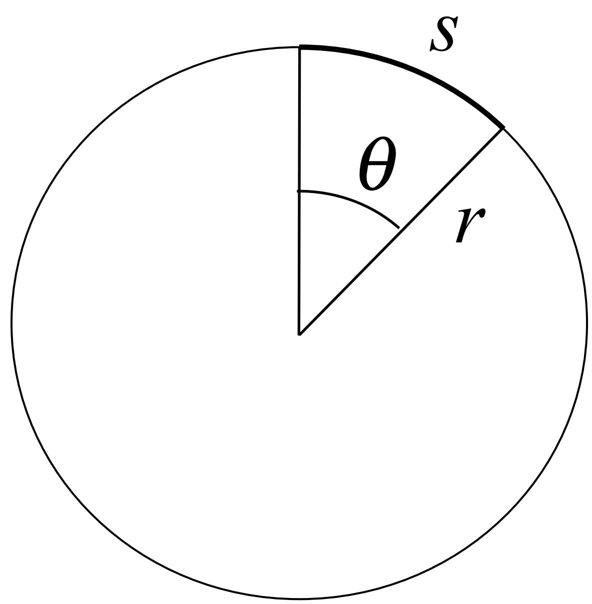
Chapter 8

Accelerated Circular Motion

8.1 Rotational Motion and Angular Displacement

A new unit, radians, is really useful for angles.



Radian measure

$$\theta(\text{radians}) = \frac{s \text{ (arc length)}}{r \text{ (radius)}}$$

$$s = r\theta$$

$(s$ in same units as $r)$

Full circle

$$\theta = \frac{s}{r} = \frac{2\pi \cancel{r}}{\cancel{r}}$$

$$= 2\pi \text{ (radians)}$$

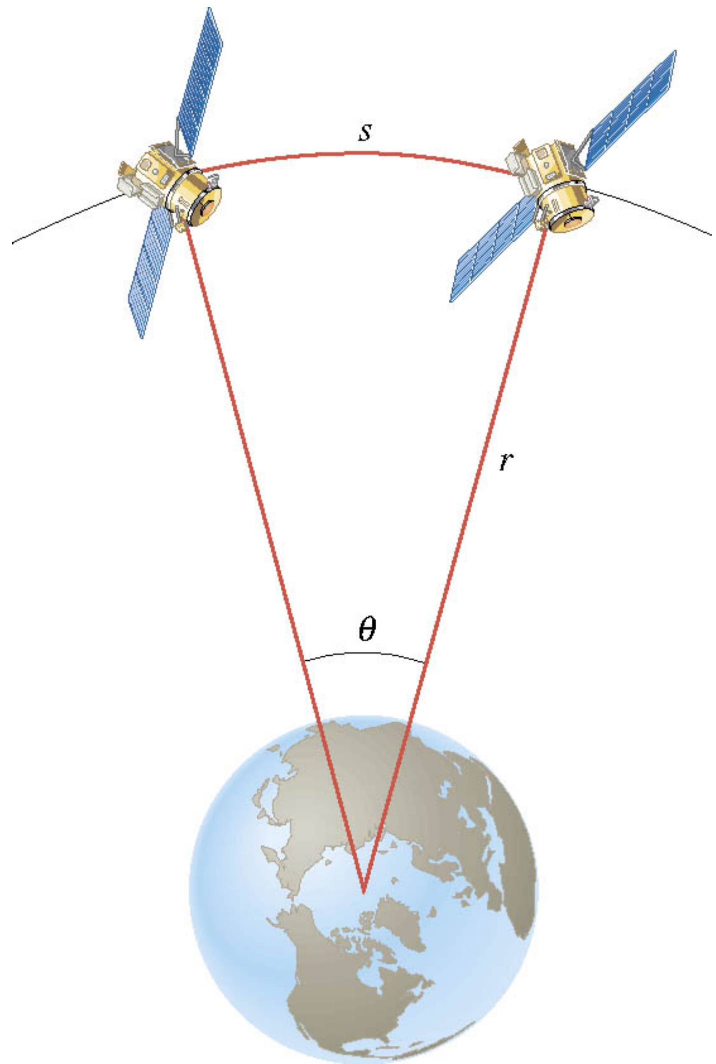
Conversion of degree to radian measure

$$\theta(\text{rad}) = \theta(\text{deg.})\left(\frac{2\pi}{360}\frac{\text{rad}}{\text{deg.}}\right)$$

$$\left(\frac{2\pi}{360}\frac{\text{rad}}{\text{deg.}}\right) = 1$$

Example: Adjacent Synchronous Satellites

Synchronous satellites are put into an orbit whose radius is 4.23×10^7m.



If the angular separation of the two satellites is 2.00 degrees, find the arc length that separates them.

Convert degree to radian measure

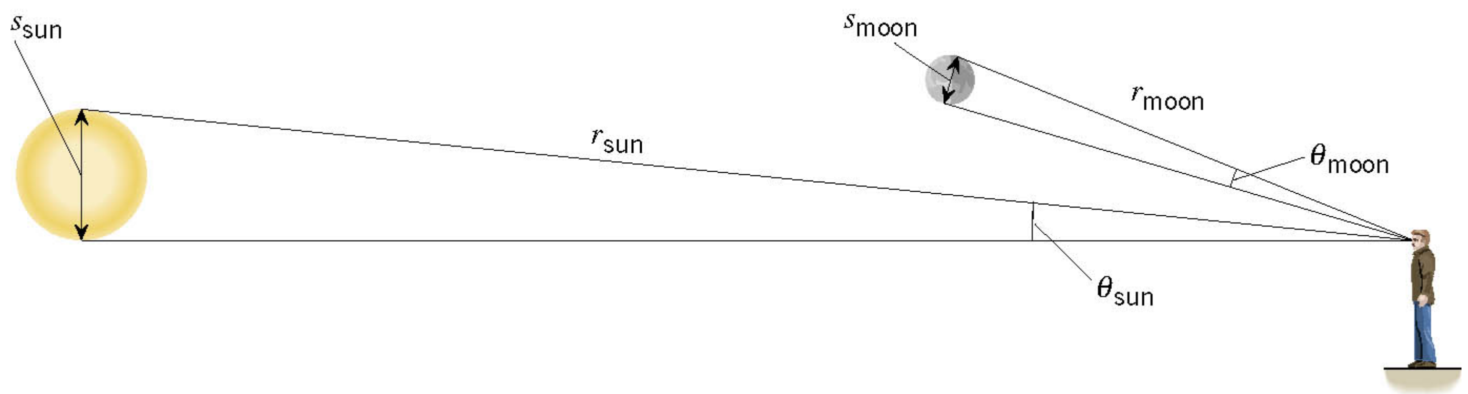
$$2.00\,\text{deg}\left(\frac{2\pi\ \text{rad}}{360\,\text{deg}}\right) = 0.0349\ \text{rad}$$

Determine arc length

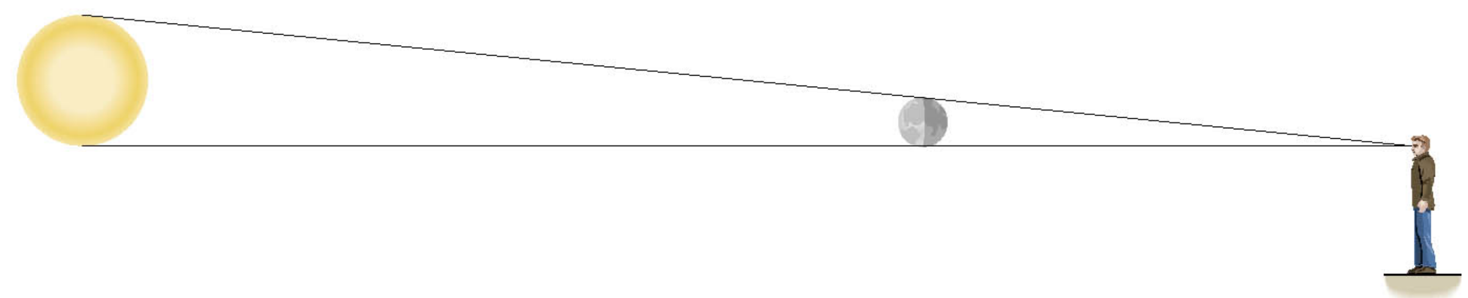
$$s = r\theta = \left(4.23\times10^{7}\,\text{m}\right)\left(0.0349\ \text{rad}\right)$$

$$= 1.48\times10^{6}\,\text{m}\ \ (920\ \text{miles})$$

8.1 Rotational Motion and Angular Displacement



(a)



(b)

For an observer on the earth, an eclipse can occur because angles subtended by the sun and the moon are the same.

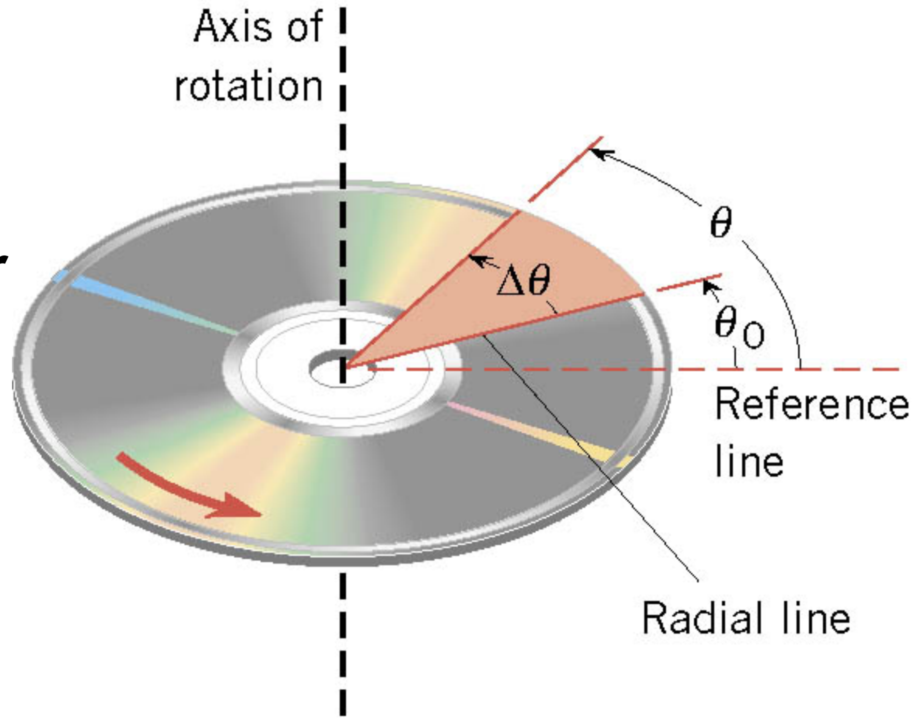
$$\theta = \frac{s_{\text{Sun}}}{r_{\text{Sun}}} \approx \frac{s_{\text{Moon}}}{r_{\text{Moon}}} \approx 9.3 \text{ mrad}$$

8.1 Rotational Motion and Angular Displacement

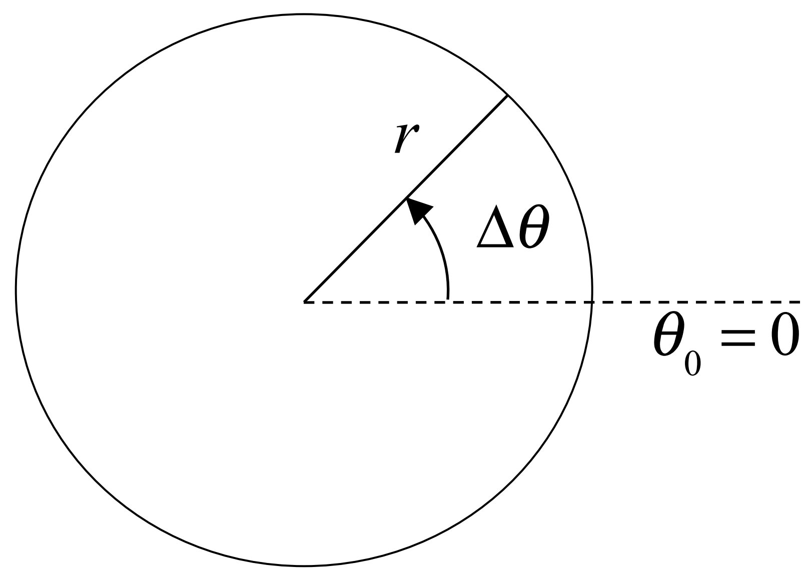
The angle through which the object rotates is called the ***angular displacement vector***

$$\Delta\theta = \theta - \theta_o$$

SI unit of angular displacement, radian (rad)



Simplified using $\theta_o = 0$, and
$\Delta\theta = \theta$, angular displacement vector.



Vector

Counter-clockwise is + displacement
Clockwise is – displacement

Clicker Question 8.1 Radian measure for angles

Over the course of a day (twenty-four hours), what is the angular displacement of the second hand of a wrist watch in radians?

a) 1440 rad

b) 2880 rad

c) 4520 rad

d) 9050 rad

e) 543,000 rad

Clicker Question 8.1 Radian measure for angles

Over the course of a day (twenty-four hours), what is the angular displacement of the second hand of a wrist watch in radians?

a) 1440 rad

b) 2880 rad

c) 4520 rad

d) 9050 rad

e) 543,000 rad

Second hand makes 1 rev. in 1 minute.

$$1 \text{ day} = (1 \text{ day})(60 \text{ min./hr})(24 \text{ hr/day})$$
$$= 1440 \text{ min.}$$
$$\#\text{ rev.} = \left(1 \text{ rev./min.}\right)(1440 \text{ min.}) = 1440 \text{ rev.}$$

$$\boxed{\begin{aligned} 1 \text{ rev.} &= 2\pi \text{ radians} \\ \text{Total } \theta \text{ displacement} &= (2\pi \text{ rad/min.})(1440 \text{ min.}) \\ &= 9048 \text{ rad} \end{aligned}}$$

DEFINITION OF AVERAGE ANGULAR VELOCITY

$$\text{Average angular velocity} = \frac{\text{Angular displacement}}{\text{Elapsed time}}$$

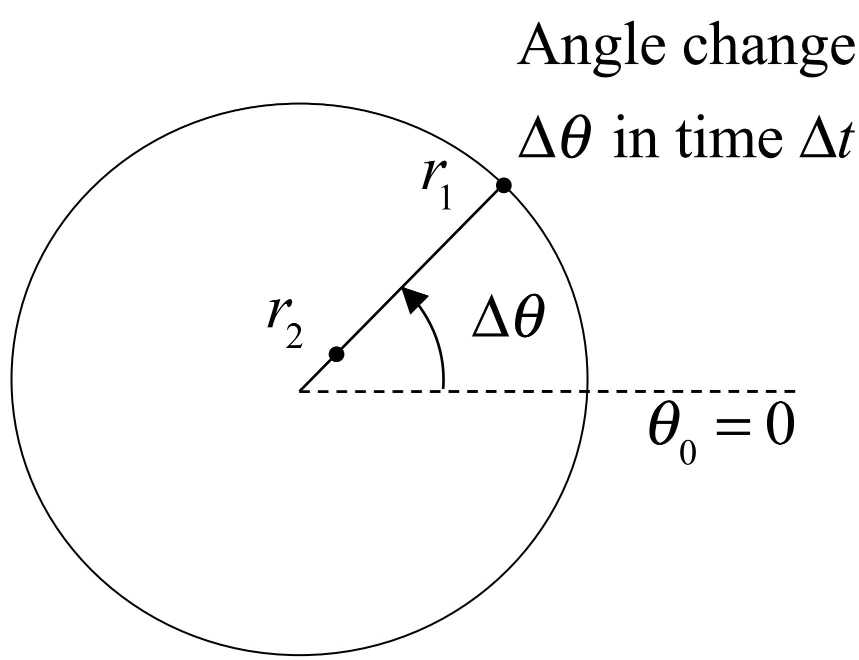
$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \qquad \text{where } \Delta t = t - t_o$$

SI Unit of Angular Velocity: radian per second (rad/s)

$\Delta\theta$ is the same at all radii.

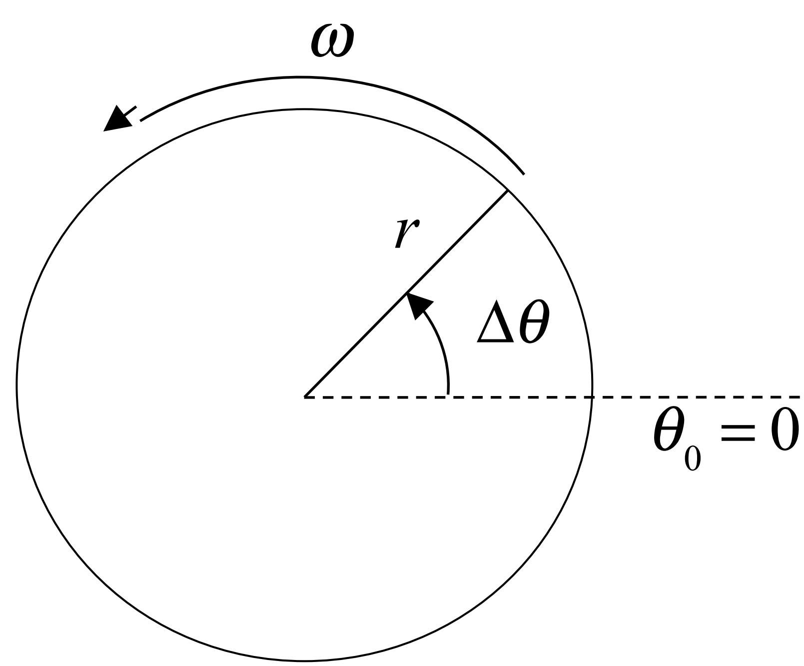
Δt is the same at all radii.

$$\boxed{\omega = \frac{\Delta\theta}{\Delta t} \text{ is the same at all radii.}}$$



8.2 Angular Velocity and Angular Acceleration

Case 1: Constant angular velocity, ω.



$$\omega = \frac{\Delta\theta}{\Delta t} \qquad \Delta\theta = \omega\,\Delta t$$

Example: A disk rotates with a constant angular velocity of +1 rad/s. What is the angular displacement of the disk in 13 seconds?

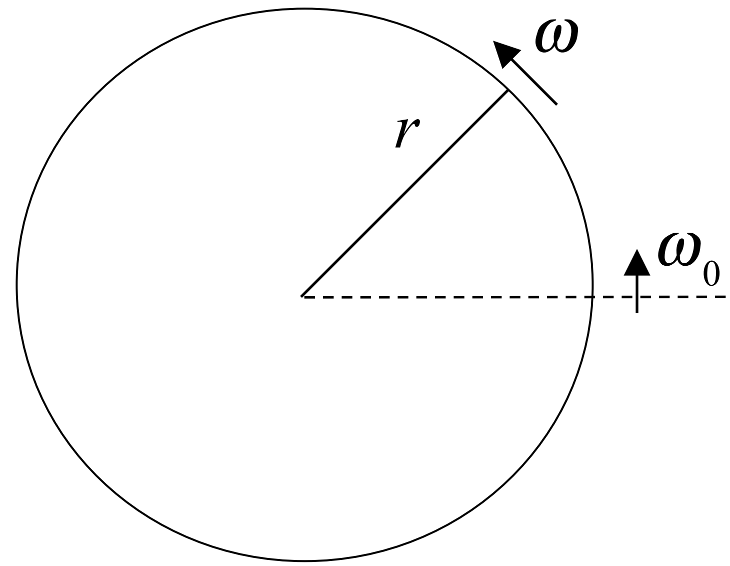
How many rotations has the disk made in that time?

$$\Delta\theta = \omega\,\Delta t = (+1\ \text{rad/s})(13\ \text{s}) = +13\ \text{rad}$$

$$2\pi\ \text{radians} = 1\ \text{rotation} \Rightarrow 2\pi\ \text{rad/rot.}$$

$$n_{rot} = \frac{\Delta\theta}{2\pi\ \text{rad/rot.}} = \frac{13\ \text{rad}}{6.3\ \text{rad/rot}} = 2.1\ \text{rot.}$$

Case 2: Angular velocity, ω, changes in time.



Instantaneous angular velocity at time t.

$$\omega = \lim_{\Delta t = 0} \frac{\Delta \theta}{\Delta t}$$

DEFINITION OF AVERAGE ANGULAR ACCELERATION

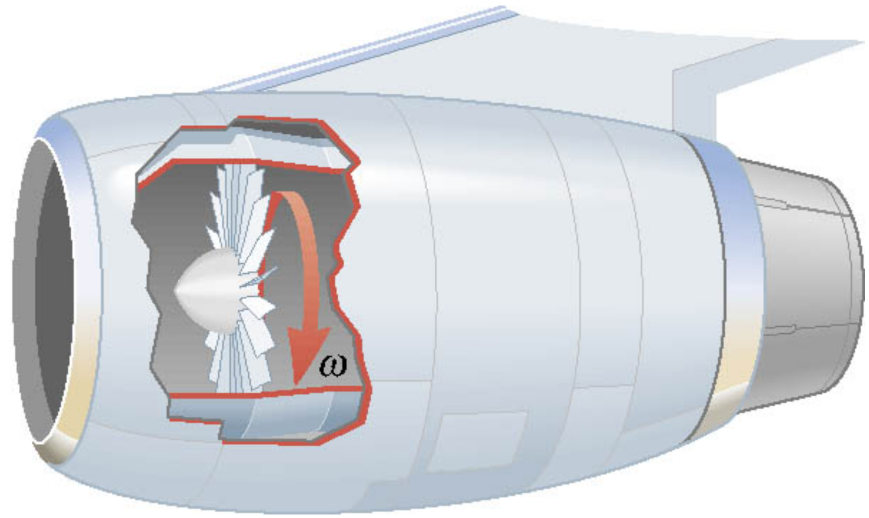
$$\text{Average angular acceleration} = \frac{\text{Change in angular velocity}}{\text{Elapsed time}}$$

$$\overline{\alpha} = \frac{\omega - \omega_o}{t - t_o} = \frac{\Delta \omega}{\Delta t}$$

SI Unit of Angular acceleration: radian per second squared (rad/s^2)

Example: A Jet Revving Its Engines

As seen from the front of the engine, the fan blades are rotating with an angular velocity of –110 rad/s. As the plane takes off, the angular velocity of the blades reaches –330 rad/s in a time of 14 s.



Rotation is clockwise (negative)

Find the angular acceleration, assuming it to be constant.

$$\bar{\alpha} = \frac{(-330\,\text{rad/s}) - (-110\,\text{rad/s})}{14\text{ s}} = -16\,\text{rad/s}^2$$

8.2 The Equations of Rotational Kinematics

The equations of rotational kinematics for constant angular acceleration:

ANGULAR ACCELERATION

ANGULAR VELOCITY

$$\omega = \omega_0 + \alpha t$$

$$\theta = \tfrac{1}{2}\left(\omega_0 + \omega\right) t$$

TIME

ANGULAR DISPLACEMENT

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

8.2 The Equations of Rotational Kinematics

Table 8.2 Symbols Used in Rotational and Linear Kinematics

Rotational Motion	Quantity	Linear Motion
θ	Displacement	x
ω_0	Initial velocity	v_0
ω	Final velocity	v
α	Acceleration	a
t	Time	t

Table 8.1 The Equations of Kinematics for Rotational and Linear Motion

Rotational Motion (α = constant)		Linear Motion (a = constant)	
$\omega = \omega_0 + \alpha t$	(8.4)	$v = v_0 + at$	(2.4)
$\theta = \frac{1}{2}(\omega_0 + \omega)t$	(8.6)	$x = \frac{1}{2}(v_0 + v)t$	(2.7)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	(8.7)	$x = v_0 t + \frac{1}{2}at^2$	(2.8)
$\omega^2 = \omega_0^2 + 2\alpha\theta$	(8.8)	$v^2 = v_0^2 + 2ax$	(2.9)

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (–).

3. Write down the values that are given for any of the five kinematic variables.

4. Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final angular velocity of one segment is the initial angular velocity for the next.

6. Keep in mind that there may be two possible answers to a kinematics problem.

Clicker Question 8.2 Rotational motion kinematics

Given the initial and final angular velocity of a disk and the total angular displacement of the disk, the angular acceleration of of the disk can be obtained with which single equation?

a) $\omega = \omega_0 + \alpha t$

b) $\theta = \frac{1}{2}(\omega + \omega_0)t$

c) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

d) $\omega^2 = \omega_0^2 + 2\alpha\theta$

e) none of the above

Clicker Question 8.2 Rotational motion kinematics

Given the initial and final angular velocity of a disk and the total angular displacement of the disk, the angular acceleration of of the disk can be obtained with which single equation?

a) $\omega = \omega_0 + \alpha t$

b) $\theta = \frac{1}{2}(\omega + \omega_0)t$

c) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

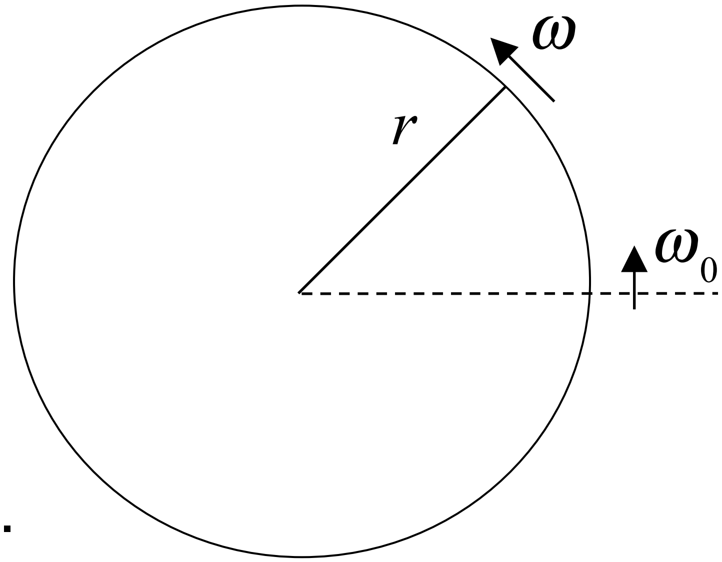
d) $\boxed{\omega^2 = \omega_0^2 + 2\alpha\theta}$

e) none of the above

No time given!

$$\boxed{\begin{aligned} \omega^2 &= \omega_0^2 + 2\alpha\theta \\ \alpha &= \frac{\omega^2 - \omega_0^2}{2\theta} \end{aligned}}$$

Example: A disk has an initial angular velocity of +375 rad/s. The disk accelerates and reaches a greater angular velocity after rotating through an angular displacement of +44.0 rad.



If the angular acceleration has a constant value of +1740 rad/s^2, find the final angular velocity of the disk.

Given: $\omega_0 = +375 \text{ rad/s}, \theta = +44 \text{ rad}, \alpha = 1740 \text{ rad/s}^2$

Want: final angular velocity, ω.

No time!

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$= (375 \text{ rad/s})^2 + 2(1740 \text{ rad/s}^2)(+44 \text{ rad})$$

$$\omega = 542 \text{ rad/s}$$

8.3 Angular Variables and Tangential Variables

$\omega =$ angular velocity - same at all radii (radians/s)

$\alpha =$ angular acceleration - same at all radii (radians/s^2)

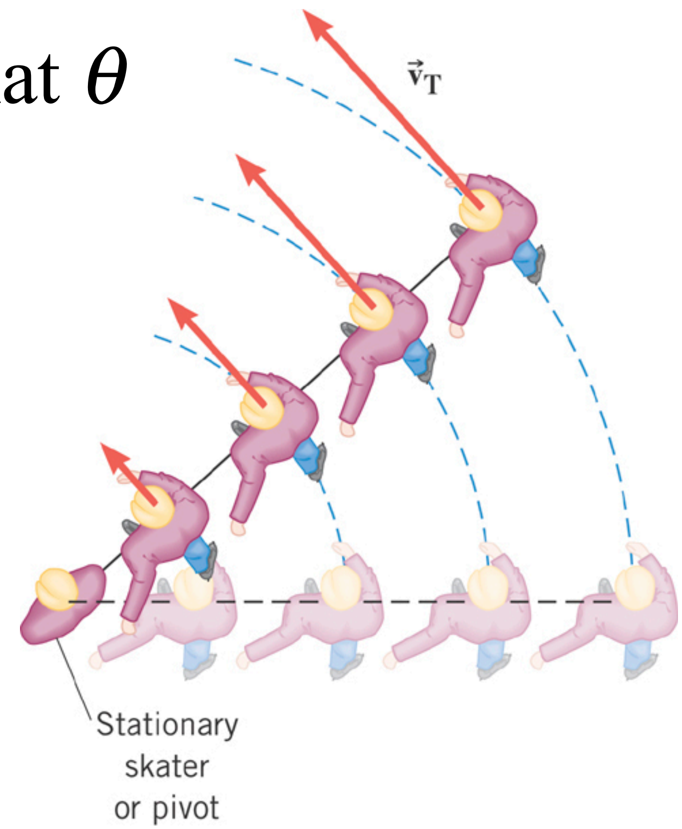
$\vec{\mathbf{v}}_T =$ tangential velocity - different at each radius

$\vec{\mathbf{a}}_T =$ tangential acceleration - different at each radius

Direction is tangent to circle at that θ

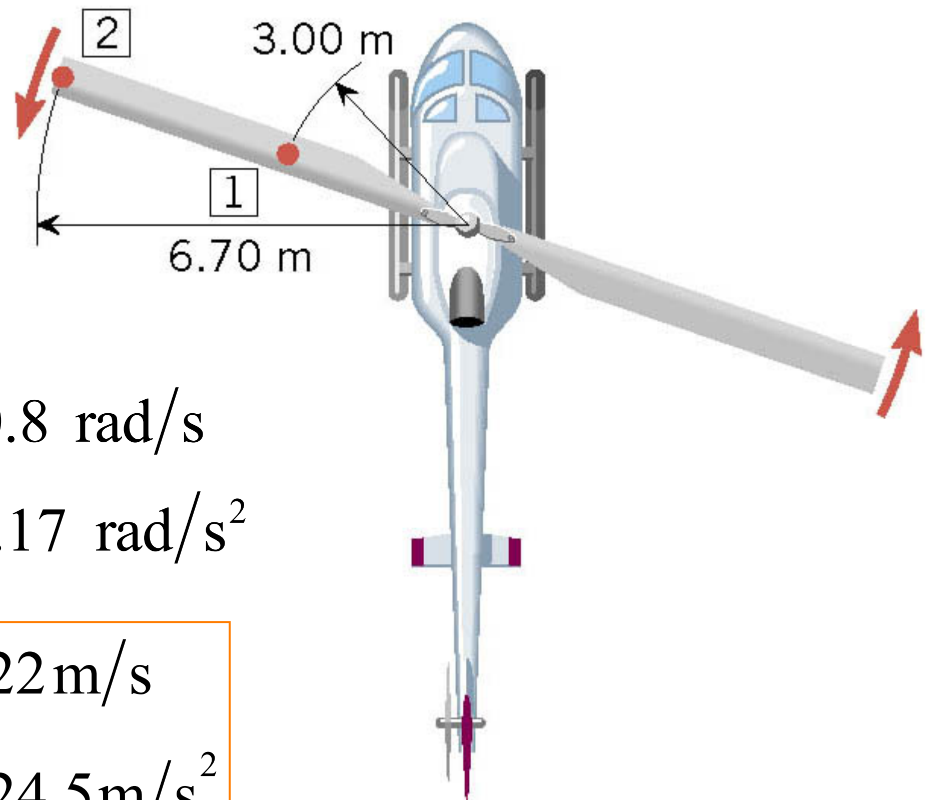
$$\vec{\mathbf{v}}_T = \omega r \qquad \vec{\mathbf{a}}_T = \alpha r$$

$\vec{\mathbf{v}}_T$ (m/s)	$\vec{\mathbf{a}}_T$ (m/s^2)
ω (rad/s)	α (rad/s^2)
r (m)	r (m)



Example: A Helicopter Blade

A helicopter blade has an angular speed of 6.50 rev/s and an angular acceleration of 1.30 rev/s^2.
For point 1 on the blade, find the magnitude of (a) the tangential speed and (b) the tangential acceleration.



Convert revolutions to radians

$$\omega = (6.50\ \text{rev/s})(2\pi\ \text{rad/rev}) = 40.8\ \text{rad/s}$$

$$\alpha = (1.30\ \text{rev/s}^2)(2\pi\ \text{rad/rev}) = 8.17\ \text{rad/s}^2$$

$$v_T = \omega r = (40.8\,\text{rad/s})(3.00\ \text{m}) = 122\,\text{m/s}$$

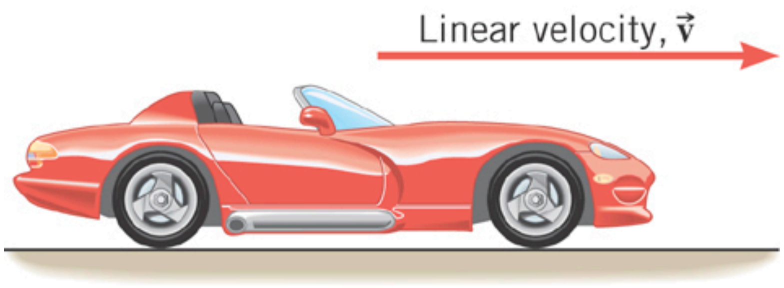
$$a_T = \alpha r = (8.17\,\text{rad/s}^2)(3.00\ \text{m}) = 24.5\,\text{m/s}^2$$

8.3 Rolling Motion

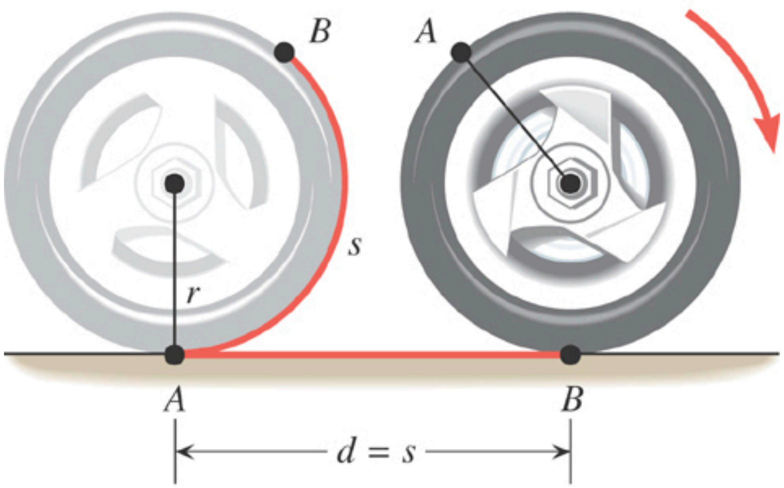
The tangential speed of a point on the outer edge of the tire is equal to the speed of the car over the ground.

$$v = v_T = \omega r$$

$$a = a_T = \alpha r$$

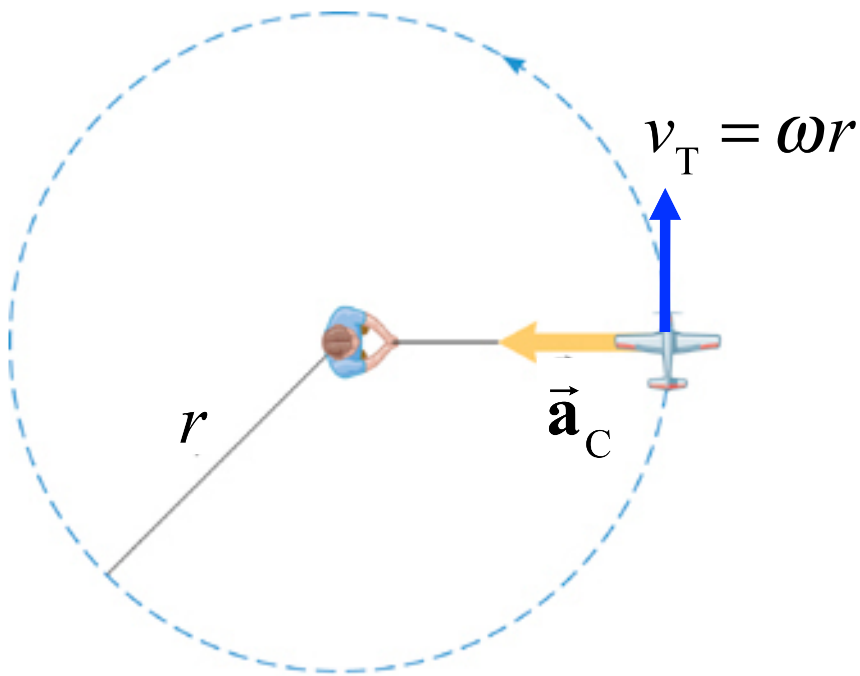


(a)

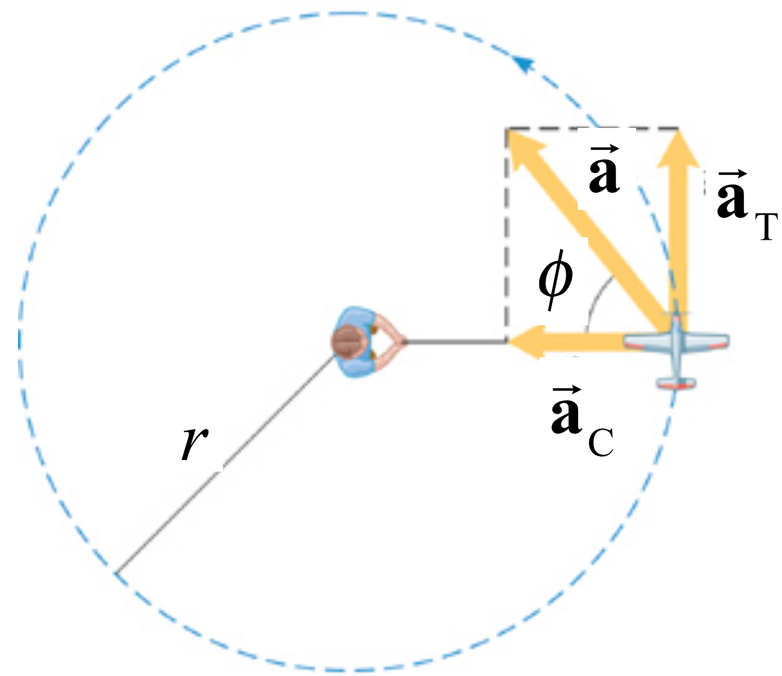


(b)

8.3 Centripetal Acceleration with Tangential Acceleration



Uniform circular motion



Non-uniform circular motion

$\underline{\omega \text{ in rad/s constant}}$

$$a_C = \frac{v_T^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$

$\underline{\text{Changing } \omega = \omega_0 + \alpha t}$

$$a_T = \alpha r$$

$$a_{\text{total}} = \sqrt{a_C^2 + \alpha^2 r^2}$$

Chapter 4.5

Force Generating Uniform Circular Motion

Newton's Second Law

When a net external force acts on an object of mass *m*, the acceleration that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force.

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} \qquad \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

Vector Equations

4.5 Centripetal Force

Thus, in uniform circular motion there must be a net force to produce the centripetal acceleration.

The centripetal force is the name given to the net force required to keep an object moving on a circular path.

The direction of the centripetal force always points toward the center of the circle and continually changes direction as the object moves.

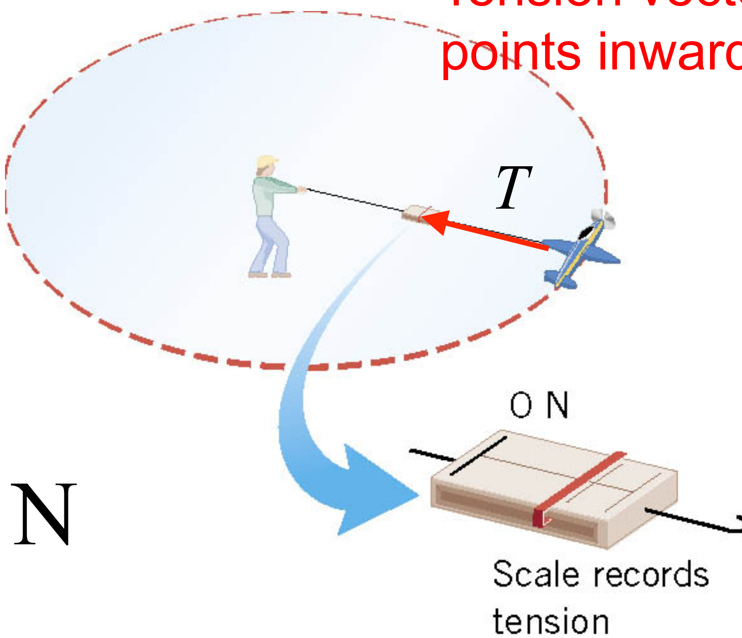
$$F_C = ma_C = m\frac{v^2}{r}$$

Magnitudes

Example: The Effect of Speed on Centripetal Force

The model airplane has a mass of 0.90 kg and moves at constant speed on a circle that is parallel to the ground. The path of the airplane and the guideline lie in the same horizontal plane because the weight of the plane is balanced by the lift generated by its wings. Find the tension in the 17 m guideline for a speed of 19 m/s.

Tension vector points inward!



$$T = F_C = m\frac{v^2}{r}$$

$$T = (0.90\,\mathrm{kg})\frac{(19\,\mathrm{m/s})^2}{17\,\mathrm{m}} = 19\,\mathrm{N}$$

Example: The Effect of Speed on Centripetal Force

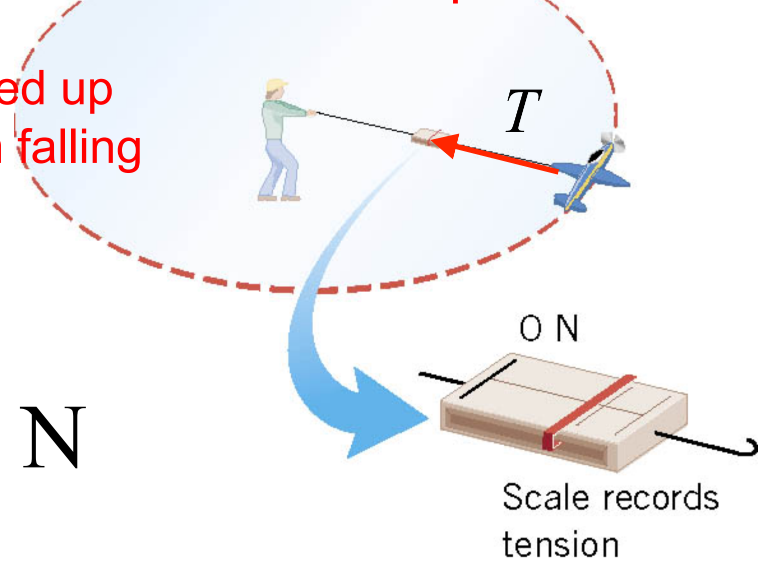
The model airplane has a mass of 0.90 kg and moves at constant speed on a circle that is parallel to the ground. The path of the airplane and the guideline lie in the same horizontal plane because the weight of the plane is balanced by the lift generated by its wings. Find the tension in the 17 m guideline for a speed of 19 m/s.

Tension is the centripetal force necessary to maintain airplane in the circle

Tension vector points inward!

$$T = F_C = m\frac{v^2}{r}$$

engine keeps speed up
wings keep it from falling



$$T = (0.90\,\text{kg})\frac{(19\,\text{m/s})^2}{17\,\text{m}} = 19\,\text{N}$$

Conceptual Example: A Trapeze Act

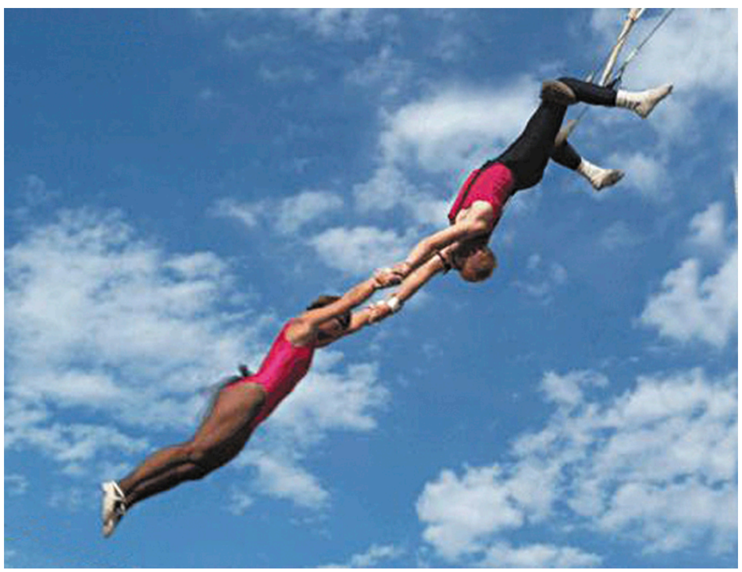
In a circus, a man hangs upside down from a trapeze, legs bent over and arms downward, holding his partner. Is it harder for the man to hold his partner when the partner hangs straight down and is stationary or when the partner is swinging through the straight-down position?

Tension in arms maintains circular motion but also must counter the gravitational force (weight)

Conceptual Example: A Trapeze Act

In a circus, a man hangs upside down from a trapeze, legs bent over and arms downward, holding his partner. Is it harder for the man to hold his partner when the partner hangs straight down and is stationary or when the partner is swinging through the straight-down position?

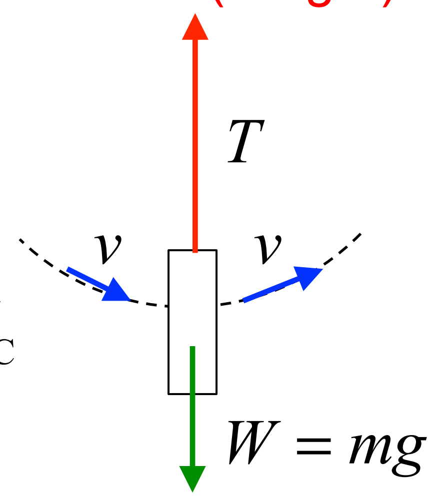
Tension in arms maintains circular motion but also must counter the gravitational force (weight)



$$F_C = m\frac{v^2}{r}$$

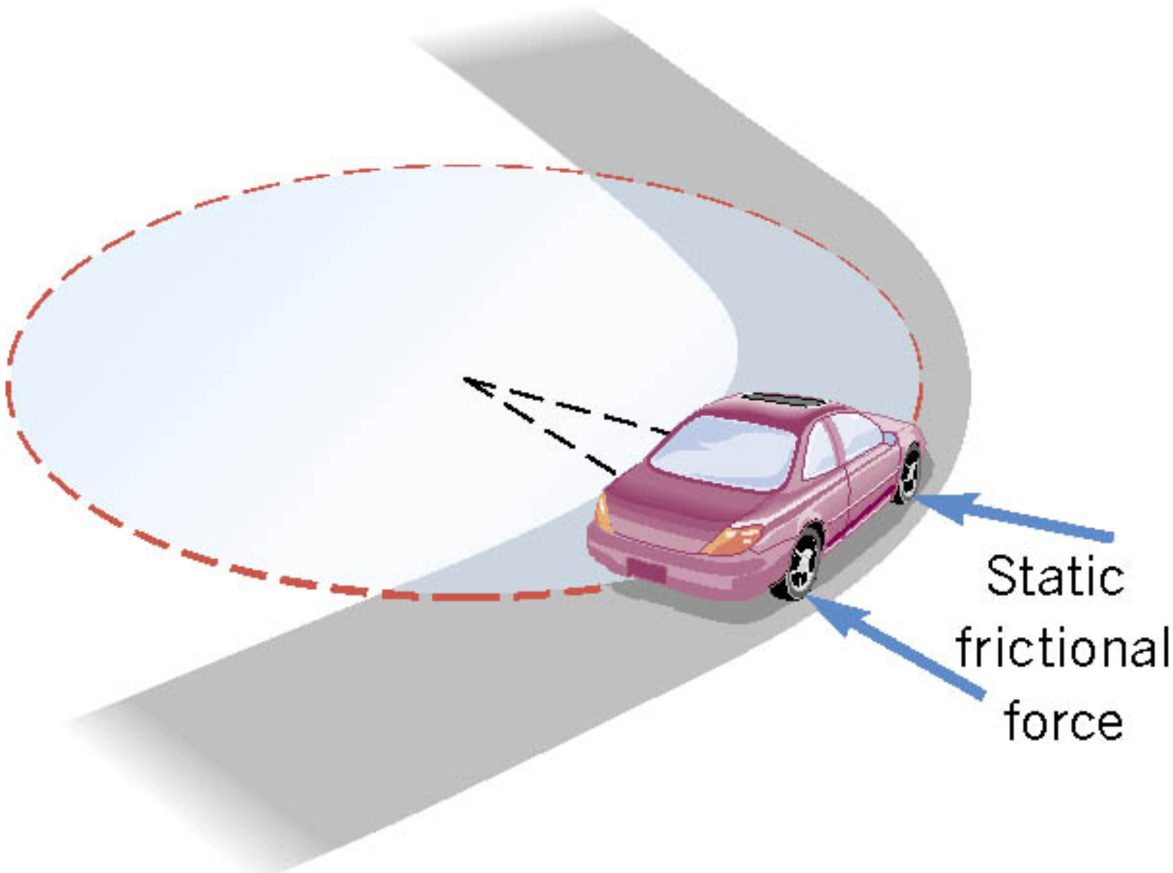
$$\sum \vec{\mathbf{F}} = +T - W = F_C$$

$$T = W + F_C$$



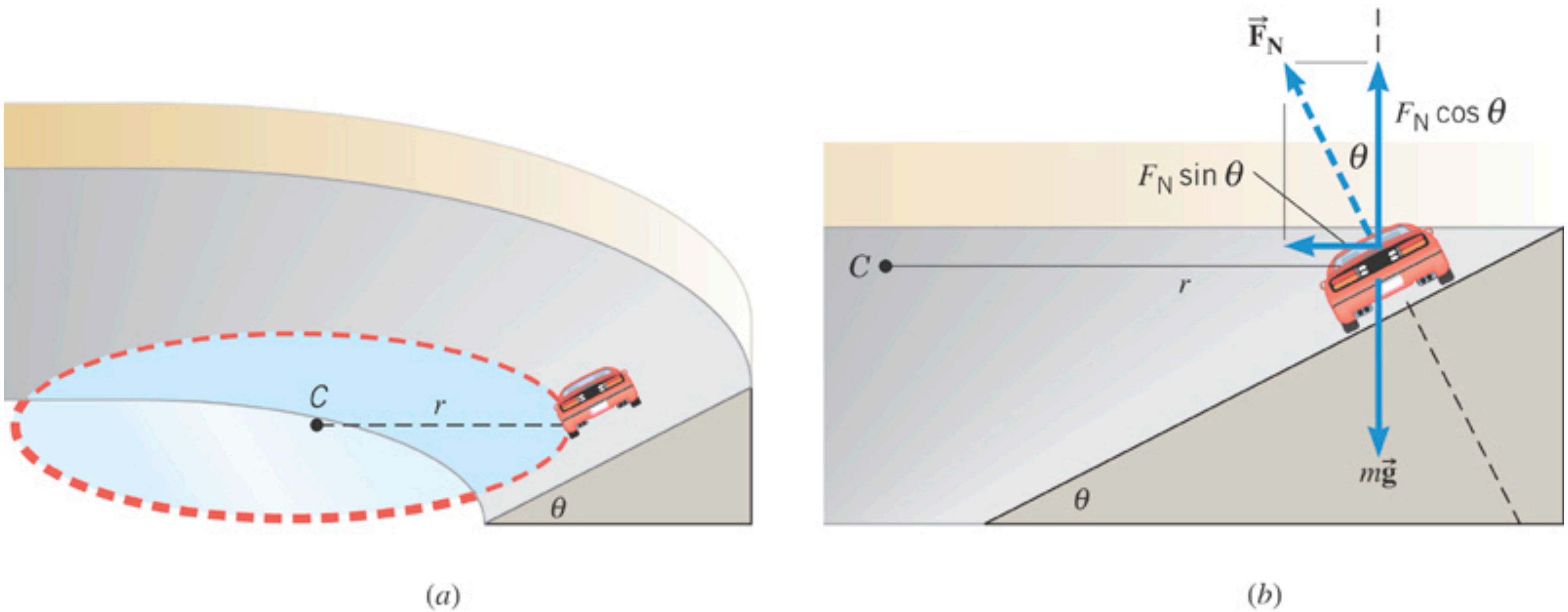
4.5 Banked Curves

On an unbanked curve, the static frictional force provides the centripetal force.



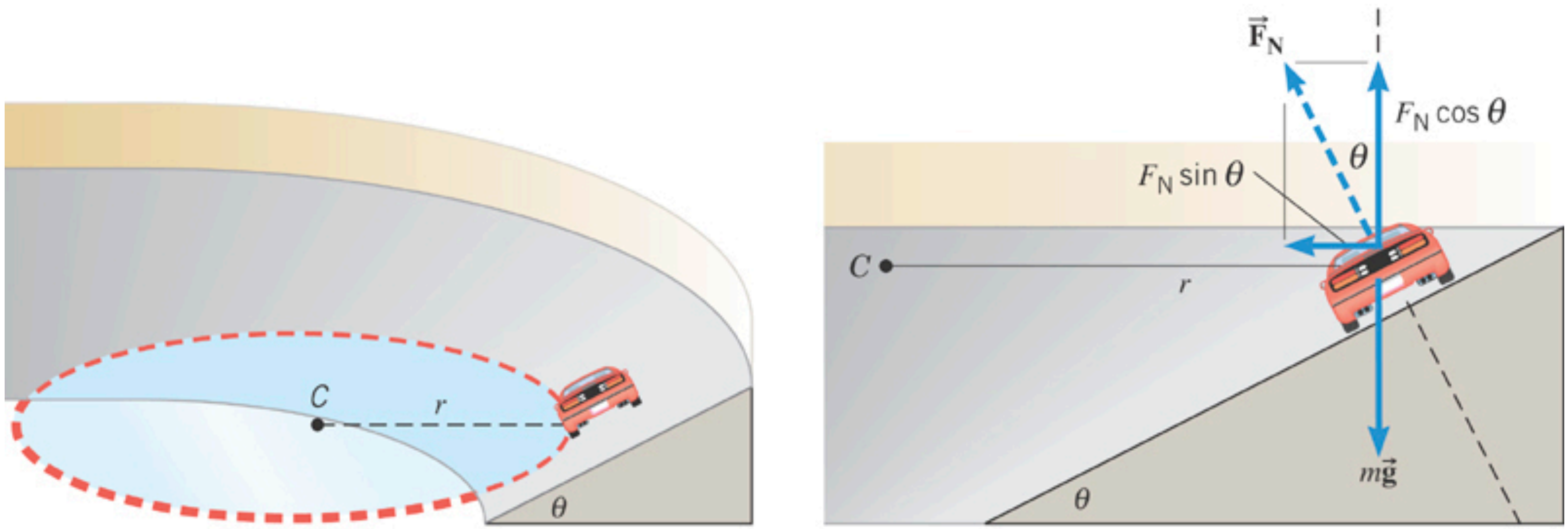
4.5 Banked Curves

On a frictionless banked curve, the centripetal force is the horizontal component of the normal force. The vertical component of the normal force balances the car's weight.



Compression of the banked road provides the normal force. The normal force pushes against the car to 1) support the weight and 2) provides the centripetal force required for the car to move in a circle.

4.5 Banked Curves



(a) (b)

$$F_C = F_N \sin\theta = m\frac{v^2}{r}$$

$$F_N \cos\theta = mg$$

Combining the two relationships can determine the speed necessary to keep the car on the track with the given angle

Example: Artificial Gravity

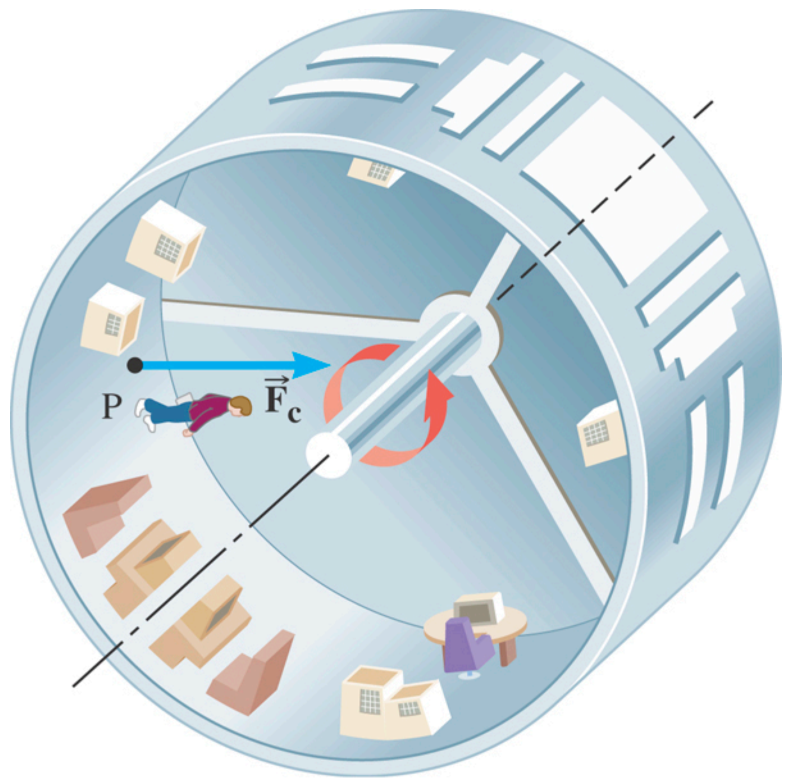
At what speed must the surface of a space station move so that an astronaut experiences a normal force on the feet equal to the weight on earth? The radius is 1700 m.

$$F_c = m\frac{v^2}{r} = mg$$

$$v = \sqrt{rg}$$

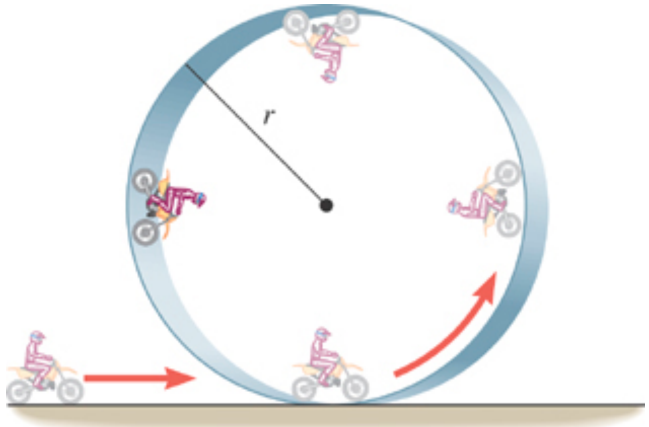
$$= \sqrt{(1700\text{ m})(9.80\,\text{m/s}^2)}$$

$$= 130\text{ m/s}$$



4.5 Vertical Circular Motion

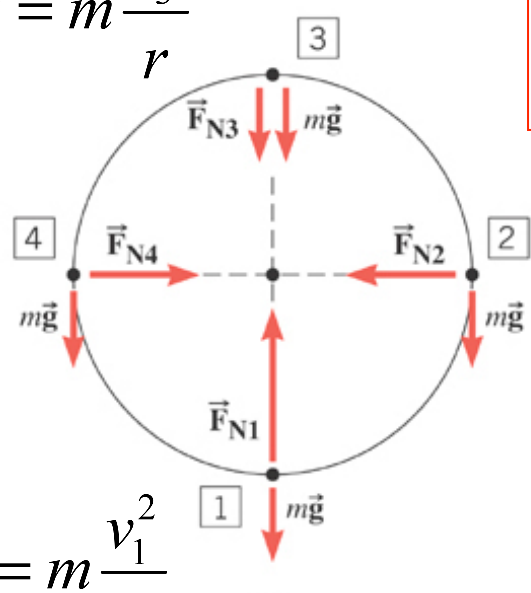
Normal forces are created by stretching of the hoop.



$$\frac{v_3^2}{r} \text{ must be} > g$$

to stay on the track

$$F_{N3} + mg = m\frac{v_3^2}{r}$$



(b)

$$F_{N4} = m\frac{v_4^2}{r}$$

$$F_{N2} = m\frac{v_2^2}{r}$$

$$F_{N1} - mg = m\frac{v_1^2}{r}$$