

Chapter 6

Impulse and Momentum

Continued

6.4 *Collisions in Two Dimensions*

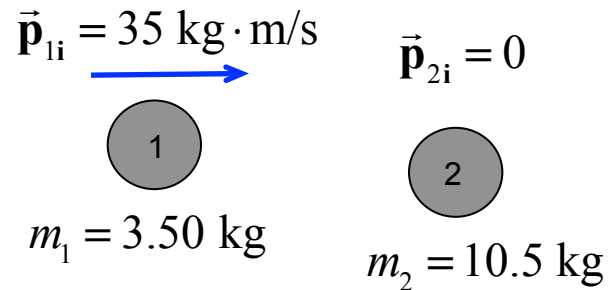
Momentum conservation can be used to solve collision problems if there are no external forces affecting the motion of the masses.

Energy conservation can be used to solve a collision problem if it is stated explicitly that the collision is ELASTIC.

If two masses in a collision stick together, the collision must be INELASTIC. **Energy conservation** cannot be used unless the energy lost or gained is provided.

6.4 Collisions in Two Dimensions

In the elastic collision, m_1 is deflected upward at 90° .

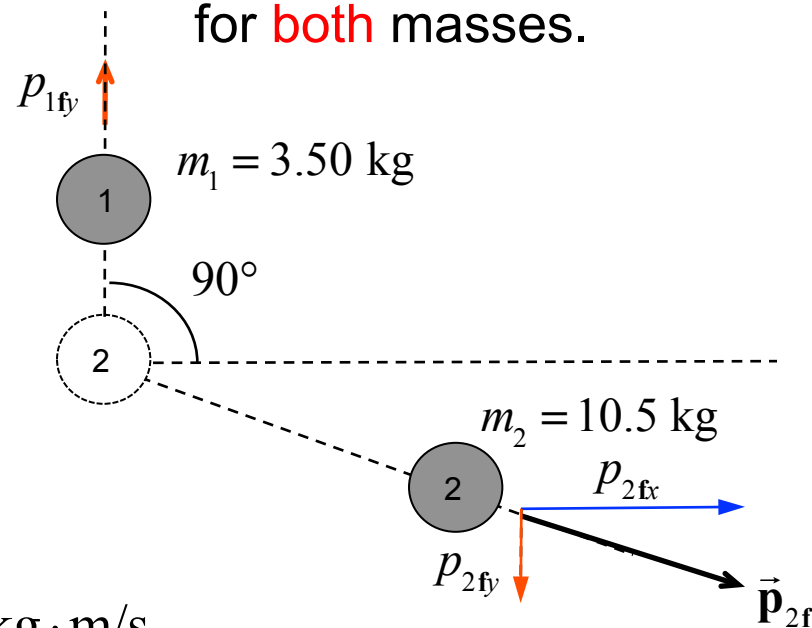


Momentum conservation

x-components: $p_{1i} = p_{2fx} = 35 \text{ kg} \cdot \text{m/s}$

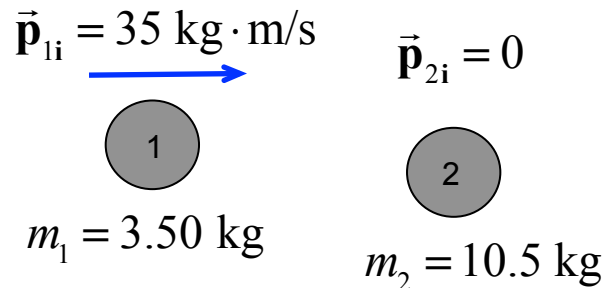
y-components: $p_{1fy} = -p_{2fy}$ (need this)

Determine the final momentum vector for **both** masses.



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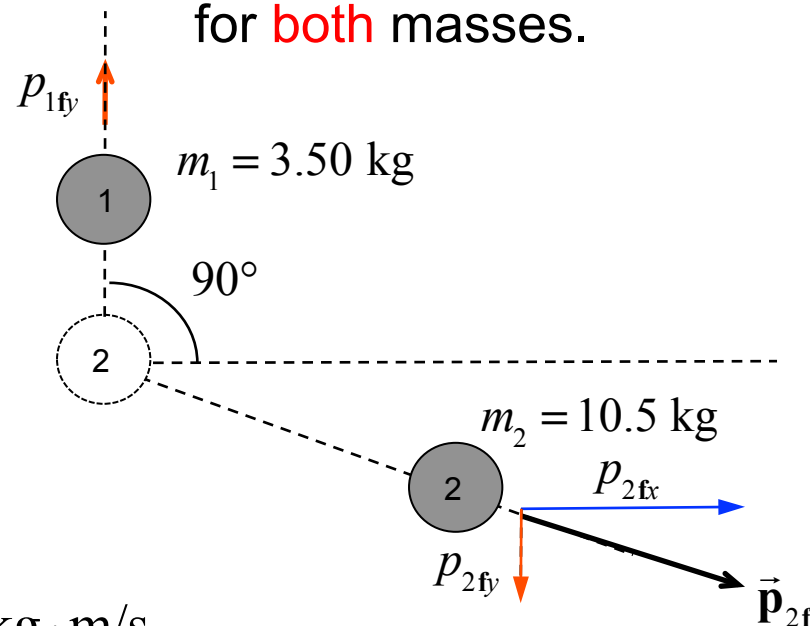
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Kinetic Energies

$$K_i = \frac{p_{1i}^2}{2m_1}$$

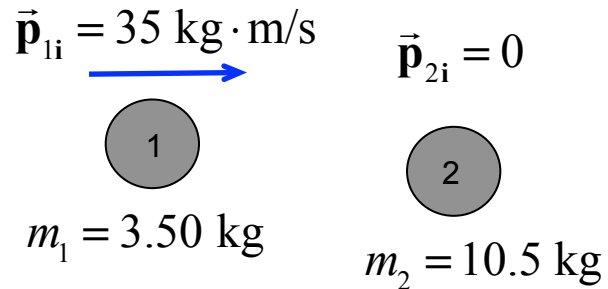
$$K_f = \frac{p_{1fy}^2}{2m_1} + \frac{p_{2fx}^2 + p_{2fy}^2}{2m_2}$$

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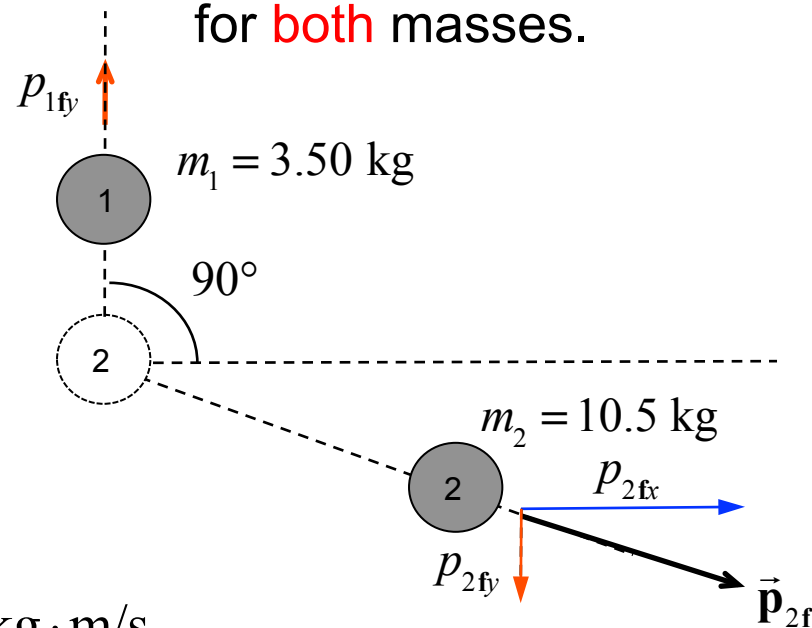
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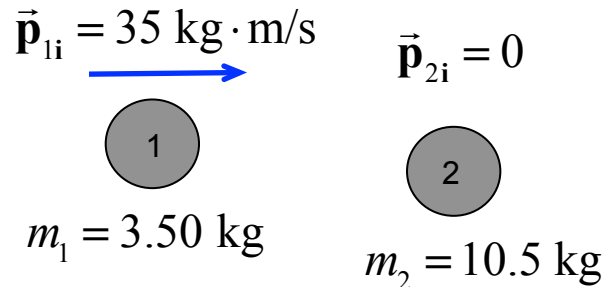
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Determine the final momentum vector for **both** masses.

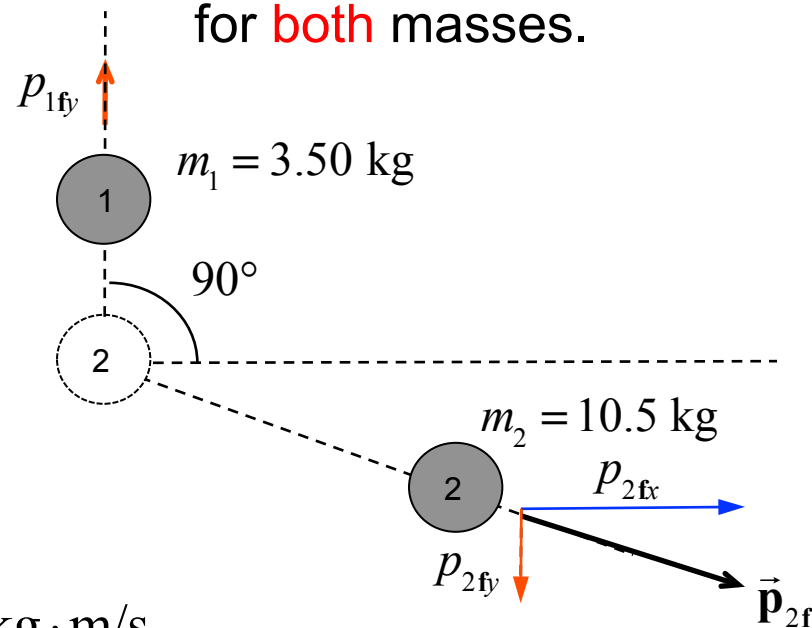


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$$\frac{p_{2fy}^2}{2m_1} + \frac{p_{1i}^2 + p_{2fy}^2}{2m_2}$$

Energy conservation

$$K_f = K_i$$

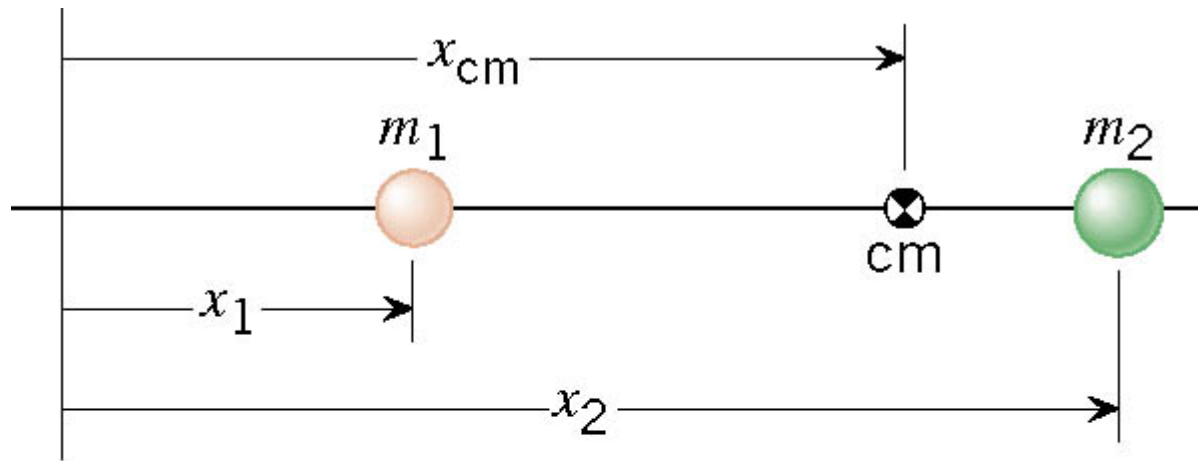
$$p_{2fy}^2 \left(\frac{1}{m_1} + \frac{1}{m_2} \right) = p_{1i}^2 \left(\frac{1}{m_1} - \frac{1}{m_2} \right)$$

$$p_{2fy} = \pm \frac{1}{\sqrt{2}} p_{1i} \Rightarrow \underline{p_{2fy} = -24.7 \text{ kg} \cdot \text{m/s}}$$

$$\underline{\text{therefore, } p_{1fy} = +24.7 \text{ kg} \cdot \text{m/s}}$$

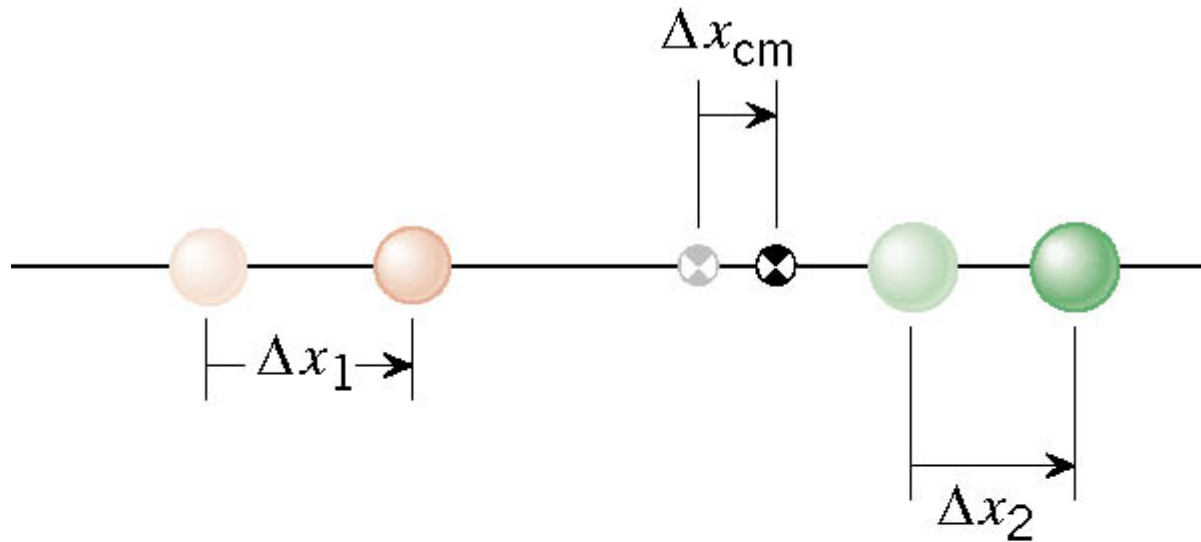
6.5 Center of Mass

The center of mass is a point that represents the average location for the total mass of a system.



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

6.5 Center of Mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2} \quad \Rightarrow \quad v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

6.5 Center of Mass

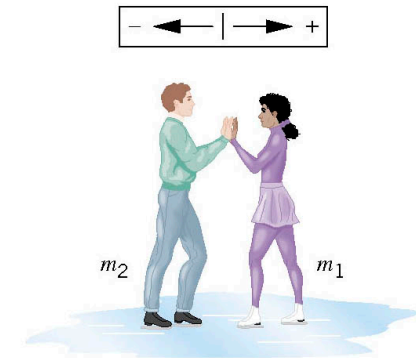
$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

6.5 Center of Mass

BEFORE

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$



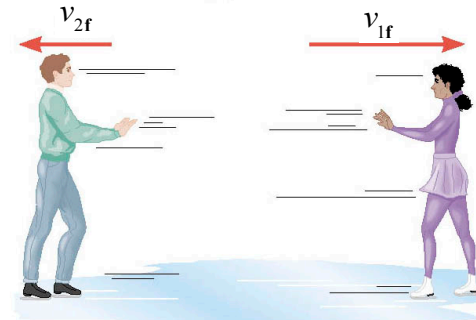
(a) Before

AFTER

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{(54 \text{ kg})(+2.5 \text{ m/s}) + (88 \text{ kg})(-1.5 \text{ m/s})}{88 \text{ kg} + 54 \text{ kg}}$$

$$= 0.00$$



(b) After

Forces and Newton's Laws of Motion

Force Magnitudes

Gravitational Force: $F_G = mg$ (magnitude), direction downward, weight

Elastic Forces: (stretched spring) $F_s = kx$ (magnitude), inward at the ends

Compression: (squeezed spring or object) normal forces, n , $\perp$ contact

Frictional: opposes motion, Static: $f_{Max} = \mu_S F_{\perp}$, then Kinetic: $f_K = \mu_K F_{\perp}$

Net Force: $\vec{F}_{Net} = \vec{F}_1 + \vec{F}_2 + \dots$ on a single object (magnitude and direction)

Newton's 1st Law: speed and direction do not change, if & only if $\vec{F}_{Net} = 0$

Newton's 2nd Law: $\vec{F}_{Net} = m\vec{a}$

Newton's 3rd Law for two masses (1,2) in contact: $\vec{F}_{12} = -\vec{F}_{21}$

Decomposition to components: $\vec{F} = F_x \hat{i} + F_y \hat{j}$, $F_x = F \cos \theta$, $F_y = F \sin \theta$

Components to magnitude angle: $F = \sqrt{F_x^2 + F_y^2}$, $\theta = \tan^{-1}(F_y / F_x)$

Summaries and Examples

Energy and energy conservation

Momentum and momentum conservation

Energy and energy conservation

$$\text{Work: } W = F(\cos \theta) \Delta x$$

$$\text{Kinetic Energy: } K = \frac{1}{2} m v^2$$

$$\text{Work changes kinetic energy: } W = K - K_0 = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$$

$$\text{Conservative forces} \Rightarrow \text{Potential Energy}$$

$$\text{Gravitational Potential Energy: } U_G = mgy$$

$$\text{Ideal Spring Potential Energy: } U_s = \frac{1}{2} kx^2$$

$$\text{Total Energy: } E = K + U$$

Work by non-conservative forces (friction, humans, explosions)

$$\text{changes total energy: } K + U = K_0 + U_0 + W_{\text{NC}}$$

If $W_{\text{NC}} = 0$, there is total energy conservation:

$$E = E_0 \Rightarrow K + U = K_0 + U_0$$

$$\text{Average power} = \text{Work/time} = (\text{Energy change})/\text{time} = F \bar{v}$$

Momentum and momentum conservation

$$\text{Impulse: } \vec{\mathbf{J}} = \vec{\mathbf{F}}t$$

$$\text{Momentum: } \vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Net average impulse changes momentum:

$$\sum \vec{\mathbf{F}}\Delta t = \vec{\mathbf{p}} - \vec{\mathbf{p}}_0 = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_0$$

$$\text{Momentum of 2 masses in collision: } \vec{\mathbf{p}}_{\text{total}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$$

No net external force active, momentum is conserved

$$\vec{\mathbf{p}}_{\text{Total},f} = \vec{\mathbf{p}}_{\text{Total},i} \Rightarrow m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} = m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i}$$

$$\text{Center of mass position: } x_{\text{cm}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \text{ and velocity } \vec{\mathbf{v}}_{\text{cm}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}$$

$$\text{Conservation of momentum} \Rightarrow \vec{\mathbf{v}}_{\text{cm}} \text{ remains constant}$$

Clicker Question 6.7

**A space probe travels with a momentum of $+6.0 \times 10^7 \text{ kg} \cdot \text{m} / \text{s}$.
A retrorocket fires for 10 s, with a force magnitude of $2.0 \times 10^6 \text{ N}$, that slows the speed of the probe. What is the momentum of the probe after the rocket ceases to fire?**

- a) $2.0 \times 10^7 \text{ kg} \cdot \text{m/s}$
- b) $4.0 \times 10^7 \text{ kg} \cdot \text{m/s}$
- c) $6.0 \times 10^7 \text{ kg} \cdot \text{m/s}$
- d) $8.0 \times 10^7 \text{ kg} \cdot \text{m/s}$
- e) $10.0 \times 10^7 \text{ kg} \cdot \text{m/s}$

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- b) $4.0 \times 10^7 \text{ kg} \cdot \text{m} / \text{s}$**
- c) $6.0 \times 10^7 \text{ kg} \cdot \text{m} / \text{s}$
- d) $8.0 \times 10^7 \text{ kg} \cdot \text{m} / \text{s}$
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Impulse changes momentum

$$\mathbf{F}\Delta t = \mathbf{p}_f - \mathbf{p}_i$$

$$\mathbf{p}_f = \mathbf{p}_i + \mathbf{F}\Delta t$$

$$\begin{aligned}\mathbf{p}_f &= \left(+6.0 \times 10^7 \text{ N} \cdot \text{s}\right) + (-2.0 \times 10^6 \text{ N})(10 \text{ s}) \\ &= +4.0 \times 10^7 \text{ N} \cdot \text{s}\end{aligned}$$

Clicker Question 6.8

Two identical 2.0-kg objects are involved in a collision. The objects are on a frictionless track. The initial velocity of object A is +12 m/s and the initial velocity of object B is – 6.0 m/s. What is the final velocity of the two objects?

- a) Object A moves at – 6.0 m/s and B moves at +12 m/s.
- b) Object A moves at +9.0 m/s and B moves at +9.0 m/s.
- c) Object A moves at – 6.0 m/s and B moves at +6.0 m/s.
- d) Object A moves at –12 m/s and B moves at +6.0 m/s.
- e) This cannot be determined without energy information.

Clicker Question 6.8

must say if elastic or inelastic

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- d) Object A moves at – 12 m/s and B moves at +6.0 m/s.
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Momentum is conserved

$$\vec{p}_{1f} + \vec{p}_{2f} = \vec{p}_{1i} + \vec{p}_{2i}$$

(all are x components) $mv_{1f} + mv_{2f} = mv_{1i} + mv_{2i}$ (m cancels in each term)

$$v_{1f} + v_{2f} = (+12\text{m/s}) + (-6\text{m/s}) = +6\text{m/s}$$

Only know the sum: $v_{1f} + v_{2f} = +6\text{m/s}$, Need energy equation

Is this an **ELASTIC** or **INELASTIC** collision?

Clicker Question 6.8

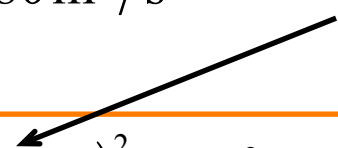
an elastic

Two identical 2.0-kg objects are involved in collision. The objects are on a frictionless track. The initial velocity of object A is +12 m/s and the initial velocity of object B is – 6.0 m/s. What is the final velocity of the two objects?

Energy is also conserved

$$v_{1f}^2 + v_{2f}^2 = v_{1i}^2 + v_{2i}^2 = 144 \text{ m}^2/\text{s}^2 + 36 \text{ m}^2/\text{s}^2 \quad (\text{energy conservation, } \frac{1}{2}m \text{ cancels})$$

$$v_{1f}^2 + v_{2f}^2 = 180 \text{ m}^2/\text{s}^2 \quad v_{2f} = +6 \text{ m/s} - v_{1f} \quad (\text{previous slide})$$


$$v_{1f}^2 + (6 \text{ m/s} - v_{1f})^2 = 2v_{1f}^2 - (12 \text{ m/s})v_{1f} + 36 \text{ m}^2/\text{s}^2$$

$$\text{Quadratic Equation: } v_{1f}^2 - (6 \text{ m/s})v_{1f} - 72 \text{ m}^2/\text{s}^2 = 0$$

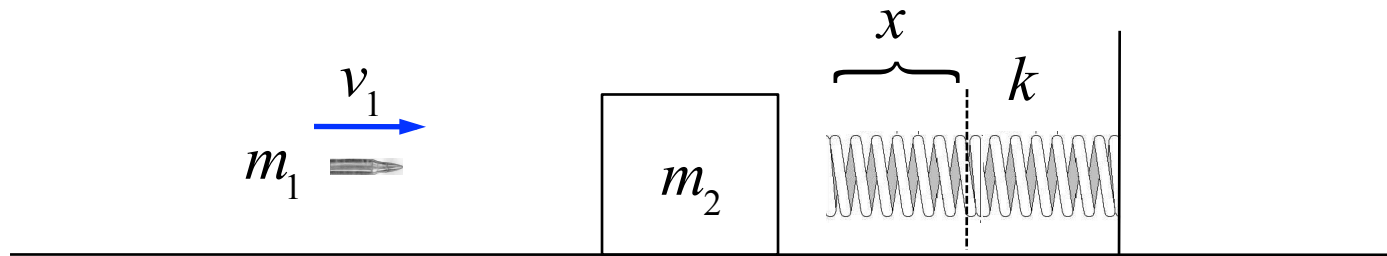
$$v_{1f} = \left(+3 \pm \sqrt{9 + 72} \right) \text{ m/s} = \left(+3 \pm 9 \right) \text{ m/s}$$

$$v_{1f} = -6 \text{ m/s} \quad (\text{other solution - } v_{1f} = +12 \text{ m/s, no collision})$$

- a) Object A moves at – 6.0 m/s and B moves at +12 m/s.**

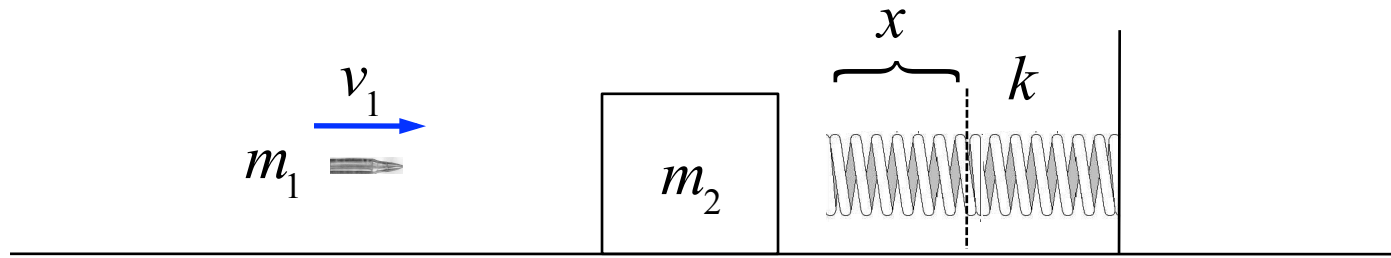
Example:

An 11-g bullet is fired horizontally into a 110-g wooden block that is initially at rest on a frictionless horizontal surface and a fixed spring having spring constant 183 N/m. The bullet becomes embedded in the block. The bullet-block system compresses the spring by a maximum amount 85 cm. What was the initial velocity of the bullet?



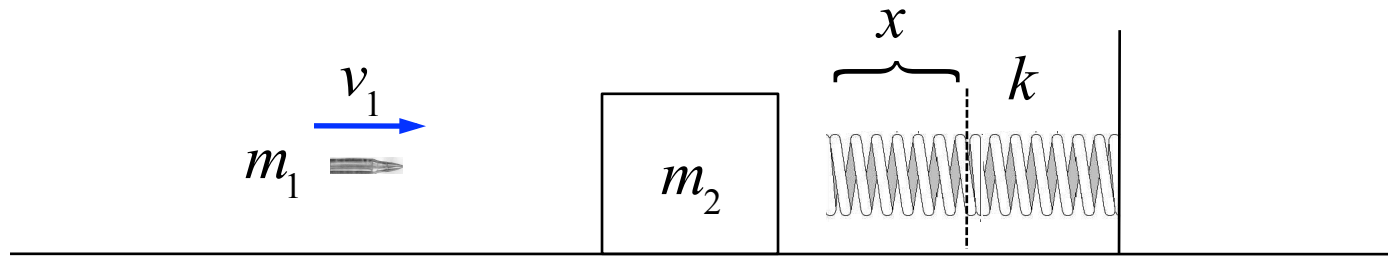
Strategy:

- 1) Momentum conservation bullet + block
- 2) Energy conservation in spring compression



1) Momentum conservation bullet + block

$$p_1 = m_1 v_1, \quad p = (m_1 + m_2) v \quad \Rightarrow \quad p_1 = p$$



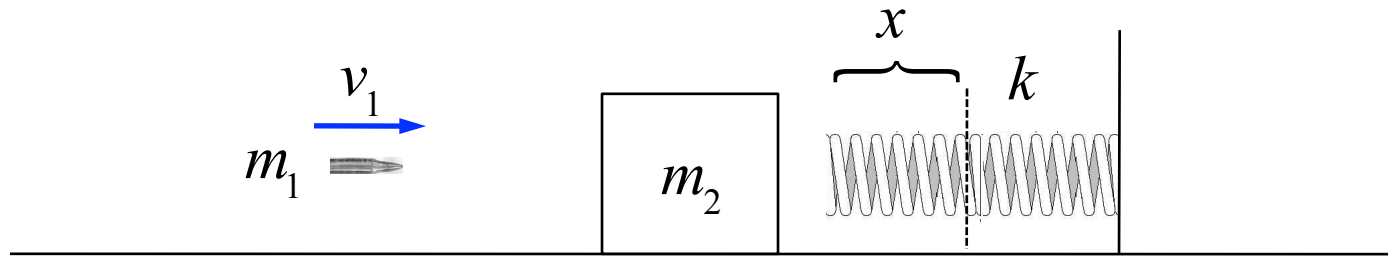
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2) Energy conservation in spring compression

Kinetic Energy of block and bullet : $K_i = p_1^2 / 2(m_1 + m_2)$

Potential Energy of compressed spring : $U = \frac{1}{2} kx^2$



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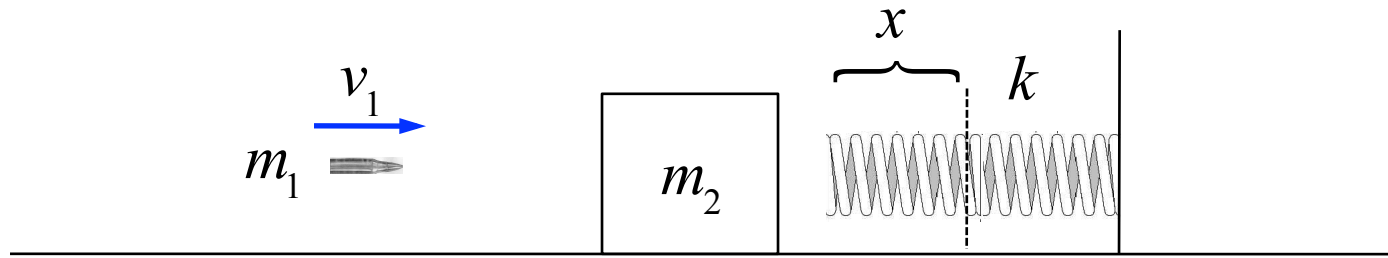
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Energy Conservation : $K + U = K_i + U_i$; $U_i = 0, K = 0$



1) Momentum conservation bullet + block

$$p_1 = m_1 v_1, \quad p = (m_1 + m_2)v \quad \Rightarrow \quad p_1 = p$$

2) Energy conservation in spring compression

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$$\text{Energy Conservation: } K + U = K_i + U_i \quad ; \quad U_i = 0, K = 0$$

$$\frac{1}{2} kx^2 = p_1^2 / 2(m_1 + m_2)$$

$$p_1 = x \sqrt{k(m_1 + m_2)}$$

$$m_1 v_1 = (0.85 \text{ m}) \sqrt{(183 \text{ N/m})(0.121 \text{ kg})} = 4.00 \text{ N} \cdot \text{s}$$

$$v_1 = 4.00 \text{ N} \cdot \text{s} / (0.011 \text{ kg}) = 364 \text{ m/s}$$