

Chapter 10

Solids & Liquids ***continued***

10.3 Pascal's Principle

PASCAL'S PRINCIPLE

Any change in the pressure applied to a completely enclosed fluid is transmitted undiminished to all parts of the fluid and enclosing walls.

$$P_2 = \frac{F_2}{A_2}; \quad P_1 = \frac{F_1}{A_1}$$

$$P_2 = P_1 + \rho gh$$

$$P_2 = P_1$$

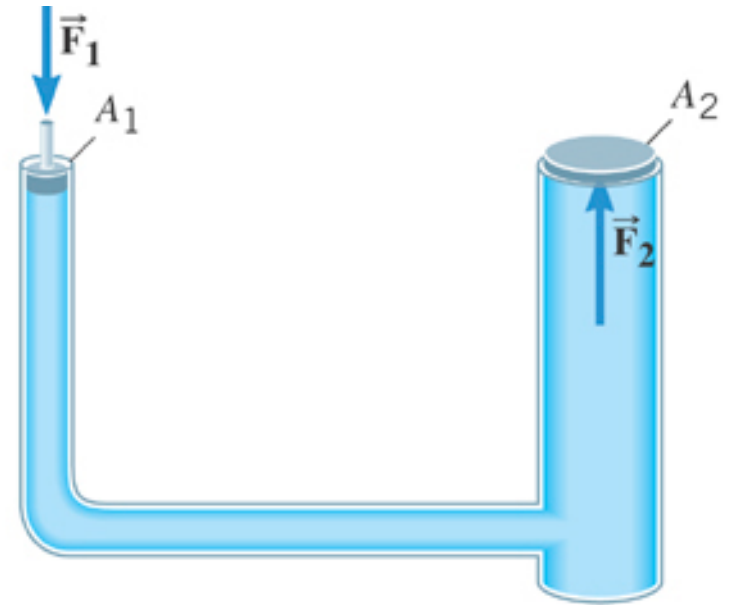
Assume weight of fluid
in the tube is negligible

$$P \gg \rho gh$$

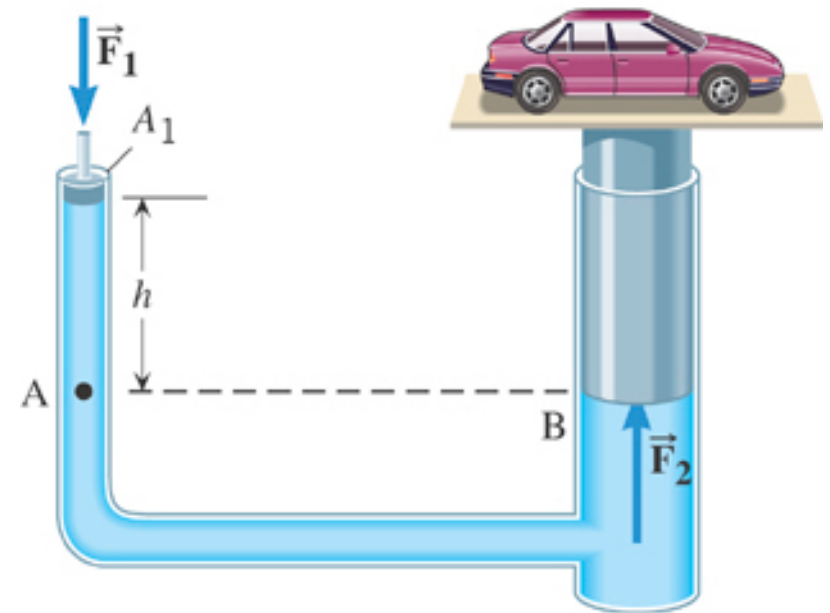
Small ratio

$$\frac{F_2}{A_2} = \frac{F_1}{A_1} \Rightarrow$$

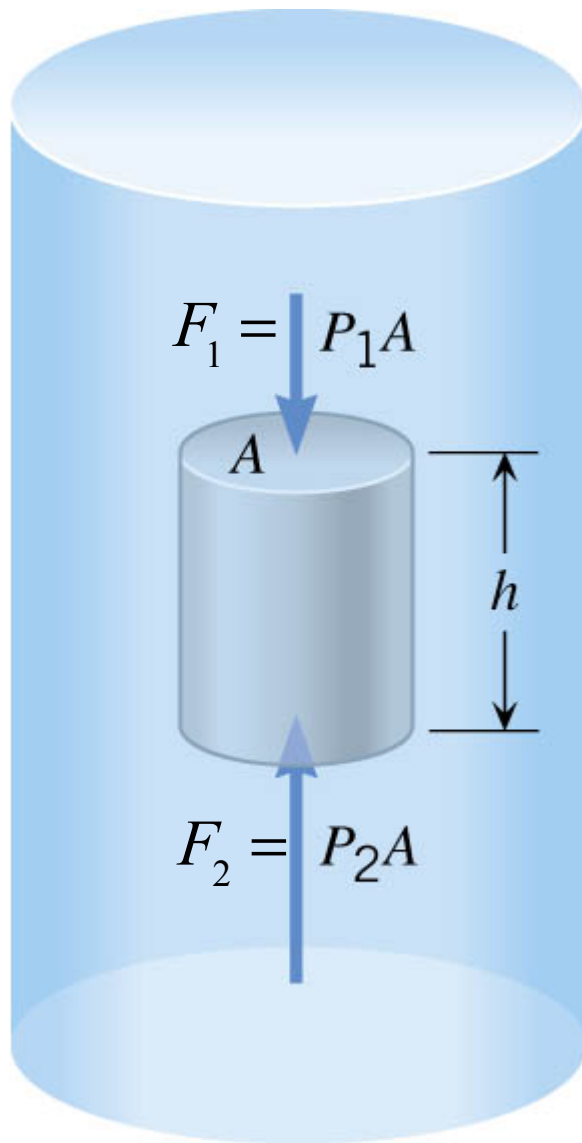
$$F_1 = F_2 \left(\frac{A_1}{A_2} \right)$$



(a)



10.4 Archimedes' Principle



$$P_2 = P_1 + \rho gh$$

Pressure grows linearly with depth.

Liquid density

$$\rho(\text{rho}) = \frac{m}{V}$$

Buoyant Force, F_B

$$F_B = F_2 + (-F_1)$$

$$= P_2 A - P_1 A = (P_2 - P_1) A$$

$$= \rho ghA$$

since $P_2 - P_1 = \rho gh$

$$= \underbrace{\rho V}_{\text{mass of displaced fluid}} g$$

and $V = hA$

Buoyant force = Weight of displaced fluid

$$F_B = \rho Vg$$

10.4 Archimedes' Principle

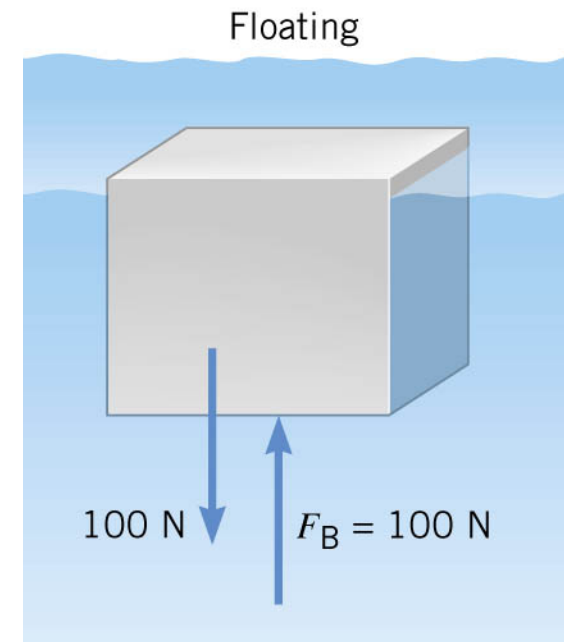
ARCHIMEDES' PRINCIPLE

Any fluid applies a buoyant force to an object that is partially or completely immersed in it; the magnitude of the buoyant force equals the weight of the fluid that the object displaces:

$$\underbrace{F_B}_{\text{Magnitude of buoyant force}} = \underbrace{W_{\text{fluid}}}_{\text{Weight of displaced fluid}}$$

CORROLARY

If an object is floating then the magnitude of the buoyant force is equal to the magnitude of its weight.



10.4 Archimedes' Principle

Any fluid applies a buoyant force to an object that is partially or completely immersed in it; the magnitude of the buoyant force equals the weight of the fluid that the object displaces:

$$F_B = \rho_{\text{Water}} V g$$

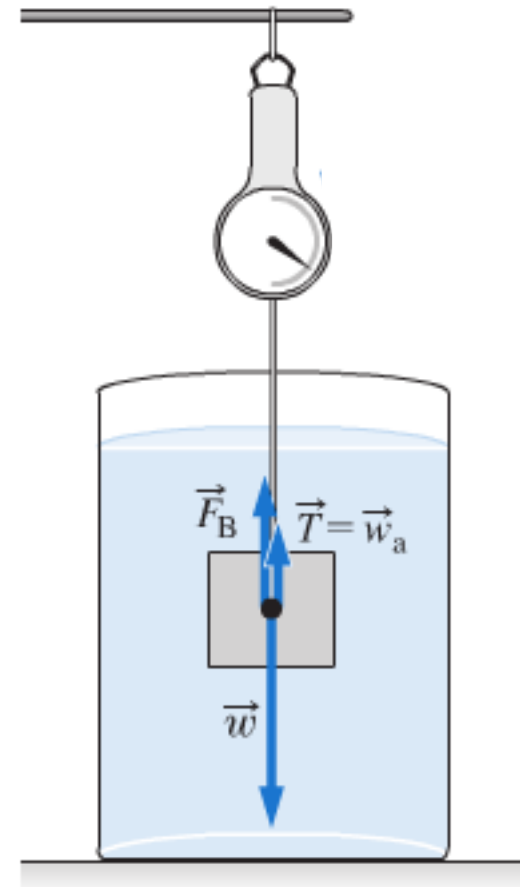
$$W = \rho_{\text{Object}} V g$$

$$F_B = W - T \quad (\text{Net force} = 0)$$

$$\frac{F_B}{W} = 1 - \frac{T}{W}$$

$$\frac{\rho_{\text{Water}} V g}{\rho_{\text{Object}} V g} = 1 - \frac{T}{W}$$

$$\rho_{\text{Object}} = \frac{\rho_{\text{Water}}}{(1 - T / W)}$$



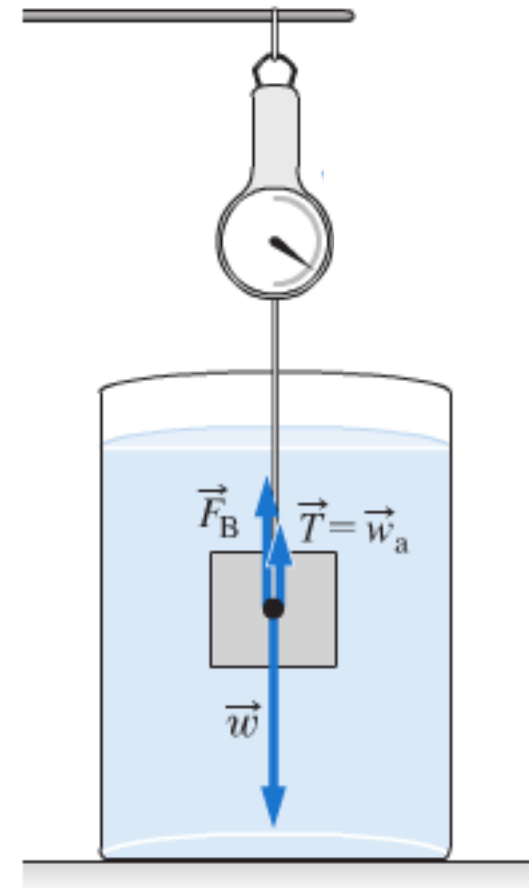
10.4 Archimedes' Principle

Use Archimedes Principle to measure the density of Aluminum

$$\rho_{\text{Object}} = \frac{\rho_{\text{Water}}}{(1 - T / W)} \quad \frac{T}{W} \sim \frac{4.5 \text{ N}}{8.5 \text{ N}} = 0.53$$
$$= \frac{1000 \text{ kg/m}^3}{1 - 0.53} = 2100 \text{ kg/m}^3$$

**Table 11.1 Mass Densities^a
of Common Substances**

Substance	Mass Density ρ (kg/m ³)
Solids	
Aluminum	2700
Brass	8470
Concrete	2200

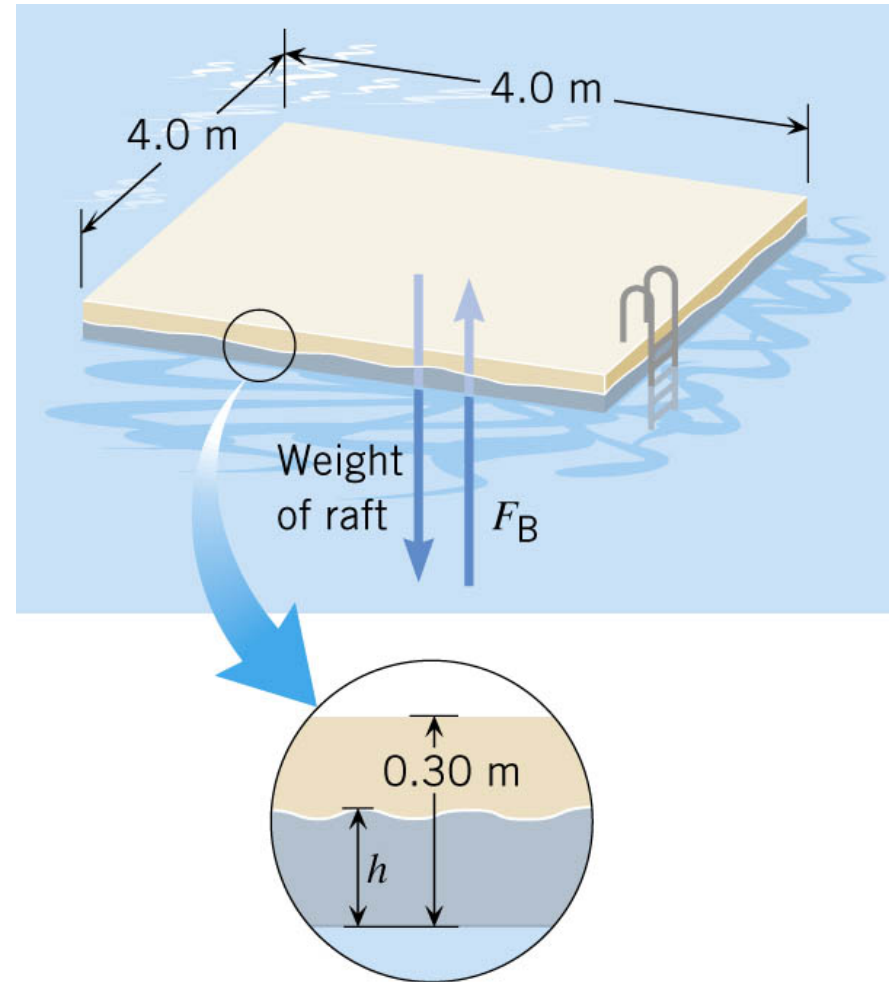


10.4 Archimedes' Principle

Example: A Swimming Raft

The raft is made of solid square pinewood.

- Determine whether the raft floats in water and if so,
- how much of the raft is beneath the surface.



10.4 Archimedes' Principle

If $W_{\text{raft}} < F_B$ submerged, **raft floats**

Weight of the Raft

$$\begin{aligned} W_{\text{raft}} &= m_{\text{raft}} g = \rho_{\text{raft}} V_{\text{raft}} g \\ &= (550 \text{ kg/m}^3)(4.8 \text{ m}^3)(9.80 \text{ m/s}^2) \\ &= 26000 \text{ N} \end{aligned}$$

Buoyant force totally submerged

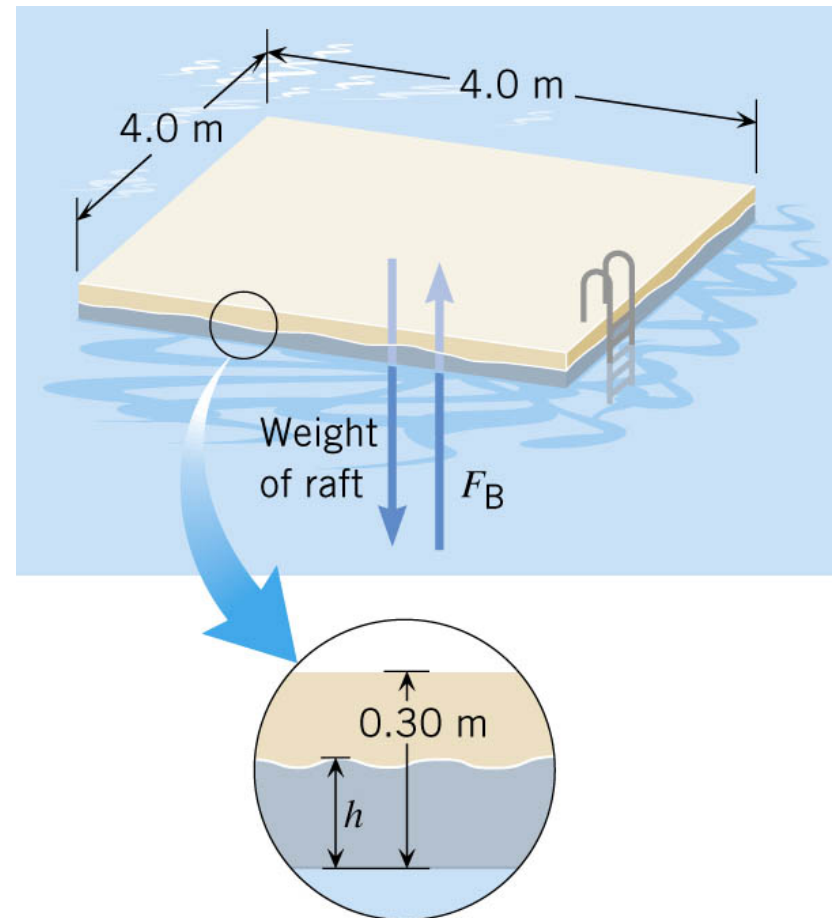
$$\begin{aligned} F_B &= \rho V g = \rho_{\text{Water}} V_{\text{Raft}} g \\ &= (1000 \text{ kg/m}^3)(4.8 \text{ m}^3)(9.80 \text{ m/s}^2) \\ &= 47000 \text{ N} \end{aligned}$$

$W_{\text{raft}} < F_B$ submerged, so **Raft floats**

Raft has these properties

$$V_{\text{raft}} = (4.0)(4.0)(0.30) \text{ m}^3 = 4.8 \text{ m}^3$$

$$\rho_{\text{raft}} = 550 \text{ kg/m}^3$$



Only part of the raft is above water

10.4 Archimedes' Principle

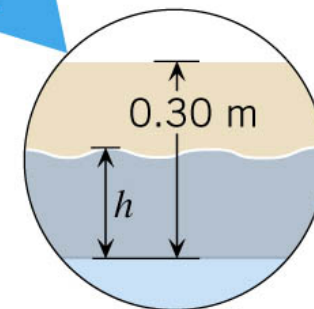
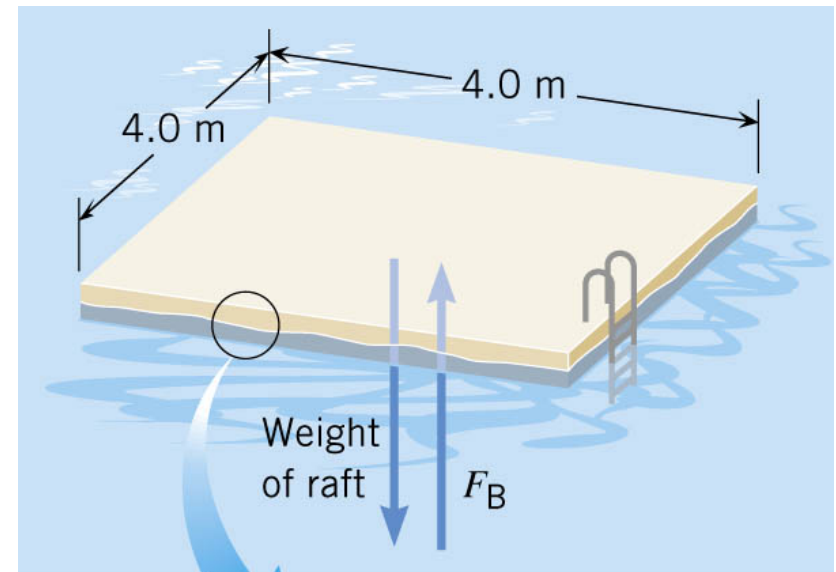
Raft has these properties

$$V_{\text{raft}} = (4.0)(4.0)(0.30)\text{m}^3 = 4.8\text{ m}^3$$

$$\rho_{\text{raft}} = 550\text{ kg/m}^3$$

An object will float in a fluid if its density is less than that of the fluid !

$$\rho_{\text{Raft}} = 550\text{ kg/m}^3 < 1000\text{ kg/m}^3 = \rho_{\text{Water}}$$



So, how much of the raft is beneath the surface?

Only part of the raft is above water

10.4 Archimedes' Principle

How much of raft below water?

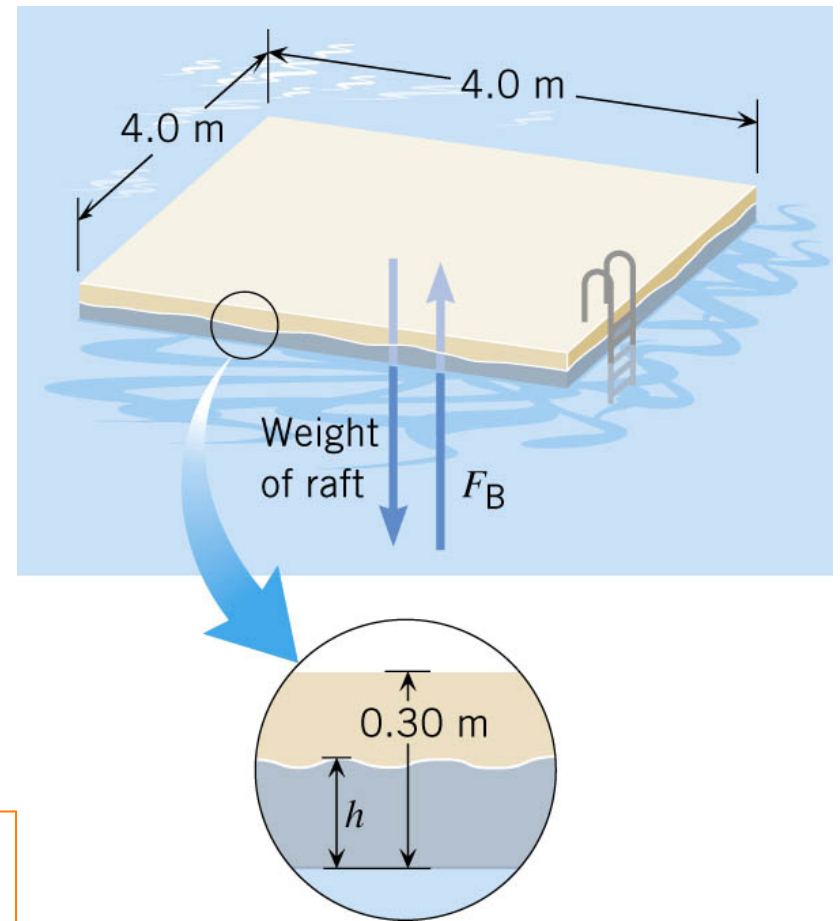
For a floating object

$$F_B = W_{\text{Raft}} = 26000 \text{ N}$$

F_B is weight of displaced water

$$\begin{aligned} F_B &= \rho_{\text{Water}} g V_{\text{Water}} (\text{displaced}) \\ &= \rho_{\text{Water}} g (A_{\text{Raft}} h) \end{aligned}$$

$$\begin{aligned} h &= \frac{W_{\text{Raft}}}{\rho_{\text{Water}} g A_{\text{Raft}}} \quad (\text{height below surface}) \\ &= \frac{26000 \text{ N}}{(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(16.0 \text{ m}^2)} \\ &= 0.17 \text{ m} \end{aligned}$$



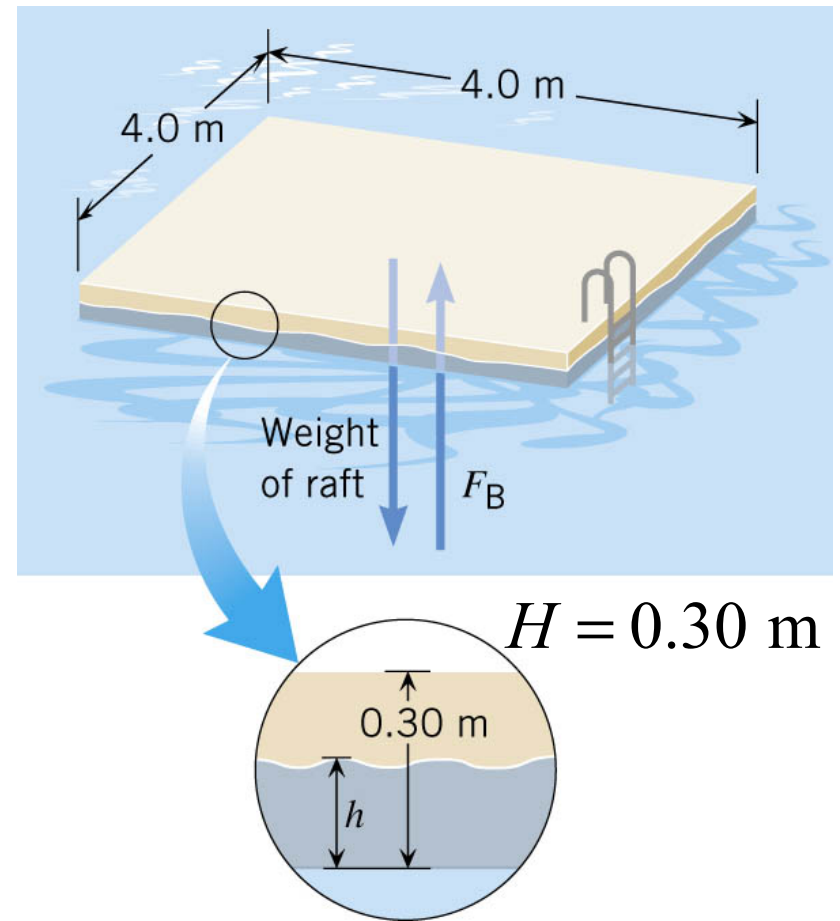
10.4 Archimedes' Principle

In General

What fraction of full height H is below the surface ?

$$h = \frac{W_{\text{Raft}}}{\rho_{\text{Water}} g A_{\text{Raft}}}$$
$$= \frac{\rho_{\text{Raft}} g A_{\text{Raft}} H}{\rho_{\text{Water}} g A_{\text{Raft}}}$$

$$\frac{h}{H} = \frac{\rho_{\text{Raft}}}{\rho_{\text{Water}}} \text{ fraction below surface}$$

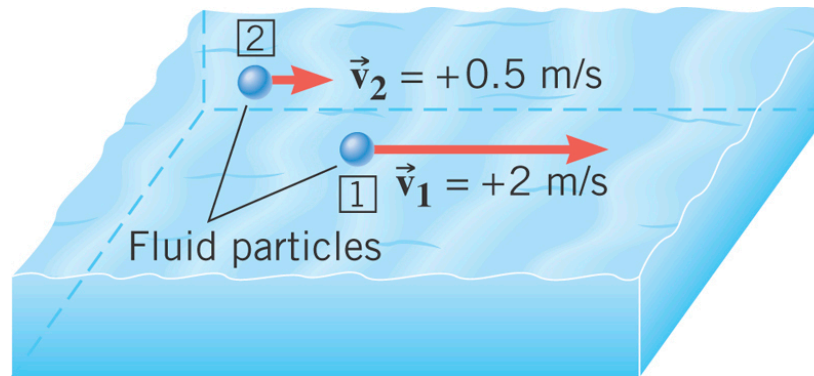


Fraction below surface is just the ratio of the density of floating object to the density of the fluid.

10.5 Fluids in Motion

In ***steady flow*** the velocity of the fluid particles at any point is constant as time passes.

Unsteady flow exists whenever the velocity of the fluid particles at a point changes as time passes.



Turbulent flow is an extreme kind of unsteady flow in which the velocity of the fluid particles at a point change erratically in both magnitude and direction.

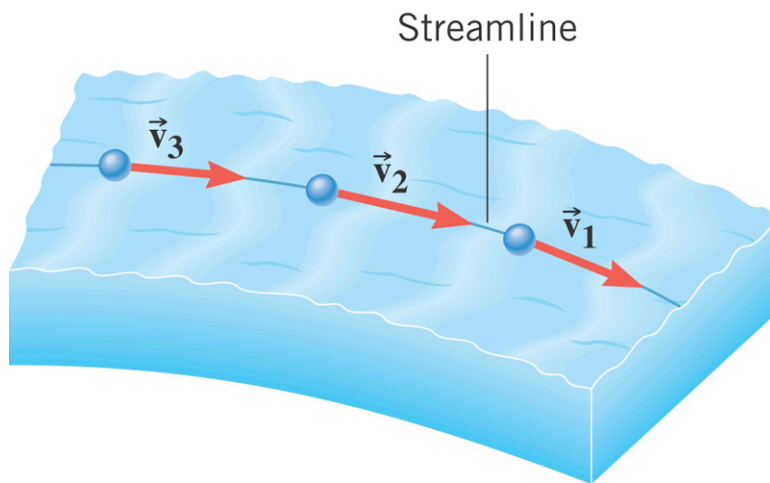
Fluid flow can be ***compressible*** or ***incompressible***. Most liquids are nearly incompressible.

Fluid flow can be ***viscous*** or ***nonviscous***.

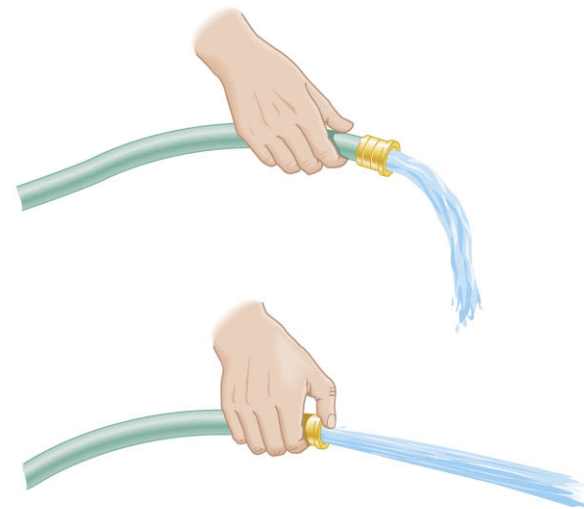
An incompressible, nonviscous fluid is called an ***ideal fluid***.

10.5 Fluids in Motion

When the flow is steady, **streamlines** are often used to represent the trajectories of the fluid particles.



The mass of fluid per second that flows through a tube is called the **mass flow rate**.



10.5 The Equation of Continuity

EQUATION OF CONTINUITY

The mass flow rate has the same value at every position along a tube that has a single entry and a single exit for fluid flow.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

SI Unit of Mass Flow Rate: kg/s



Incompressible fluid:

$$\rho_1 = \rho_2$$

$$A_1 v_1 = A_2 v_2$$

Volume flow rate Q :

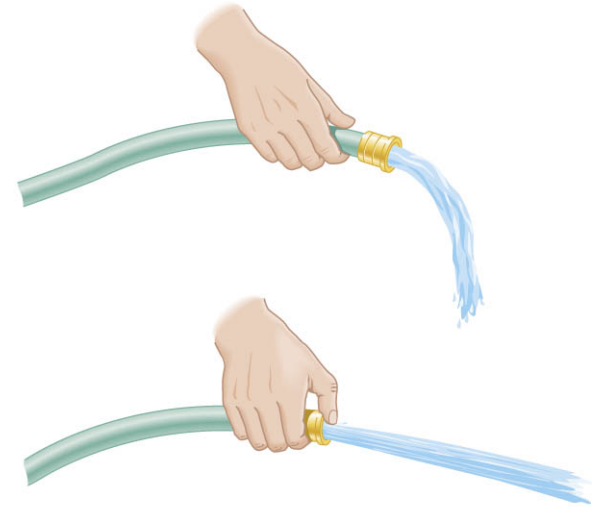
$$Q = Av$$

10.5 The Equation of Continuity

Example: A Garden Hose

A garden hose has an unobstructed opening with a cross sectional area of $2.85 \times 10^{-4} \text{m}^2$. It fills a bucket with a volume of $8.00 \times 10^{-3} \text{m}^3$ in 30 seconds.

Find the speed of the water that leaves the hose through (a) the unobstructed opening and (b) an obstructed opening with half as much area.



$$\text{a) } Q = Av$$

$$v = \frac{Q}{A} = \frac{(8.00 \times 10^{-3} \text{m}^3) / (30.0 \text{ s})}{2.85 \times 10^{-4} \text{m}^2} = 0.936 \text{m/s}$$

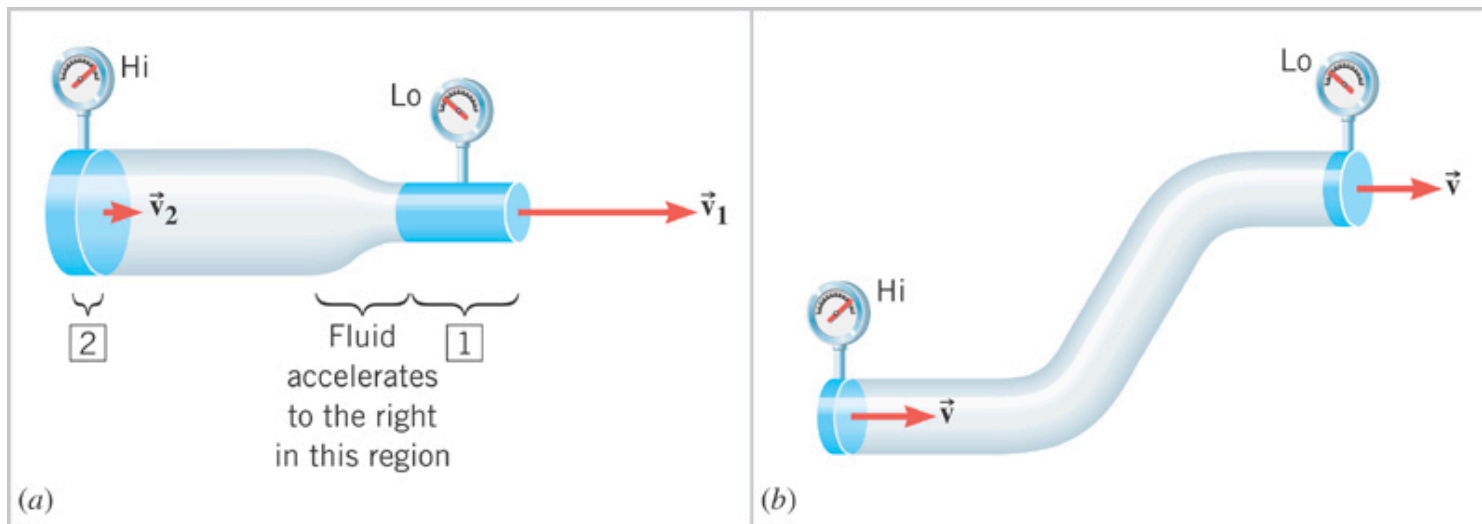
$$\text{b) } A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 = (2)(0.936 \text{m/s}) = 1.87 \text{m/s}$$

10.5 Bernoulli's Equation

The fluid accelerates toward the lower pressure regions.

According to the pressure-depth relationship, the pressure is lower at higher levels, provided the area of the pipe does not change.



Apply Work-Energy theorem
to determine relationship between
pressure, height, velocity, of the fluid.

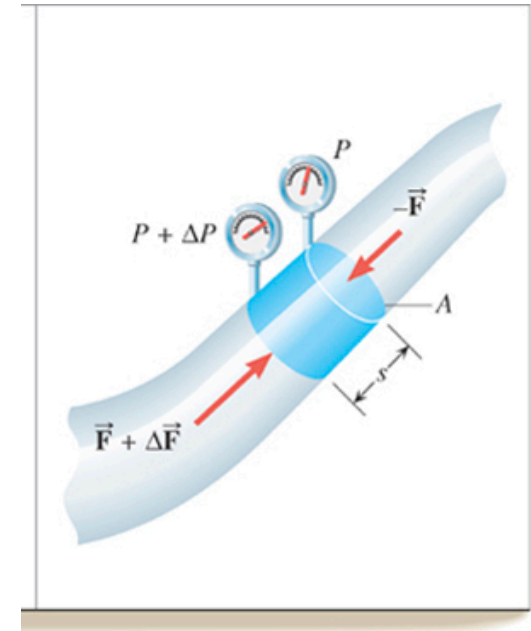
10.5 Bernoulli's Equation

Work done by tiny pressure “piston”

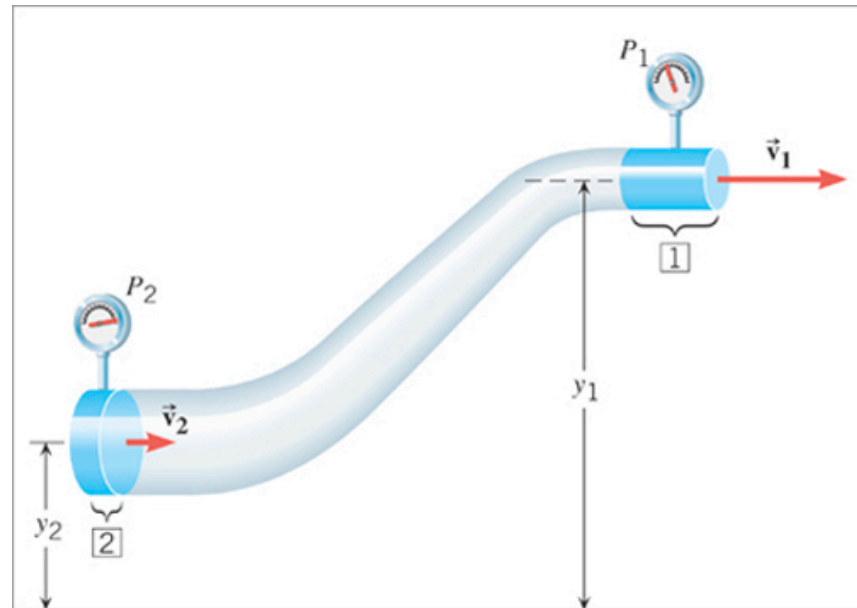
$$W_{\Delta P} = \left(\sum F \right) s = (\Delta F) s = (\Delta P A) s; \quad V = A s$$

Work (NC) done by pressure difference from 2 to 1

$$W_{\text{NC}} = (P_2 - P_1) V$$



$$E_2 = \frac{1}{2} m v_2^2 + m g y_2$$



$$E_1 = \frac{1}{2} m v_1^2 + m g y_1$$

$$W_{\text{NC}} = E_1 - E_2 = \left(\frac{1}{2} m v_1^2 + m g y_1 \right) - \left(\frac{1}{2} m v_2^2 + m g y_2 \right)$$

10.5 Bernoulli's Equation

$$W_{\text{NC}} = (P_2 - P_1)V$$

$$W_{\text{NC}} = E_1 - E_2 = \left(\frac{1}{2}mv_1^2 + mgy_1\right) - \left(\frac{1}{2}mv_2^2 + mgy_2\right)$$

NC Work yields a total Energy change.

Equating the two expressions for the work done,

$$(P_2 - P_1)V = \left(\frac{1}{2}mv_1^2 + mgy_1\right) - \left(\frac{1}{2}mv_2^2 + mgy_2\right) \quad m = \rho V$$

$$(P_2 - P_1) = \left(\frac{1}{2}\rho v_1^2 + \rho gy_1\right) - \left(\frac{1}{2}\rho v_2^2 + \rho gy_2\right)$$

Rearrange to obtain Bernoulli's Equation

BERNOULLI'S EQUATION

In steady flow of a nonviscous, incompressible fluid, the pressure, the fluid speed, and the elevation at two points are related by:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gy_2$$

10.5 Applications of Bernoulli's Equation

Conceptual Example: Tarpaulins and Bernoulli's Equation

When the truck is stationary, the tarpaulin lies flat, but it bulges outward when the truck is speeding down the highway.

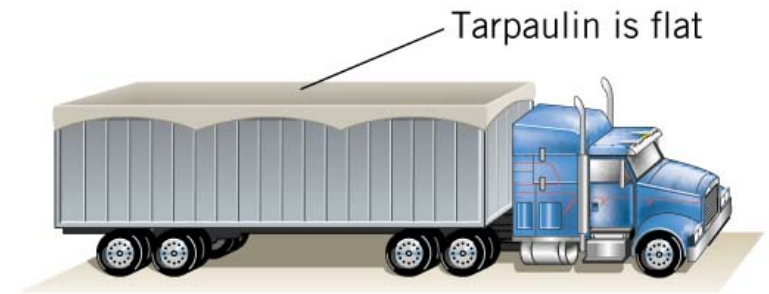
Account for this behavior.

Bernoulli's Equation

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

$$P_1 = P_2 + \frac{1}{2} \rho v_2^2$$

$$P_1 > P_2$$

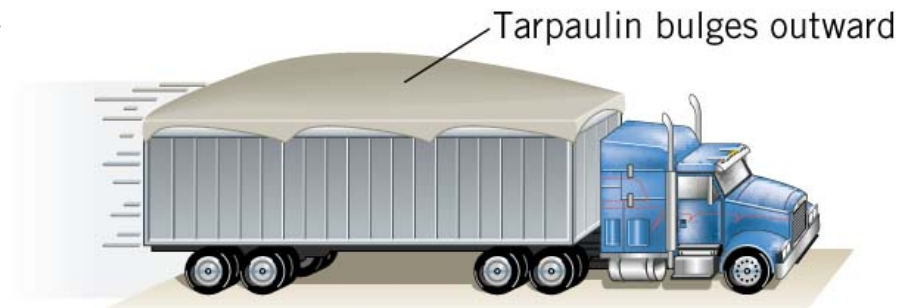


Stationary

Relative to moving truck

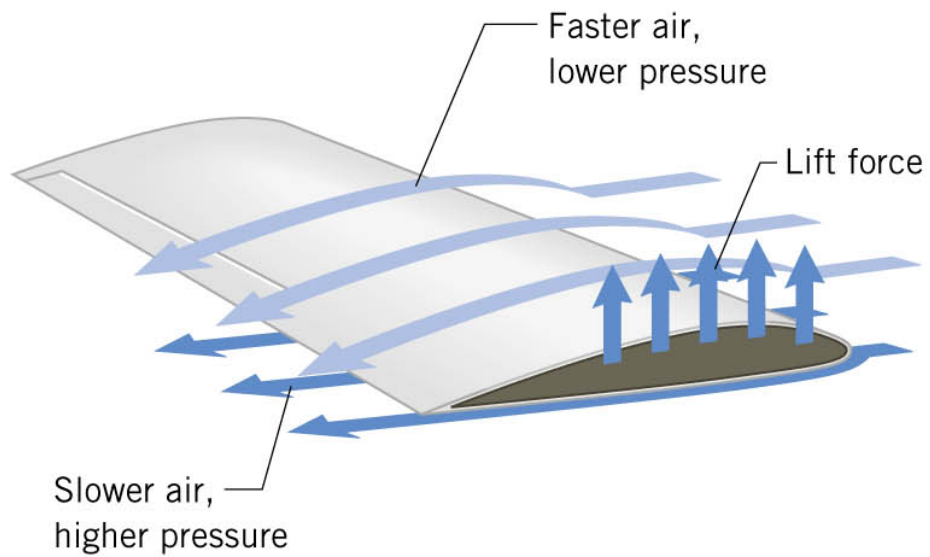
$v_1 = 0$ under the tarp

v_2 air flow over top

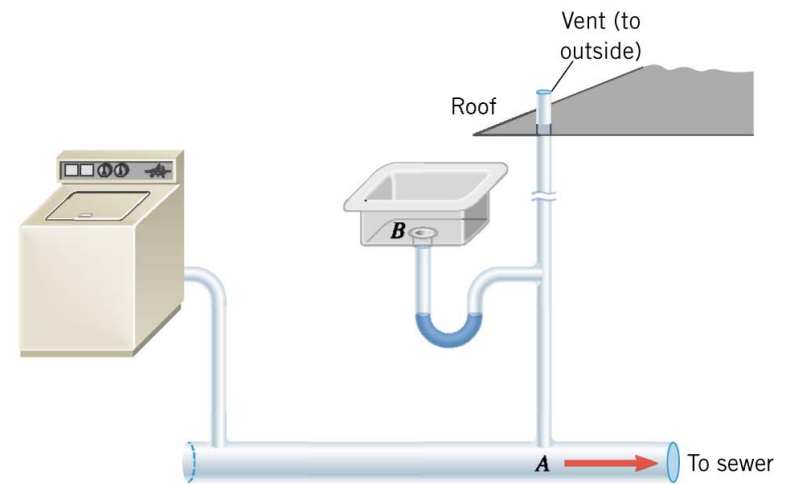


Moving

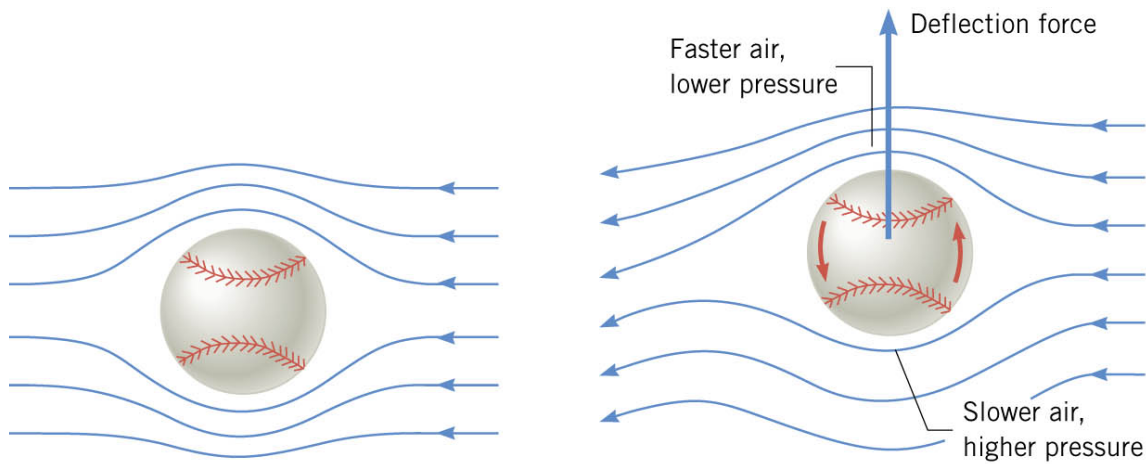
10.5 Applications of Bernoulli's Equation



(a)

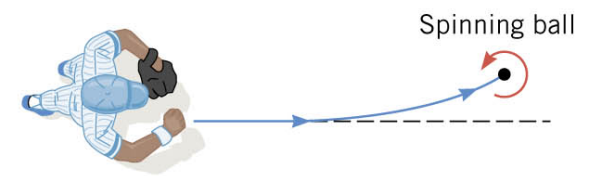


(b) With vent



(a) Without spin

(b) With spin



(c)

10.5 Applications of Bernoulli's Equation

Example: Efflux Speed

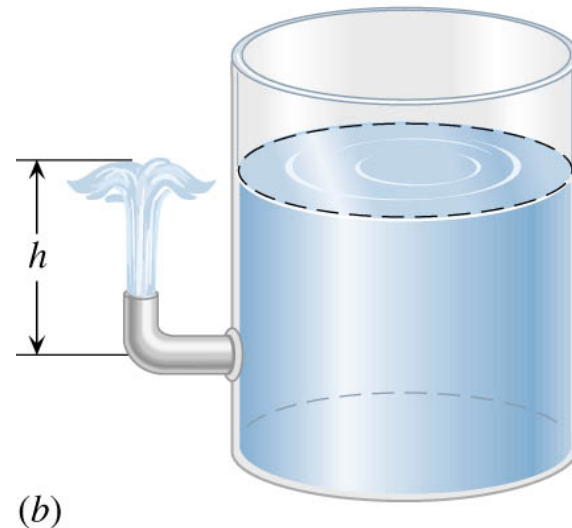
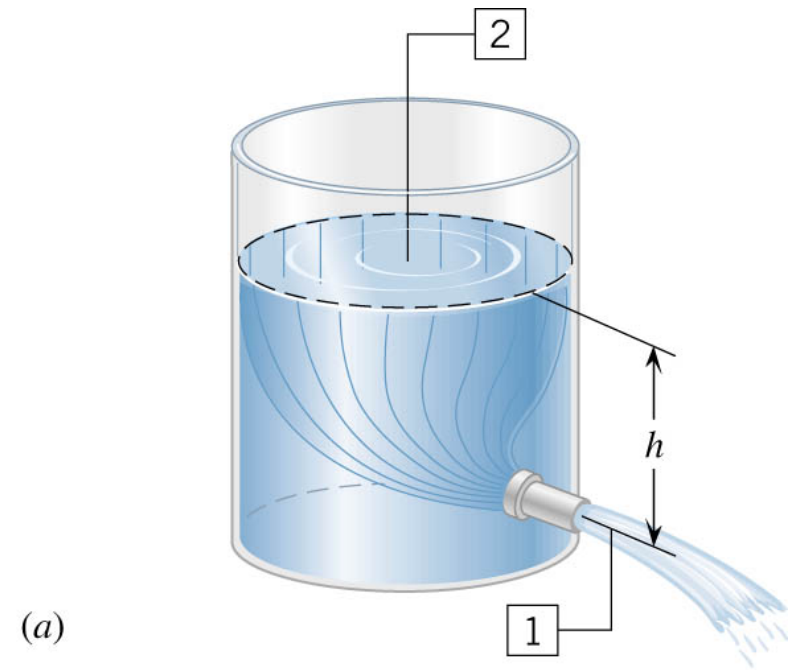
The tank is open to the atmosphere at the top. Find an expression for the speed of the liquid leaving the pipe at the bottom.

$$P_1 = P_2 = P_{\text{atmosphere}} \quad (1 \times 10^5 \text{ N/m}^2)$$
$$v_2 = 0, \quad y_2 = h, \quad y_1 = 0$$

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

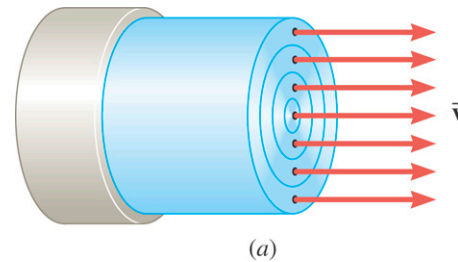
$$\frac{1}{2} \rho v_1^2 = \rho g h$$

$$v_1 = \sqrt{2gh}$$

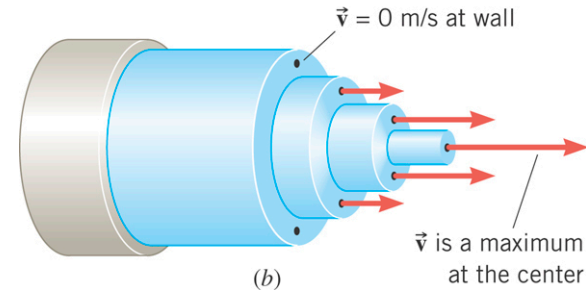


10.6 Viscous Flow

Flow of an ideal fluid.



Flow of a viscous fluid.



FORCE NEEDED TO MOVE A LAYER OF VISCOUS FLUID WITH CONSTANT VELOCITY

The magnitude of the tangential force required to move a fluid layer at a constant speed is given by:

$$F = \frac{\eta A v}{y}$$

η , is the coefficient of viscosity

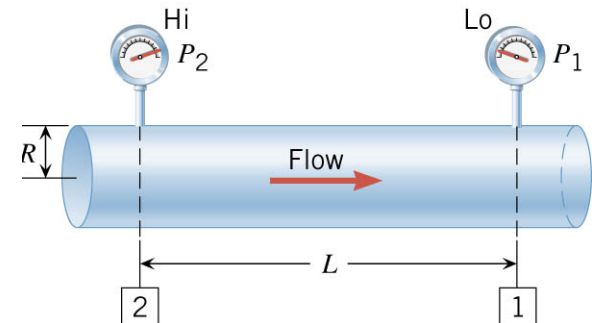
SI Unit: $\text{Pa} \cdot \text{s}$; 1 poise (P) = $0.1 \text{ Pa} \cdot \text{s}$

POISEUILLE'S LAW (flow of viscous fluid)

The volume flow rate is given by:

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta L}$$

Pressure drop in a straight uniform diameter pipe.

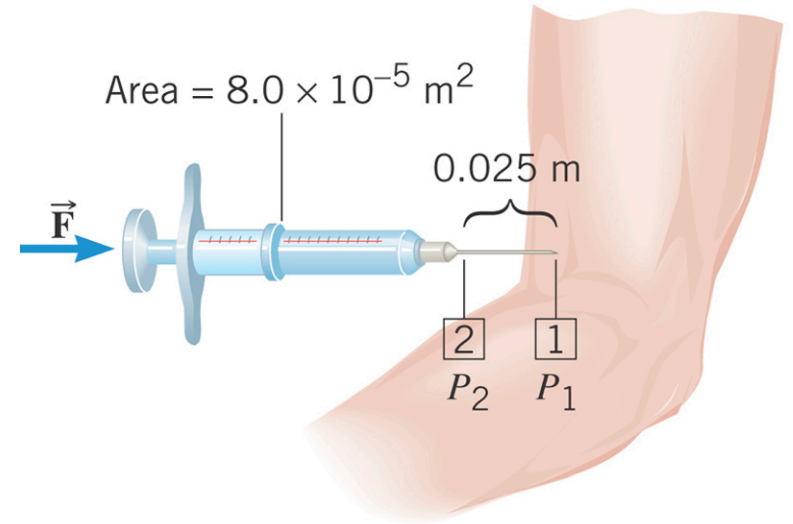


11.11 Viscous Flow

Example: Giving and Injection

A syringe is filled with a solution whose viscosity is $1.5 \times 10^{-3} \text{ Pa} \cdot \text{s}$. The internal radius of the needle is $4.0 \times 10^{-4} \text{ m}$.

The gauge pressure in the vein is 1900 Pa. What force must be applied to the plunger, so that $1.0 \times 10^{-6} \text{ m}^3$ of fluid can be injected in 3.0 s?



$$P_2 - P_1 = \frac{8\eta LQ}{\pi R^4}$$
$$= \frac{8(1.5 \times 10^{-3} \text{ Pa} \cdot \text{s})(0.025 \text{ m})(1.0 \times 10^{-6} \text{ m}^3 / 3.0 \text{ s})}{\pi(4.0 \times 10^{-4} \text{ m})^4} = 1200 \text{ Pa}$$

$$P_2 = (1200 + P_1) \text{ Pa} = (1200 + 1900) \text{ Pa} = 3100 \text{ Pa}$$

$$F = P_2 A = (3100 \text{ Pa})(8.0 \times 10^{-5} \text{ m}^2) = 0.25 \text{ N}$$