \frac{p_{2\mathbf{f}x}^2 + p_{2\mathbf{f}y}^2}{2m_2}$$

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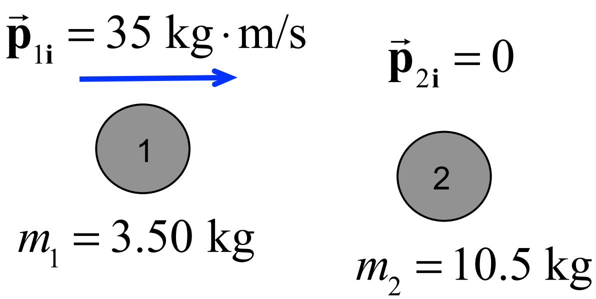
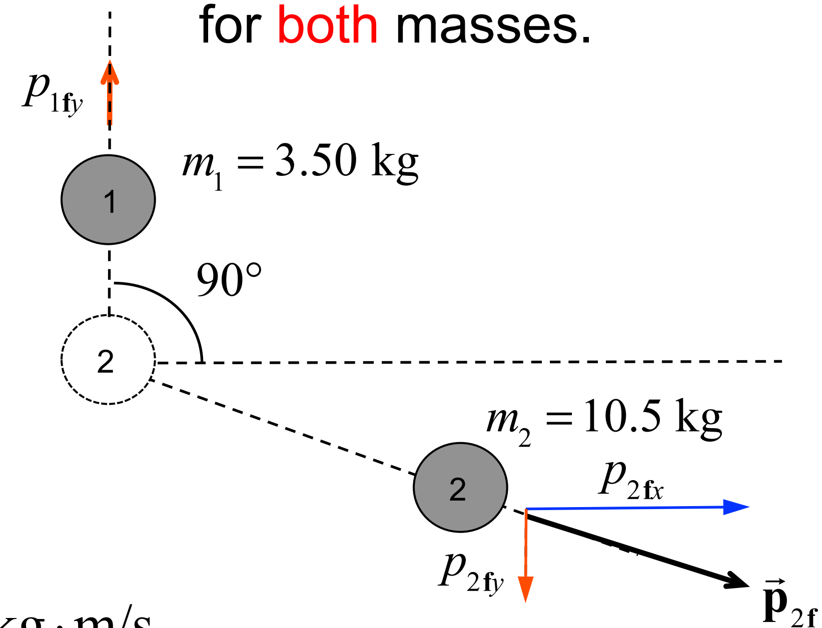
In the elastic collision, m_1 is deflected upward at 90°.





Momentum conservation

x-components: $p_{1\mathrm{i}} = p_{2\mathrm{f}x} = 35\ \mathrm{kg \cdot m/s}$

y-components: $p_{1\mathrm{f}y} = -p_{2\mathrm{f}y}$ (need this)

Kinetic Energies

$$K_\mathrm{i} = \frac{p_{1\mathrm{i}}^2}{2m_1}$$

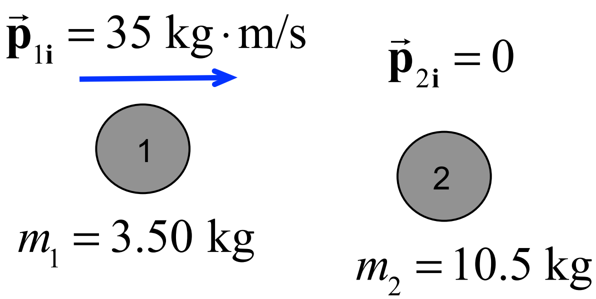
$$K_\mathrm{f} = \frac{p_{1\mathrm{f}y}^2}{2m_1} + \frac{p_{2\mathrm{f}x}^2 + p_{2\mathrm{f}y}^2}{2m_2}$$

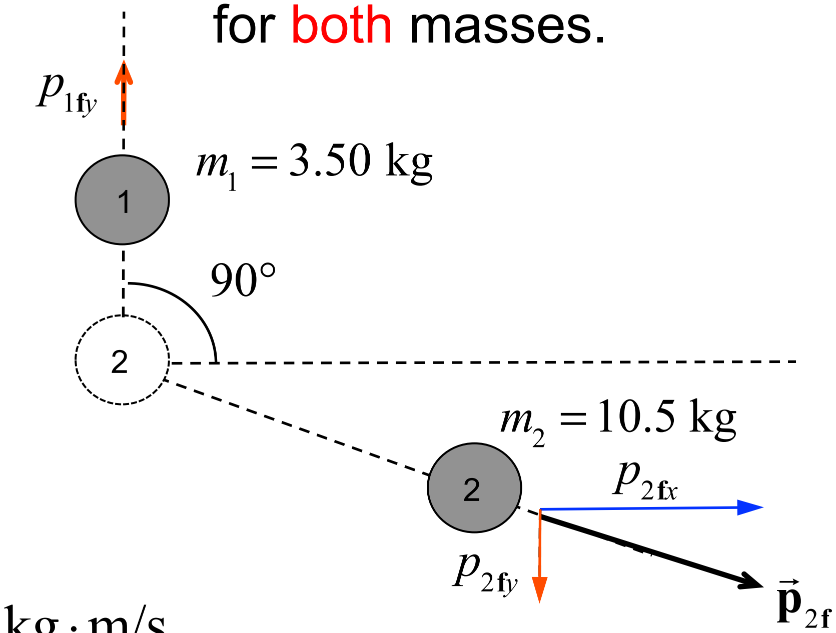
$$\frac{p_{2\mathrm{f}y}^2}{2m_1} + \frac{p_{1\mathrm{i}}^2 + p_{2\mathrm{f}y}^2}{2m_2}$$

6.4 Collisions in Two Dimensions

In the elastic collision, m_1 is deflected upward at 90°.

Determine the final momentum vector for both masses.





Momentum conservation

x-components: $p_{1\mathrm{i}} = p_{2\mathrm{f}x} = 35\ \mathrm{kg \cdot m/s}$

y-components: $p_{1\mathrm{f}y} = -p_{2\mathrm{f}y}$ (need this)

Kinetic Energies

$$K_\mathrm{i} = \frac{p_{1\mathrm{i}}^2}{2m_1}$$

$$K_\mathrm{f} = \frac{p_{1\mathrm{f}y}^2}{2m_1} + \frac{p_{2\mathrm{f}x}^2 + p_{2\mathrm{f}y}^2}{2m_2}$$

$$\frac{p_{2\mathrm{f}y}^2}{2m_1} + \frac{p_{1\mathrm{i}}^2 + p_{2\mathrm{f}y}^2}{2m_2}$$

Energy conservation

$$K_\mathrm{f} = K_i$$

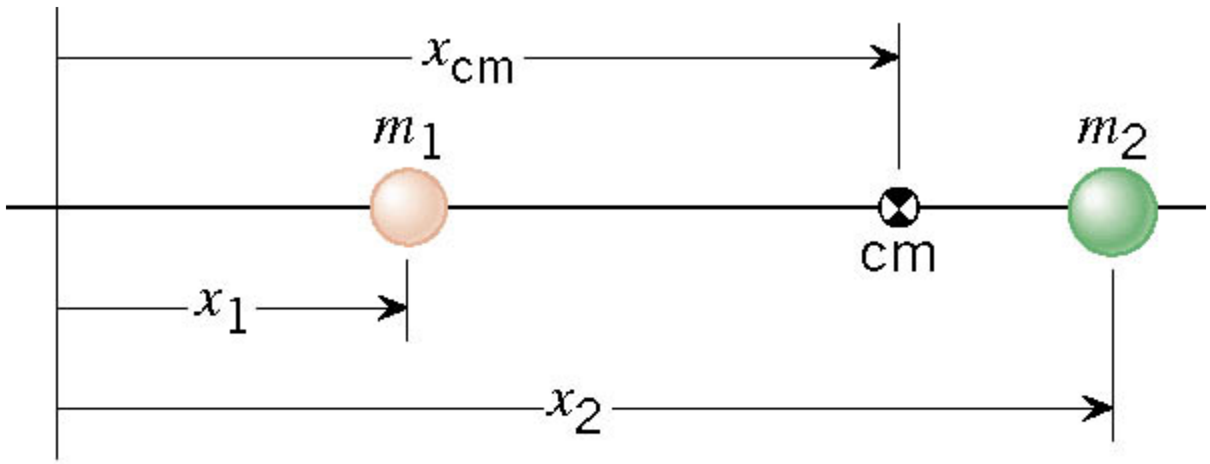
$$p_{2\mathrm{f}y}^2\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = p_{1\mathrm{i}}^2\left(\frac{1}{m_1} - \frac{1}{m_2}\right)$$

$$p_{2\mathrm{f}y} = \pm\frac{1}{\sqrt{2}}p_{1\mathrm{i}} \quad \Rightarrow \quad p_{2\mathrm{f}y} = -24.7\ \mathrm{kg \cdot m/s}$$

therefore, $p_{1\mathrm{f}y} = +24.7\ \mathrm{kg \cdot m/s}$

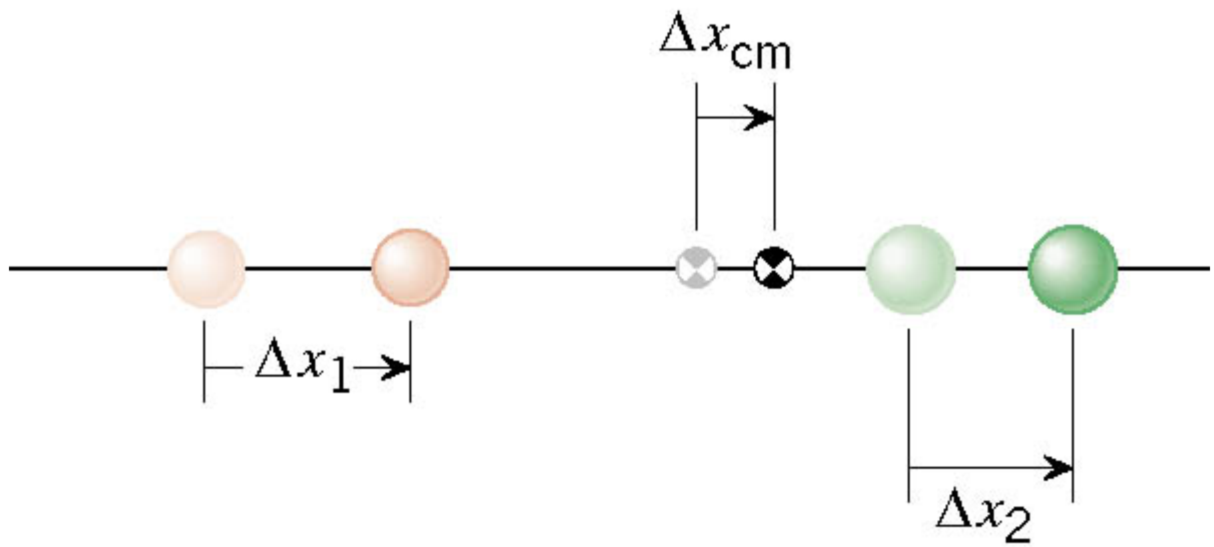
6.5 Center of Mass

The center of mass is a point that represents the average location for the total mass of a system.



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

6.5 Center of Mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} \quad \Rightarrow \quad v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

6.5 Center of Mass

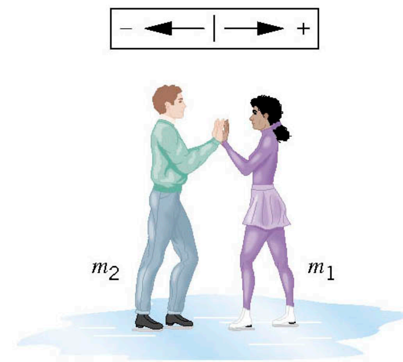
$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

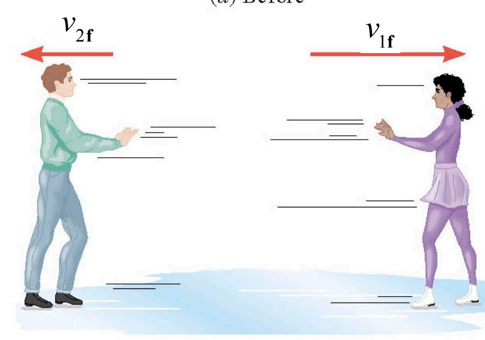
6.5 Center of Mass

BEFORE

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$



(*a*) Before



(*b*) After

AFTER

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{(88\ \text{kg})(-1.5\,\text{m/s}) + (54\ \text{kg})(+2.5\,\text{m/s})}{88\ \text{kg} + 54\ \text{kg}}$$

$$= 0.00$$