

Experiment 7

Rotational Motion: Moment of Inertia

7.1 Objectives

- Familiarize yourself with the concept of moment of inertia, I , which plays the same role in the description of the rotation of a rigid body as mass plays in the description of linear motion.
- Investigate how changing the moment of inertia of a body affects its rotational motion.

7.2 Introduction

In physics, we encounter various types of motion, primarily linear or rotational. We have already learned how linear motion works and the relevant quantities we need to look at in order to understand it. Today we will investigate rotational motion and measure one of the most important quantities pertaining to that: the moment of inertia. The way mass is distributed greatly affects how easily an object can rotate. For example, if you are sitting in an office chair and start spinning around, you can notice that if you extend your arms away from your body, you will begin to rotate slower than when you started. If you then pull your arms back in as close as possible, you will start to rotate much faster than you just were with your

arms extended. This gives us evidence of the reliance that the moment of inertia has on mass and how it is distributed.

7.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics.¹ Look for keywords: moment of inertia, torque, angular acceleration

7.4 Theory

If we apply a single unbalanced force, F , to an object, the object will undergo a linear acceleration, a , which is determined by the unbalanced force acting on the object and the mass of the object. The mass is a measure of an object's inertia, or its resistance to being accelerated. Newton's Second Law expresses this relationship:

$$F = ma$$

If we consider rotational motion, we find that a single unbalanced torque

$$\tau = (\text{Force})(\text{lever arm})$$

produces an *angular* acceleration, α , which depends not only on the mass of the object but on how that mass is *distributed*². The equation which is analogous to $F = ma$ for an object that is rotationally accelerating is

$$\tau = I\alpha \tag{7.1}$$

where the Greek letter tau (τ) represents the **torque** in Newton-meters, α is the **angular acceleration** in radians/sec², and I is the **moment of inertia** in kg-m². The moment of inertia is a measure of the way the mass is distributed on the object and determines its resistance to angular acceleration.

Every rigid object has a definite moment of inertia about any particular axis of rotation. Here are a couple of examples of the expression for I for two special objects:

¹<http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html>

²In this lab the lever arm will be the radius at which the force is applied (the radius of the axle). This is due to the fact that the forces will be applied tangentially, i.e., perpendicular to the radius. The general form of this relationship is $\tau = (\text{force})(\text{lever arm})(\sin(\theta))$ where θ is the angle between the force and the lever arm. However, in this experiment θ is 90° and $\sin(90^\circ) = 1$.)

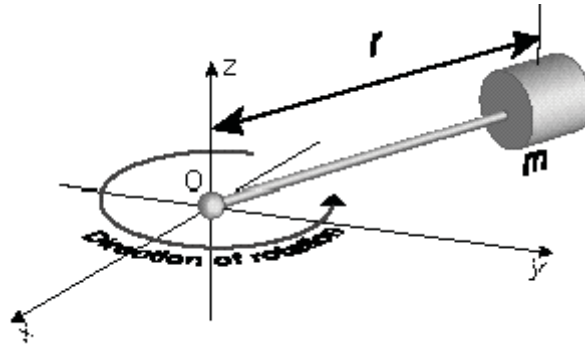


Figure 7.1: One point mass m on a weightless rod of radius r ($I = mr^2$).

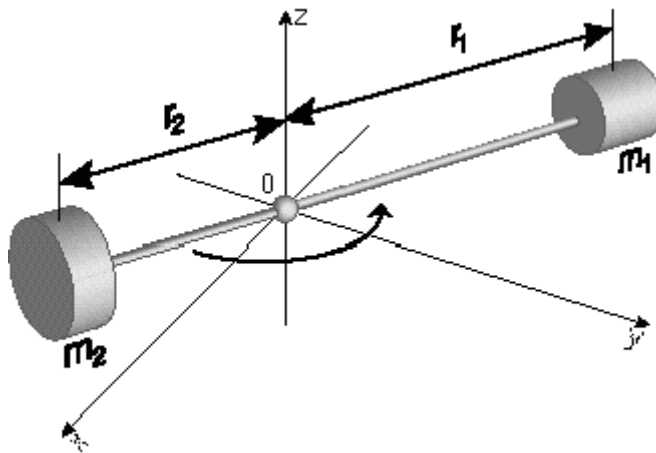


Figure 7.2: Two point masses on a weightless rod ($I = m_1r_1^2 + m_2r_2^2$).

To illustrate, we will calculate the moment of inertia for a mass of 2 kg at the end of a massless rod that is 2 m in length (Fig. 7.1 above):

$$I = mr^2 = (2 \text{ kg})(2 \text{ m})^2 = 8 \text{ kg m}^2$$

If a force of 5 N were applied to the mass *perpendicular* to the rod (to make the lever arm equal to r) the torque is given by:

$$\tau = Fr = (5 \text{ N})(2 \text{ m}) = 10 \text{ N m}$$

7. ROTATIONAL MOTION: MOMENT OF INERTIA

By equation 7.1 we can now calculate the angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{10 \text{ N m}}{8 \text{ kg m}^2} = 1.25 \frac{\text{rad}}{\text{sec}^2}$$

Note: The moment of inertia of a complicated object is found by adding up the moments of each individual piece (Figure 7.2 above is the sum of two Figure 7.1 components).

7.5 In today's lab

Today we will measure the moment of inertia for multiple mass distributions. We will plot our data and determine the relationship of the moment of inertia and the radii that our masses were placed at.

7.6 Equipment

- 2 Cylindrical Masses
- Hanger
- Small Masses
- Main Axle
- String

In our case, the rigid body consists of two cylinders, which are placed on a metallic rod at varying radii from the axis of rotation. The cylinders and rod are supported by a rotating platform attached to a central pulley and nearly frictionless air bearings. A side view of the apparatus is shown in Figure 7.3 and a top view of the central pulley is shown in Figure 7.4.

In this experiment, we will change the moment of inertia of the rotating body by changing how the mass is distributed on the rotating body. We will place the two cylindrical masses at four different radii such that $r = r_1 = r_2$ in each of the four cases. We will then use our measurements to calculate the moment of inertia (I) for each of the four radial positions of the cylindrical masses (r). The sum of the two cylindrical masses ($m_1 + m_2$) can then be found from a graph of I versus r^2 .

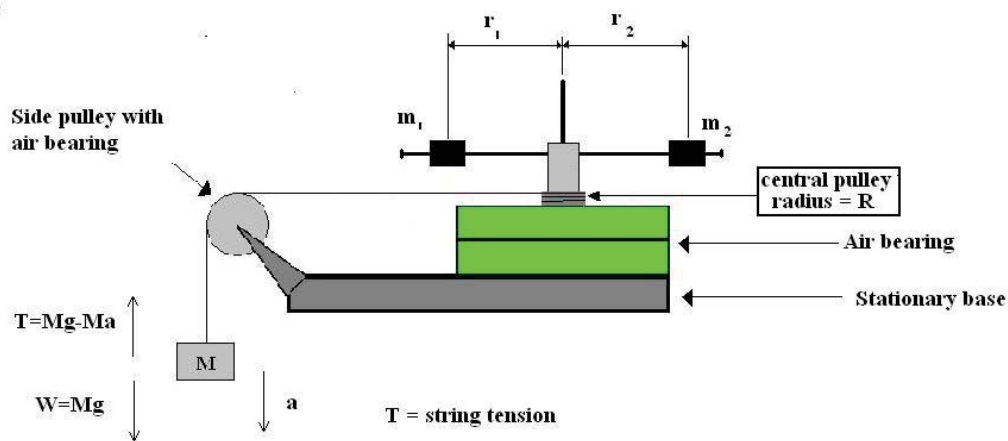


Figure 7.3: Moment of Inertia Apparatus

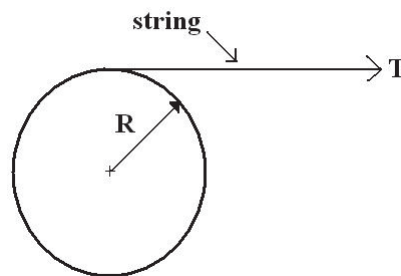


Figure 7.4: Central Pulley (axle)

To set up your rigid body, wrap the string around the central pulley (axle) and run it over the side pulley to a known weight as shown in Figure 7.3.

Consider the following steps:

If we release the weight from rest, the tension in the string will exert a torque on the rigid body causing it to rotate with a constant angular acceleration α . The angular acceleration of the rigid body is related to the linear acceleration of the falling mass by:

$$\alpha = \frac{\text{Linear acceleration}}{\text{Radius of axle}} = \frac{a}{R}$$

7. ROTATIONAL MOTION: MOMENT OF INERTIA

or

$$a = R\alpha \quad (7.2)$$

From Figure 7.3 and Newton's Second Law, the tension in the string is:

$$T = Mg - Ma \quad (7.3)$$

The tension in the string causes a net torque on the rigid body. Since Torque = (Lever arm) (Force), the net torque on the rigid body is given by:

$$\tau = R \times T \quad (7.4)$$

The moment of inertia of the rigid body is then found from equation 7.1 ($\tau = I\alpha$).

7.7 Procedure

1. Measure and record the masses of the hanging mass (M) and the two cylinders (m_1 and m_2).
2. Place the cylinders on the horizontal rod such that the axes of the cylinders are along the horizontal rod (as shown in Figure 7.5). Make sure the thumbscrew on each cylinder is tightened. The center of mass of each cylinder must be the same distance (r) from the axis of rotation (i.e. $r_1 = r_2$ in Figure 7.3). Estimate the uncertainty in r (called δr). This should include both the uncertainty in reading your ruler and the uncertainty in locating the cylinder's center of mass.
3. With the air supply on, attach the hanging mass (M) to one end of a string and wind the other end around the central pulley. The string should also pass over the side pulley such that the hanging mass is just below the side pulley. Hold the hanging mass stationary and measure its elevation (y) using the floor as your reference level. Record this elevation in your spreadsheet and assign an appropriate uncertainty to this measurement. Then release the hanging mass and simultaneously start the desktop timer. When the mass hits the floor, stop the timer. For the uncertainty in this time (δt), use the standard deviation of a measurement (denoted by s) from the Reaction Time experiment.

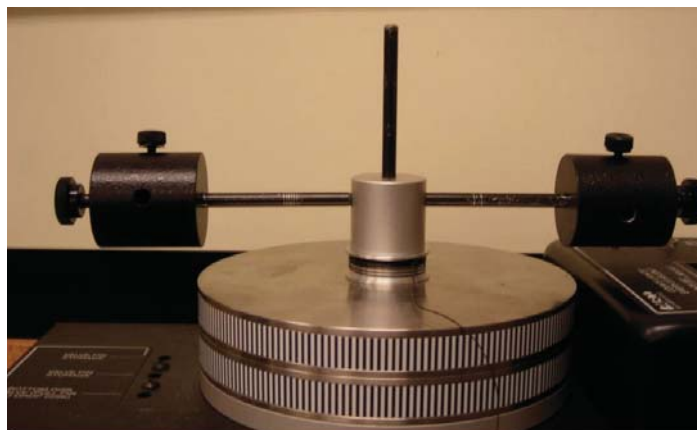


Figure 7.5: View of main axle with 2 masses at same radius r

4. The position, y , of an object released from rest a distance h above the floor is found using: $y = h - \frac{at^2}{2}$. The final position of the mass is $y = 0$, so the acceleration is found using: $a = \frac{2h}{t^2}$.

Calculate the linear acceleration of the falling mass (M) and use $\delta a = a \left(2\frac{\delta t}{t} + \frac{\delta h}{h} \right)$ to calculate its uncertainty.

5. Use equation 7.3 to calculate the tension in the string (T) and use $\delta T = T \left(\frac{\delta M}{M} + \frac{\delta a}{g-a} \right)$ to calculate its uncertainty.
6. Use $R = 1.27 \pm 0.01$ cm for the radius of the central pulley and equation 7.2 to calculate the angular acceleration of the rotating apparatus. In addition, use $\delta \alpha = \alpha \left(\frac{\delta a}{a} + \frac{\delta R}{R} \right)$ to calculate its uncertainty.
7. Use equation 7.4 to calculate the unbalanced torque on the rotating apparatus and use $\delta \tau = \tau \left(\frac{\delta T}{T} + \frac{\delta R}{R} \right)$ to calculate the uncertainty in this torque. (**Note:** in this equation the Greek letter τ (tau) is the torque and T is the tension in the string.)
8. Use equation 7.1 to calculate the moment of inertia of the rotating apparatus; the uncertainty in moment of inertia is given by: $\delta I = I \left(\frac{\delta \tau}{\tau} + \frac{\delta \alpha}{\alpha} \right)$. Calculate r^2 and its uncertainty, $\delta r^2 = 2r\delta r$.

7. ROTATIONAL MOTION: MOMENT OF INERTIA

9. Repeat steps 2–8 for two additional (non-zero) values of r . Make sure that these values differ by at least 2 cm.
10. We would like to place the two cylinders at $r = 0$. To do this, we will use the vertical bar on the support (see Figure 7.6). When you place the cylinders on the vertical bar, make sure they are oriented the same way as in your previous trials, i.e. with the axes of the two cylinders perpendicular to the vertical bar. As before, make sure to tighten the thumbscrews on the cylinders. Follow the procedure in steps 3–8 to calculate the moment of inertia of the body with the two cylinders at $r = 0$. Include this data in your data table.

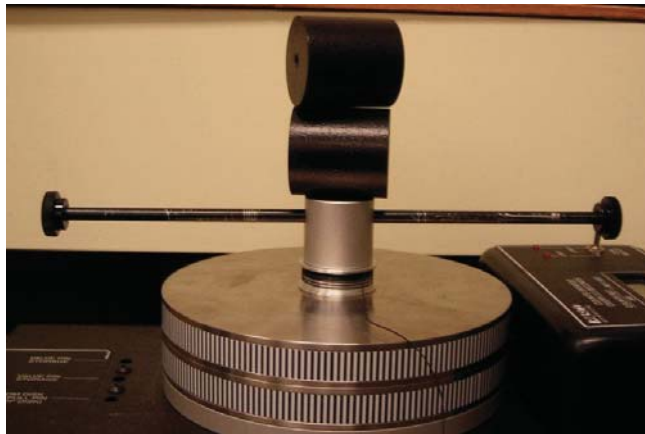


Figure 7.6: View of main axle with 2 masses at radius $r = 0$

11. Transfer your data into KaleidaGraph and make a plot of I vs. r^2 . Your data points should have both horizontal and vertical error bars. Also, fit your data with a best fit line, display its equation with the uncertainties in the slope and intercept. When the two cylinders are placed *on the axis of rotation*, the measured moment of inertia I_0 is the moment of inertia of the rotating apparatus alone plus the moment of inertia of each of the two cylinders about an axis through their own centers of mass.

$$I = I_0 \tag{7.5}$$

If the two masses are now each placed a distance r from the axis of rotation then equation 7.5 becomes:

$$I = (m_1 + m_2)r^2 + I_0 \quad (7.6)$$

If you compare equation 7.6 to the form of an equation for a straight line:

$$y = mx + b$$

You can see that a plot of I vs. r^2 should be a straight line. The slope of this line is the sum of the masses $(m_1 + m_2)$ and the intercept is I_0 .

7.8 Checklist

1. Excel Sheets
2. Plot of I vs. r^2 with proper error bars and fit line.
3. Questions

7.9 Questions

1. The moment of inertia of a body depends not only on its mass, but also on how the mass is distributed. Does your data support this? Why or why not?
2. Discuss the consistency of the slope of the plot of I vs. r^2 with the value you measured for $(m_1 + m_2)$. If they are not consistent, suggest possible sources of error.

7. ROTATIONAL MOTION: MOMENT OF INERTIA

3. In your plot of I vs. r^2 , why did you use r^2 and not r in the plot? What are the units of the slope of I vs. r^2 ?
4. In step 6 of the procedure, you were given that $R = 1.27 \pm 0.01$ cm. Using only the experimental apparatus and a meter stick, how would you verify this radius with an uncertainty of less than or equal to 0.01 cm? (**Hint:** You cannot get this uncertainty by holding the meter stick next to the axle and measuring the diameter. Also note that the string is a part of the apparatus.)