Experiment 9

The Spring: Hooke's Law and Oscillations

9.1 Objectives

- Investigate how a spring behaves when it is stretched under the influence of an external force. To verify that this behavior is accurately described by Hooke's Law.
- Measure the spring constant (k) in two independent ways.

9.2 Introduction

Springs appear to be very simple tools we use everyday for multiple purposes. We have springs in our cars to make the ride less bumpy. We have springs in our pens to help keep our pockets/backpacks ink free. It turns out that there is a lot of physics involved in this simple tool. Springs can be used as harmonic oscillators and also as tools for applying a force to something. Today we will learn about the physics involved in a spring, and why the spring is such an interesting creation.

9.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics.[1] Look for keywords: Hooke's Law, oscillation

9.4 Theory

Hooke's Law

An ideal spring is remarkable in the sense that it is a system where the generated force is **linearly dependent** on how far it is stretched. Hooke's law describes this behavior, and we would like to verify this in lab today. In order to extend a spring by an amount Δx from its previous position, one needs a force F which is determined by $F = k\Delta x$. Hooke's Law states that:

$$F_S = -k\Delta x \tag{9.1}$$

Here k is the **spring constant**, which is a quality particular to each spring, and Δx is the distance the spring is stretched or compressed. The force F_S is a restorative force and its direction is opposite to the direction of the spring's displacement Δx.

To verify Hooke's Law, we must show that the spring force F_S and the distance the spring is stretched Δx are proportional to each other (that just means linearly dependant on each other), and that the constant of proportionality is $-k$.

In our case the external force is provided by attaching a mass (m) to the end of the spring. The mass will of course be acted upon by gravity, so the force exerted downward on the spring will be $F_g = mg$ (see Figure 9.1). Consider the forces exerted on the attached mass. The force of gravity (mg) is pointing downward. The force exerted by the spring ($-k\Delta x$) is pulling upwards. When the mass is attached to the spring, the spring will stretch until it reaches the point *where the two forces are equal but pointing in opposite directions*:

$$F_S - F_g = 0 \text{ or } mg = -k\Delta x \tag{9.2}$$

This point where the forces balance each other out is known as the **equilibrium point**. The spring + mass system can stay at the equilibrium

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

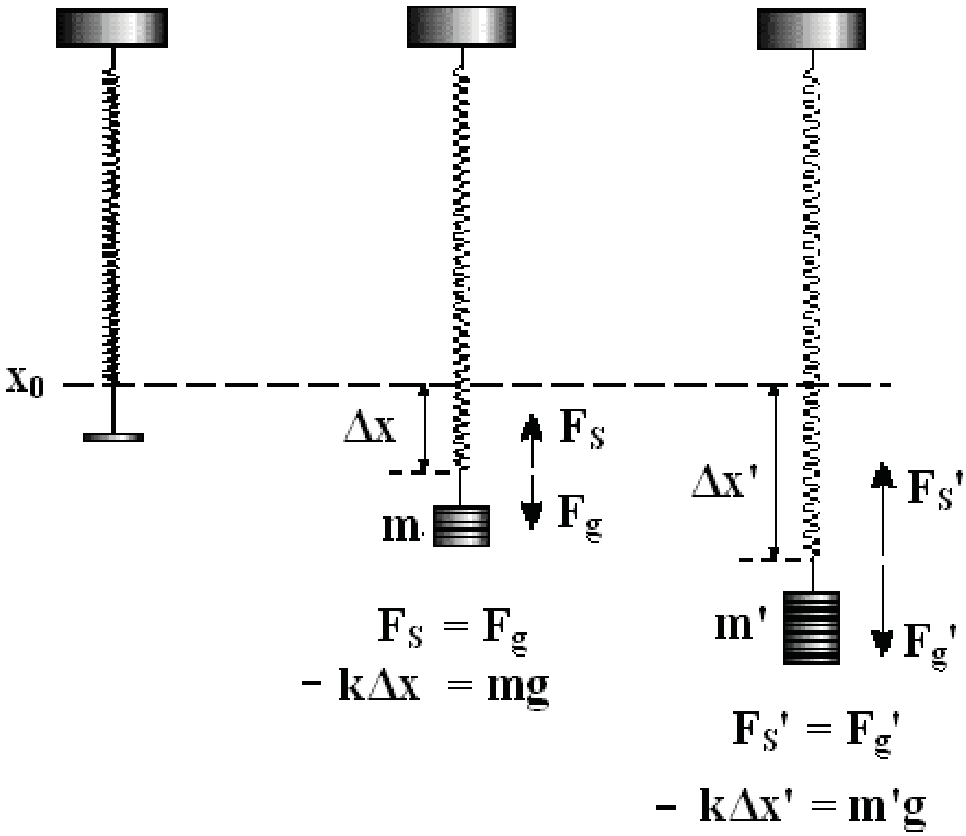


Figure 9.1: Force diagram of a spring in gravity.

point indefinitely as long as no additional external forces come to be exerted on it. The relationship in Eq. 9.2 allows us to determine the spring constant k when m, g, and Δx are known or can be measured. This is one way in which we will determine k today.

Oscillation

The position where the mass is at rest is called the equilibrium position ($x = x_0$). As we now know, the downward force due to gravity $F_g = mg$ and the force due to the spring pulling upward $F_S = -k\Delta x$ cancel each other. This is shown in the first part of Figure 9.2. However, if the string is stretched *beyond its equilibrium point* by pulling it down and then releasing it, the mass will accelerate upward ($a > 0$), because the force due to the spring is *larger* than gravity pulling down. The mass will then pass through the equilibrium point and continue to move upward. Once above the equilibrium position, the motion will slow because the net force acting on the mass is now downward (i.e. the downward force due to gravity is constant while the upwardly directed spring force is getting smaller). The mass and spring will stop and then its downward acceleration will cause it to move back down again. The result of this is that the mass will oscillate around the equilibrium position. These steps and the forces (F), accelerations (a), and velocities (v) are illustrated in Figure 9.2 for the first complete cycle of an oscillation. The oscillation will proceed with a characteristic period, τ, which is determined by the spring constant and the total attached mass. This period is the time it takes for the spring to complete one oscillation, or the time necessary to return to the point where the cycle starts repeating (the points where x, v, and a are the same). One complete cycle is shown in Figure 9.2 and the time of each position is indicated in terms of the period τ, where

$$\tau = 2\pi\sqrt{\frac{m}{k}} \tag{9.3}$$

By measuring the period for given masses the spring constant can be determined. This is the second way we will determine k today. You will use this value of k to verify that the proportionality constant you determined for Hooke's Law in the first part is indeed the correct k for the spring.

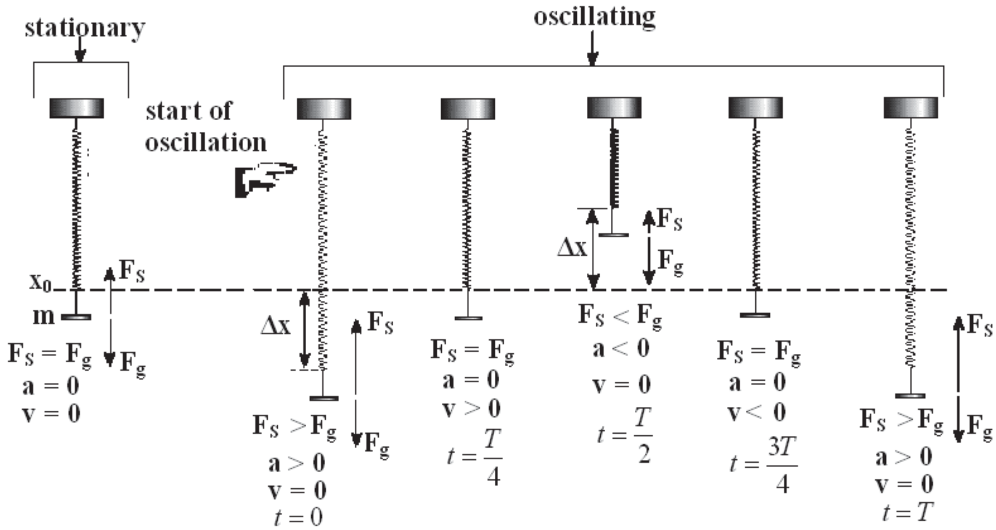


Figure 9.2: Oscillation of a spring.

9.5 In today's lab

Today we will measure the spring constant of a given spring in two ways. First we will add mass gradually to the spring and measure the displacement, then plot the results to find the spring constant. Then, we will find the period of oscillation for the spring after attaching varying mass to the bottom. Once again, we will plot these results to find the spring constant a different way.

9.6 Equipment

- Spring
- Photogate
- Masses
- Hanger

9.7 Procedure

Part I: Hooke's Law

1. Record the mass of the mass hanger, $m_H = 50.0$ g, in your data sheet.
2. Measure the rest length (nothing on the end) of the spring and record it in your data sheet.
3. Attach the **empty** mass hanger to the spring and measure the position X_0 of the end of the spring with the zero end of the meter stick **on the table**. Be sure to include a reasonable uncertainty.
4. Increase the **total mass** on the end of the spring to 120 g (this includes the mass of the hanger). Measure the height of the spring now and record it in your data sheet.
5. Increase the mass by 10 g increments, making sure to measure and record the height at each step, until you reach 220 g.
6. Calculate $\Delta m = m - m_H$, $\Delta X = X - X_0$, $\delta(X - X_0) = 2\delta X_0$, and $F_S = -k\Delta X = \Delta mg$ (we are measuring the distances when the spring is in equilibrium) for each trial on your data sheet.
7. Graph F_S vs. ΔX in KaleidaGraph. Include horizontal error bars and a best fit line. If you have a straight line, this verifies Hooke's Law already. Here, the slope will tell you the spring constant and it's uncertainty.

Part II: Period of Oscillation

1. Set the photogate to the `PEND` setting.
2. Starting at a mass of 120 g on the end of the spring, measure the period of oscillation by causing the masses to oscillate through the photogate. You can adjust the height of the photogate and the height of the spring to allign the equilibrium position with the photogate. When displacing the mass for oscillation, this should be a small displacement; **do not** stretch the spring more than 5 cm. Do this in 20 g intervals up to 220 g.

3. Calculate the mass of the spring using the given spring density and the rest length of the spring. Record this value in your data sheet.

4. Calculate τ^2 in Excel for each trial.

   This gives us an equation in the same form as a straight line $y = mx+b$ with intercept $b = 0$. The value m in Equation 9.4 is the total mass felt by the spring.

5. Calculate the total mass using the formula $m = m_H + m + \frac{\text{spring mass}}{3}$ in Excel. Note that this is a different m than you used in Part 1. Here the total mass experienced by the spring is the mass of the hanger, the masses added to the hanger, and 1/3 of the mass of the spring.

6. Make a plot of τ^2 vs. m in KaleidaGraph. Be sure to include a best fit line on this plot. In the question you will use the slope of your graph to find the spring constant. Note that squaring both sides of Equation 9.3 we get

$$\tau^2 = \frac{4\pi^2}{k}m \tag{9.4}$$

9.8 Checklist

1. Excel Sheets
2. Plot of F_S vs. ΔX
3. Plot of τ^2 vs. m
4. Questions

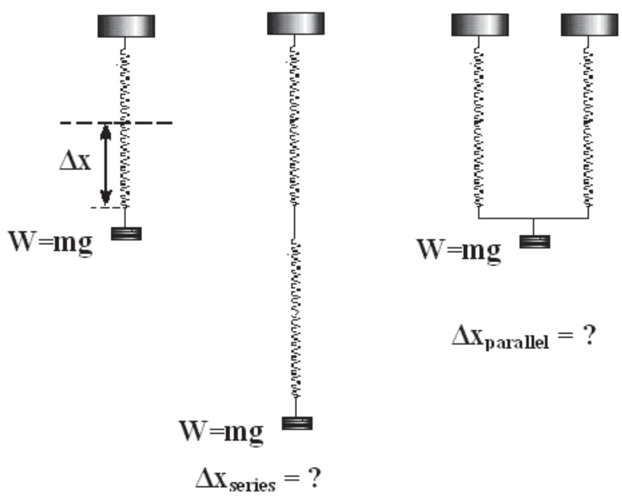
9.9 Questions

1. Is your data consistent with Hooke's Law? Specifically, is the spring force linearly dependent on how much the spring is stretched? Is it a restorative force? Why or why not? From the slope of your graph, what is the spring constant and it's uncertainty?

2. Calculate the spring constant and its uncertainty using the information obtained from your graph of τ^2 vs. m. **Hint:** The fractional uncertainty in the spring constant is equal to the fractional uncertainty in the slope $(\frac{\delta k}{k} = \frac{\delta \text{slope}}{\text{slope}})$.

3. You obtained the spring constant in two independent ways. Discuss the consistency of your two measurements of the spring constant. If they are not consistent, suggest a possible source of error.

4. When a mass m is attached to a spring it exerts a force $W = mg$ on the spring and the length of the spring is changed by Δx. If the single spring is replaced with a) two identical springs in series, what happens to Δx_{series} compared to the case of a single spring? b) If the single spring is replaced by two identical springs in parallel, what happens to $\Delta x_{\text{parallel}}$ compared to the case of a single spring? Assume all springs have the same spring constant and always compare to the single spring case. Answer each question by stating if Δx increases, decreases or remains unchanged. Also, what are Δx_{series} and $\Delta x_{\text{parallel}}$ in terms of Δx for the single spring case? **Hint:** draw a force diagram of the system – the net force on the mass must be zero.



Δx
W=mg
W=mg
Δx_series = ?
W=mg
Δx_parallel = ?