

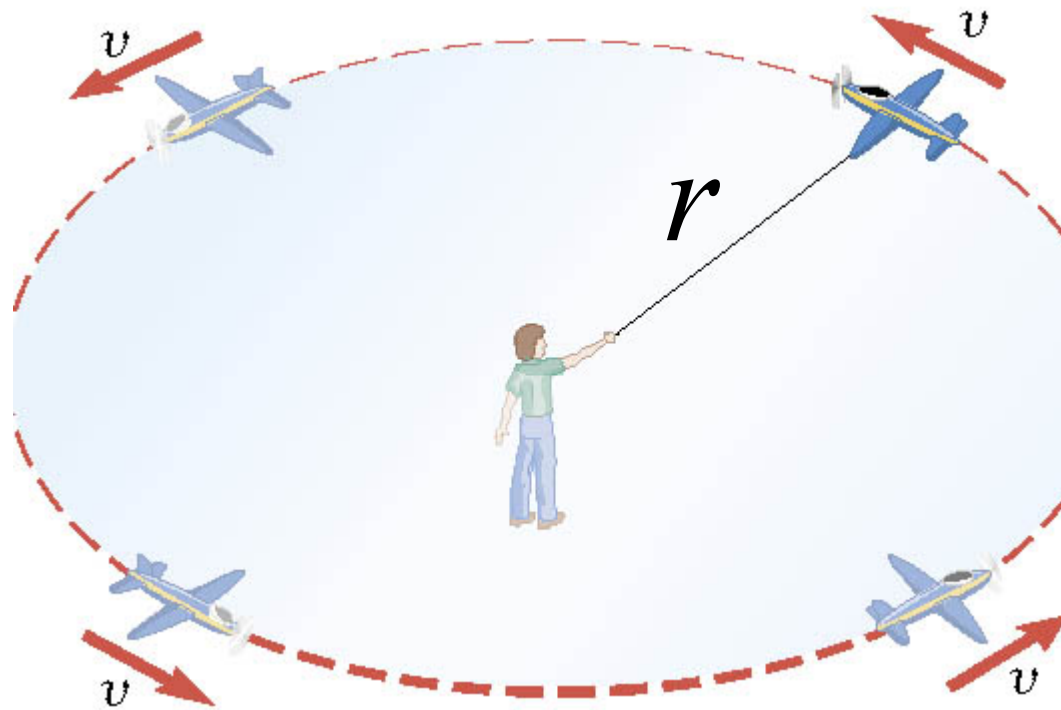
Chapter 3.5

Uniform Circular Motion

3.5 Uniform Circular Motion

DEFINITION OF UNIFORM CIRCULAR MOTION

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.

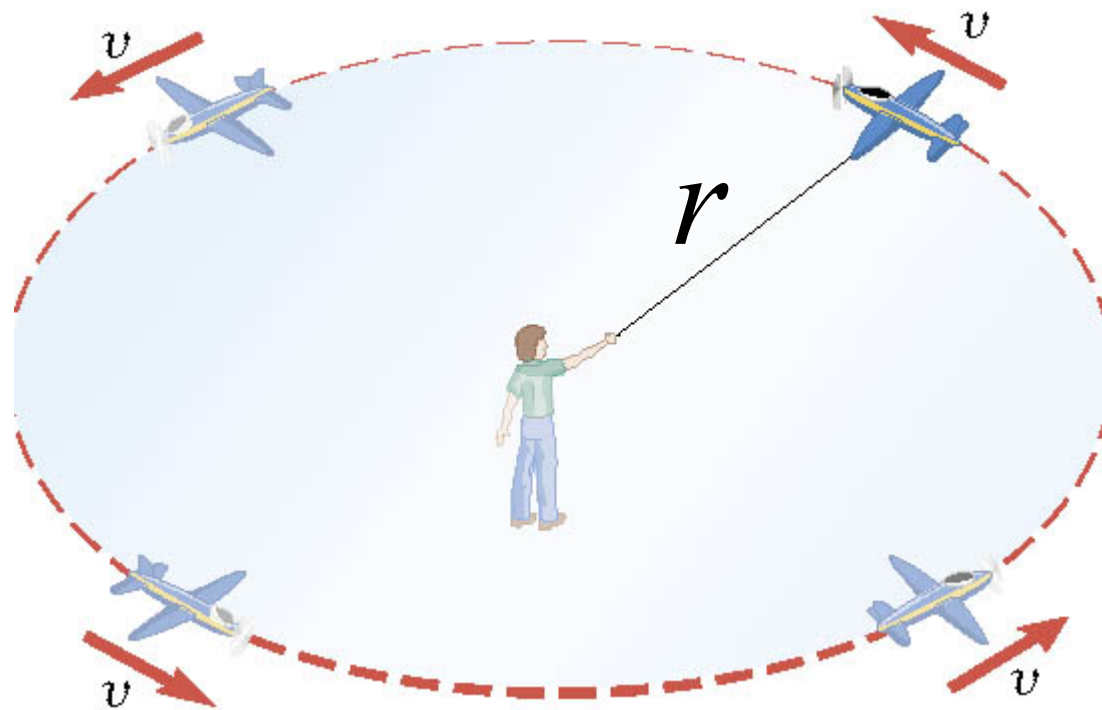


Circumference of the circle is $2\pi r$.

3.5 Uniform Circular Motion

The time it takes the object to travel once around the circle is T (a.k.a. the period)

Speed around the circle is, $v = \frac{2\pi r}{T}$.



3.5 Uniform Circular Motion

Example: A Tire-Balancing Machine

The wheel of a car has a radius of 0.29m and it being rotated at 830 revolutions per minute on a tire-balancing machine. Determine the speed at which the outer edge of the wheel is moving.

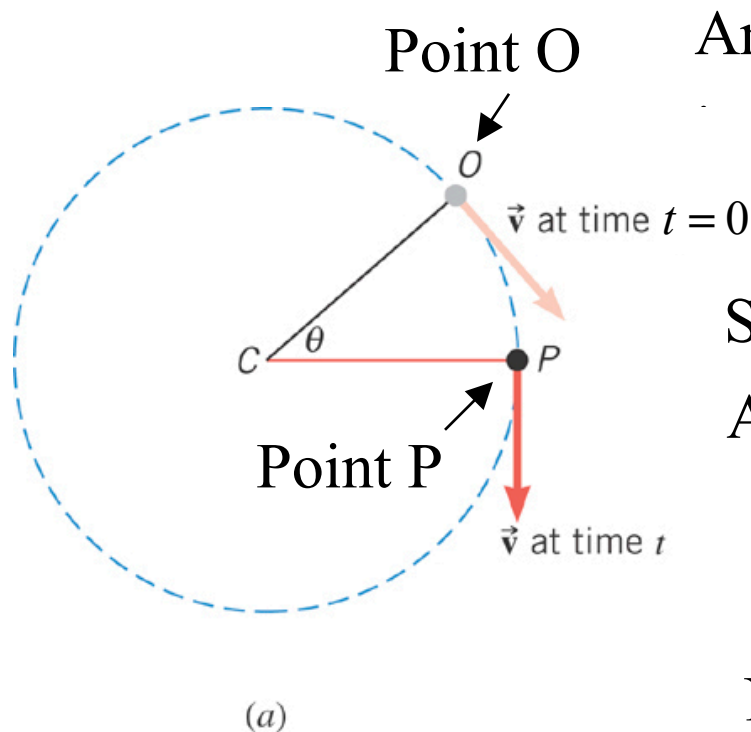
$$\frac{1}{830 \text{ revolutions/min}} = 1.2 \times 10^{-3} \text{ min/revolution}$$

$$T = 1.2 \times 10^{-3} \text{ min} = 0.072 \text{ s}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.29 \text{ m})}{0.072 \text{ s}} = 25 \text{ m/s}$$

3.5 Centripetal Acceleration

In uniform circular motion, the **speed** is *constant*, but the direction of the **velocity vector** is *not constant*.



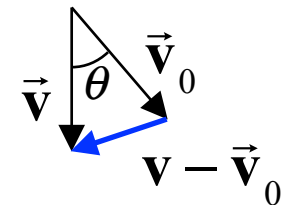
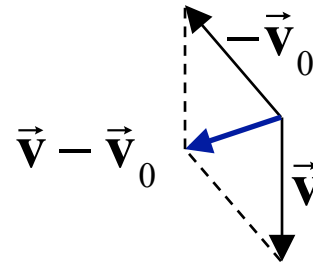
Angle between point O and point P
the same as between $\vec{v}_0$ and $\vec{v}$.

Since velocity vector changes direction
Acceleration vector is **NOT ZERO**.

$$\vec{a} = \frac{\vec{v} - \vec{v}_0}{t}$$

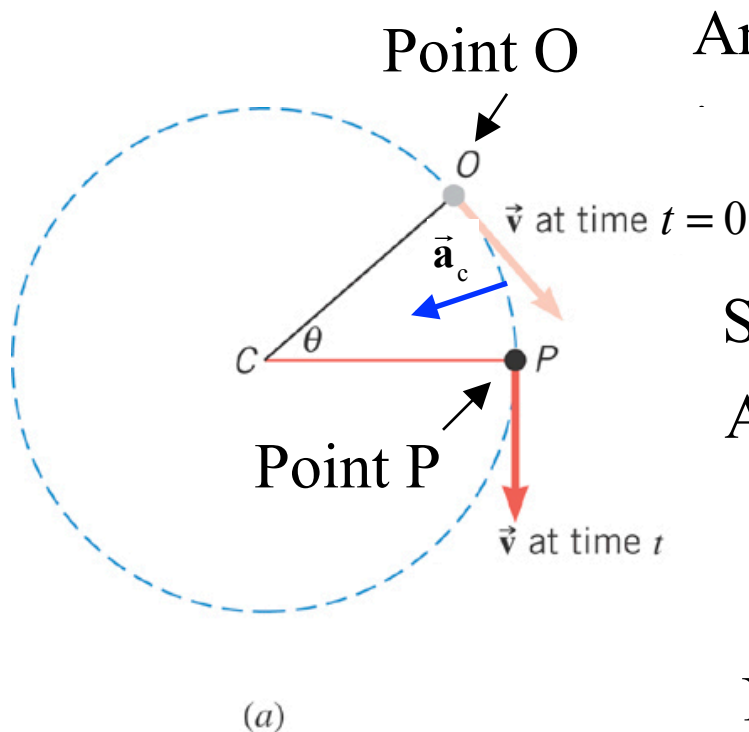
Need to understand: $\vec{v} - \vec{v}_0$

NOTE: $\vec{v} - \vec{v}_0$ and $\vec{a}$ point
in toward center of circle!



3.5 Centripetal Acceleration

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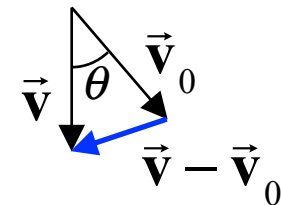
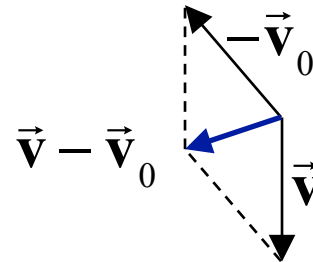
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Need to understand: $\vec{v} - \vec{v}_0$

NOTE: $\vec{v} - \vec{v}_0$ and $\vec{a}$ point
in toward center of circle!



3.5 Centripetal Acceleration

Compare geometry of velocity vectors and the portion of the circle.

$$\theta = \frac{\Delta v}{v}$$

$$\theta = \frac{vt}{r}$$



Magnitudes

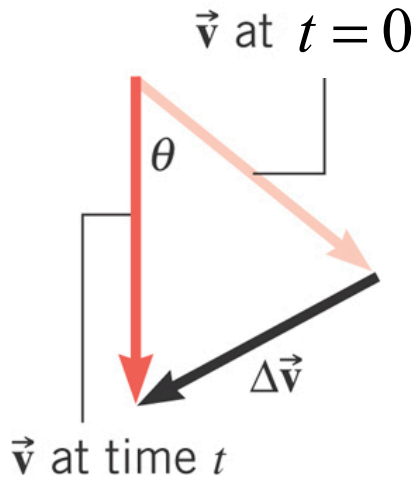
$$\frac{\Delta v}{v} = \frac{vt}{r}$$



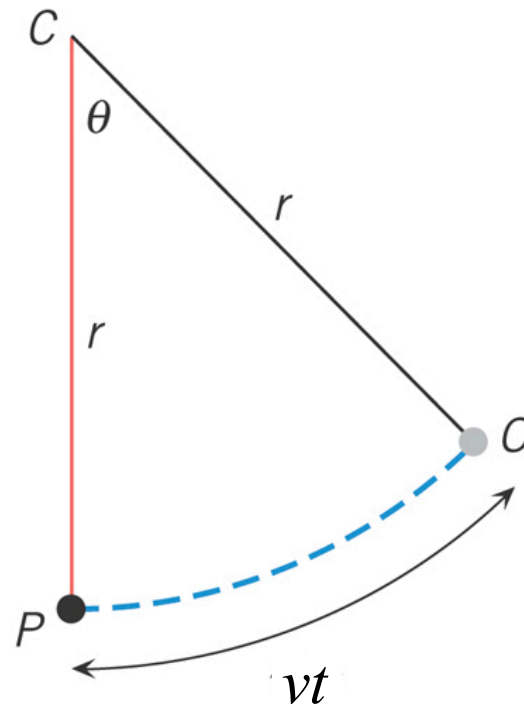
$$a_c = \frac{\Delta v}{t} = \frac{v^2}{r}$$



$$a_c = \frac{v^2}{r}$$



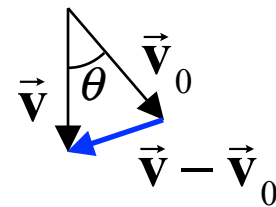
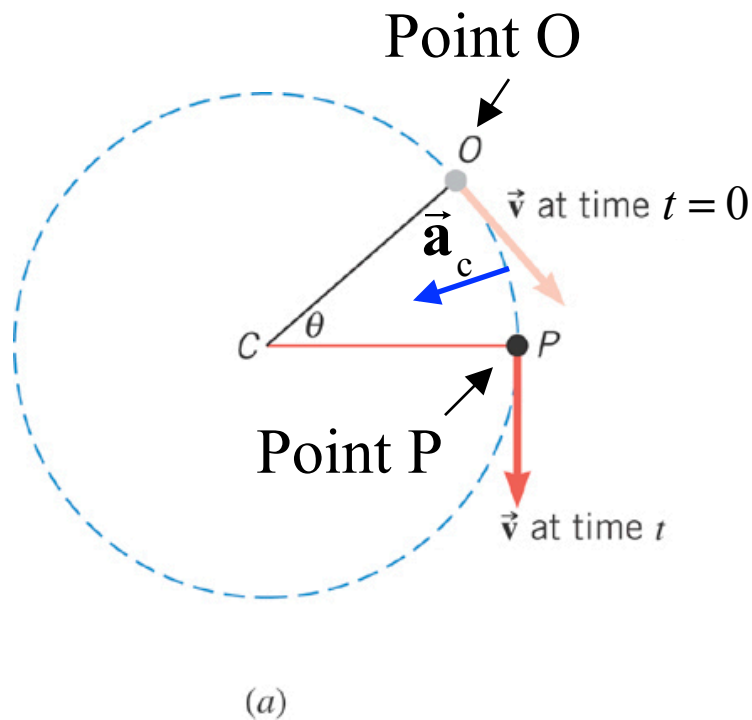
(a)



(b)

3.5 Centripetal Acceleration

The direction of the centripetal acceleration is towards the center of the circle; in the same direction as the change in velocity.



$$a_c = \frac{v^2}{r}$$

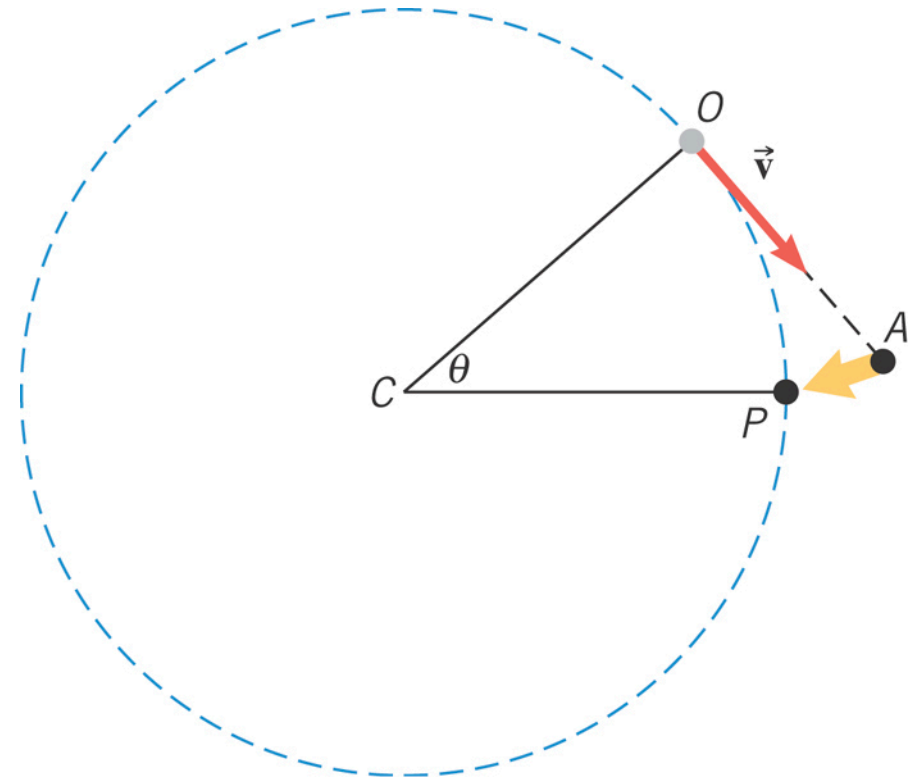
Centripetal acceleration
vector points *inward*
at ALL points on the circle

3.5 Centripetal Acceleration

Conceptual Example: Which Way Will the Object Go?

An object ($\bullet$) is in uniform circular motion. At point O it is released from its circular path.

Does the object move along the
(A) Straight path between O and A
or
(B) Along the circular arc between points O and P ?

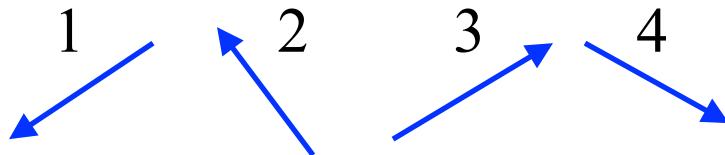


3.5 Centripetal Acceleration

Example: The Effect of Radius on Centripetal Acceleration

The bobsled track contains turns with radii of 33 m and 24 m.

Match the acceleration vector directions below to the points A,B,C,D.

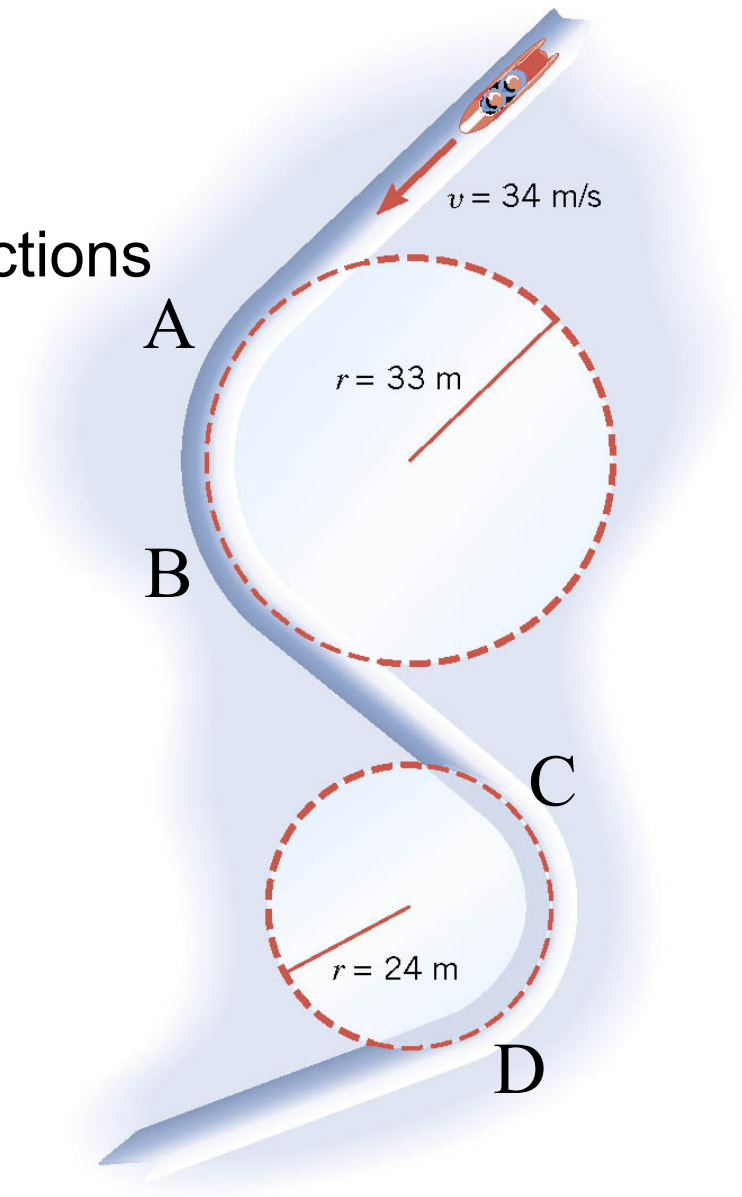


A –

B –

C –

D –

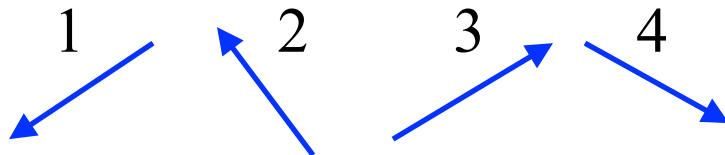


3.5 Centripetal Acceleration

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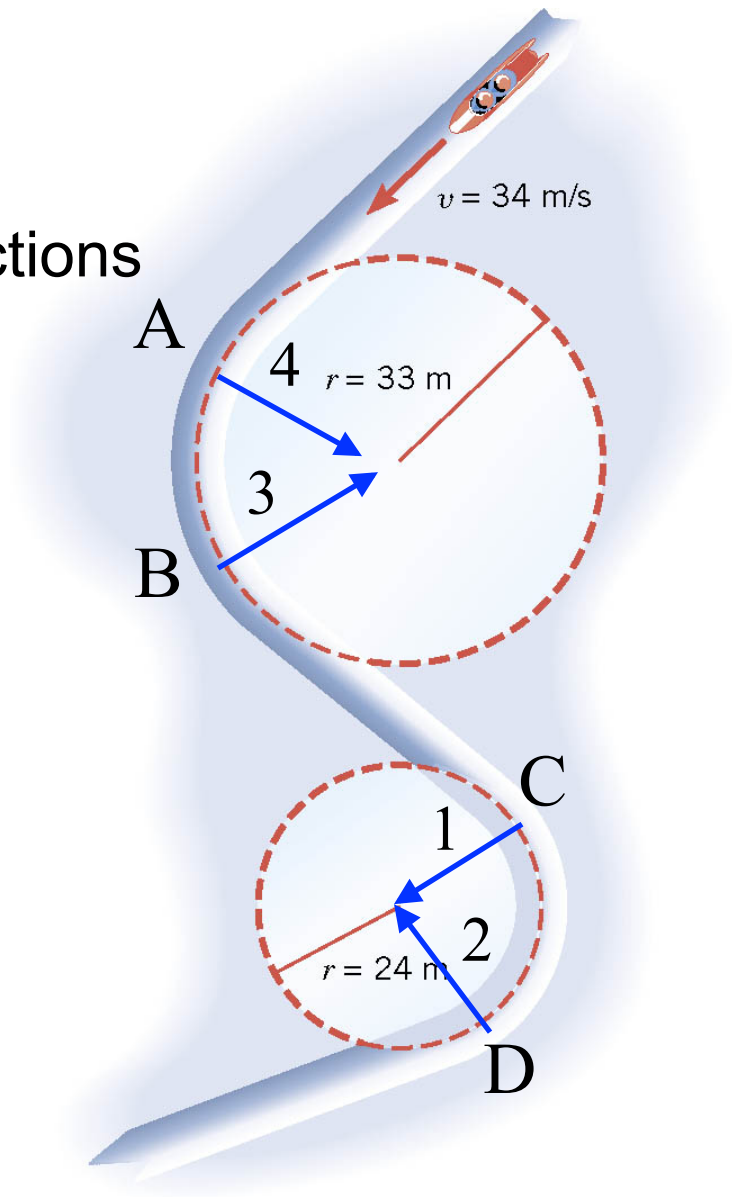


A – 4

B – 3

C – 1

D – 2



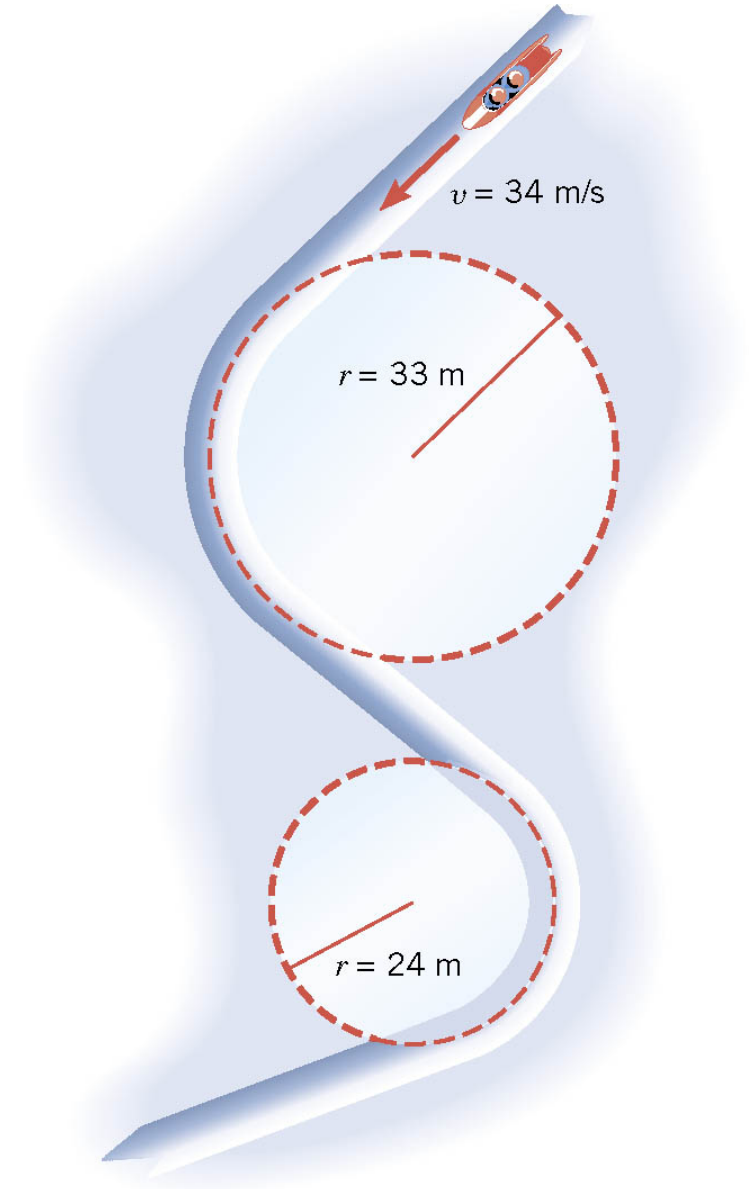
3.5 Centripetal Acceleration

$$a_c = v^2 / r$$

Find the centripetal acceleration at each turn for a speed of 34 m/s. Express answers as multiples of $g = 9.8 \text{ m/s}^2$.

$$a_c = \frac{(34 \text{ m/s})^2}{33 \text{ m}} = 35 \text{ m/s}^2 = 3.6g$$

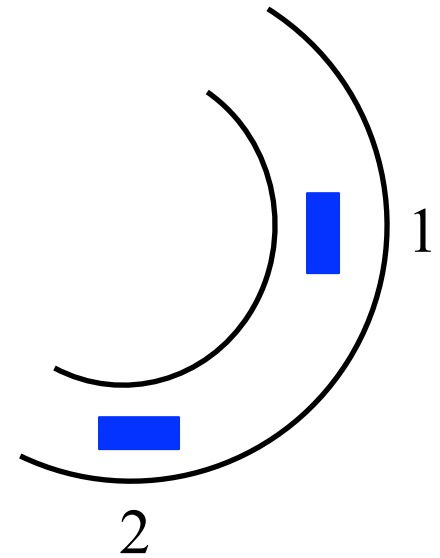
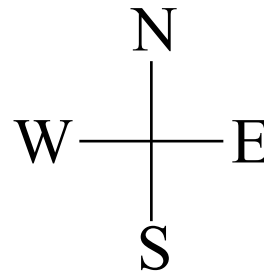
$$a_c = \frac{(34 \text{ m/s})^2}{24 \text{ m}} = 48 \text{ m/s}^2 = 4.9g$$



Clicker Question 3.5.1

A car is moving counter-clockwise around a circular section of road at constant speed. What are the directions of its velocity and acceleration at position 1.

- | | v | a |
|----|----------|----------|
| a) | N | S |
| b) | N | E |
| c) | N | W |
| d) | N | N |
| e) | S | E |



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	$\mathbf{v}$	$\mathbf{a}$
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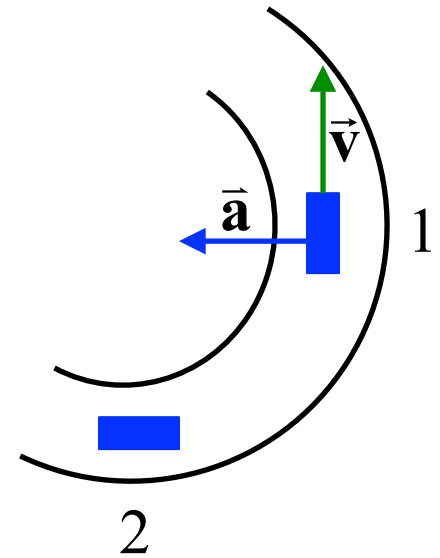
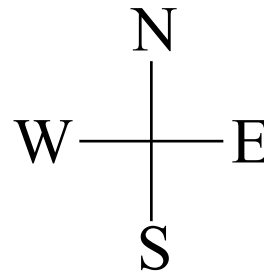
a)	N	S
----	---	---

b)	N	E
----	---	---

c)	N	W
----	---	---

d)	N	N
----	---	---

e)	S	E
----	---	---

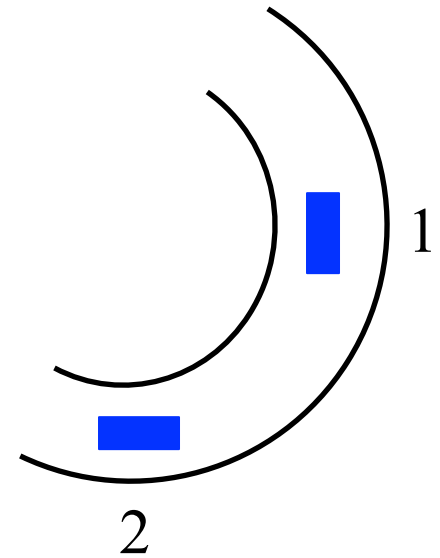
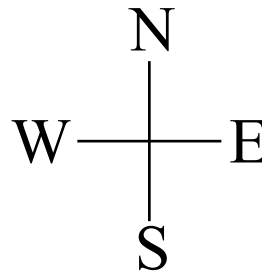


Clicker Question 3.5.2

A car is moving counter-clockwise around a circular section of road at constant speed. What are the directions of its velocity and acceleration

at position 2?

- | | <u>v</u> | <u>a</u> |
|----|----------|----------|
| a) | E | S |
| b) | E | E |
| c) | E | N |
| d) | E | W |
| e) | W | S |

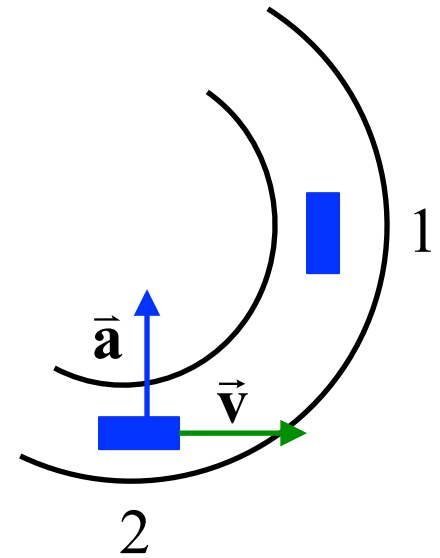
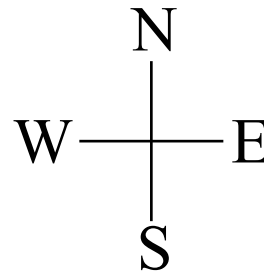


Clicker Question 3.5.2

A car is moving counter-clockwise around a circular section of road at constant speed. What are the directions of its velocity and acceleration

at position 2?

- | | $\vec{v}$ | $\vec{a}$ |
|----|-----------|-----------|
| a) | E | S |
| b) | E | E |
| c) | E | N |
| d) | E | W |
| e) | W | S |



Chapter 8

Accelerated Circular Motion

8.1 *Rotational Motion and Angular Displacement*

Why are there 360 degrees in a circle?

Why are there 60 minutes in an hour?

Why are there 60 seconds in a minute?

Because the Greeks, who invented these units were enamored with numbers that are divisible by most whole numbers, 12 or below (except 7 and 11).

Strange, because later it was the Greeks who discovered that the ratio of the radius to the circumference of a circle was a number known as 2π .

A new unit, radians, is really useful for angles.

8.1 Rotational Motion and Angular Displacement

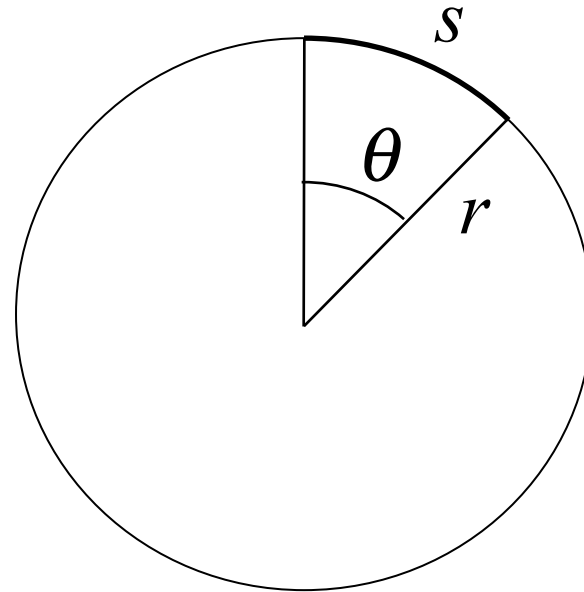
A new unit, radians, is really useful for angles.

Radian measure

$$\theta(\text{radians}) = \frac{s \text{ (arc length)}}{r \text{ (radius)}}$$

$$s = r\theta$$

(s in same units as r)



Full circle

$$\begin{aligned}\theta &= \frac{s}{r} = \frac{2\pi r}{r} \\ &= 2\pi \text{ (radians)}\end{aligned}$$

Conversion of degree to radian measure

$$\begin{aligned}\theta(\text{rad}) &= \theta(\text{deg.}) \left(\frac{2\pi \text{ rad}}{360 \text{ deg.}} \right) \\ \left(\frac{2\pi \text{ rad}}{360 \text{ deg.}} \right) &= 1\end{aligned}$$