

## *Experiment 10*

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# **Vibration Modes of a String: Standing Waves**

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### **10.1 Objectives**

- Observe resonant vibration modes on a string, i.e. conditions for the creation of standing wave patterns.
- Determine how resonant frequencies are related to the number of nodes, tension of the string, length of the string, and string density.
- Determine the velocity of transverse waves in the string.

### **10.2 Introduction**

Everything you can see is due to waves. A wave is defined as an oscillation through space. The reason we can see things is because photons oscillate through space in the form of a wave and enter our eyes. This sends a signal to our brain and thus we can see. Today we will investigate waves on a much large scale. When you apply an oscillation to the end of a tight string, it begins to form waves with given frequencies. The **resonant frequency** is the frequency at which the wave oscillates freely without constructive interference adding to its amplitude.

### 10.3 Key Concepts

As always, you can find a summary on-line at Hyperphysics.<sup>1</sup> Look for keywords: standing waves on a string, resonance, transverse waves

### 10.4 Theory

A wave in a string can be characterized by its **wavelength**,  $\lambda$ , just like a sound wave or a light wave. For a string that is fixed on both ends, a **standing wave** can develop *if an integer number of half wavelengths fit into the length,  $L$ , of the string*:

$$n \left( \frac{\lambda_n}{2} \right) = L \quad (10.1)$$

Here  $n$  refers to the number of maxima (also called antinodes) in the wave pattern as demonstrated in Figure 10.1. The **resonant frequency**,  $f_n$ , for wavelength  $\lambda_n$  with wave speed  $c$  is<sup>2</sup>

$$f_n = \frac{c}{\lambda_n} \quad (10.2)$$

Combining equations 10.1 and 10.2 we obtain:

$$f_n = \left( \frac{c}{2L} \right) n \quad (10.3)$$

If a force acts on a string with a resonant frequency, the amplitude of the vibration will grow very large. This is a common behavior in many physical systems. An example of such behavior is pushing a child on a swing. A swing oscillates with a **characteristic frequency**. If someone exerts a push on the child with that frequency, after several cycles the amplitude of the swing becomes large, even if the pushes are gentle. If pushes are given with a different frequency, some of the pushes will be out of phase; meaning that the child will be pushed against his motion and the amplitude will

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<sup>1</sup><http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html>

<sup>2</sup>The pitch of musical instruments is determined by the resonant frequency, whether it is a string instrument, a wind instrument or a percussion instrument. Since instruments are not driven at a fixed frequency, the vibrations are composed of a mixture of several harmonic frequencies.

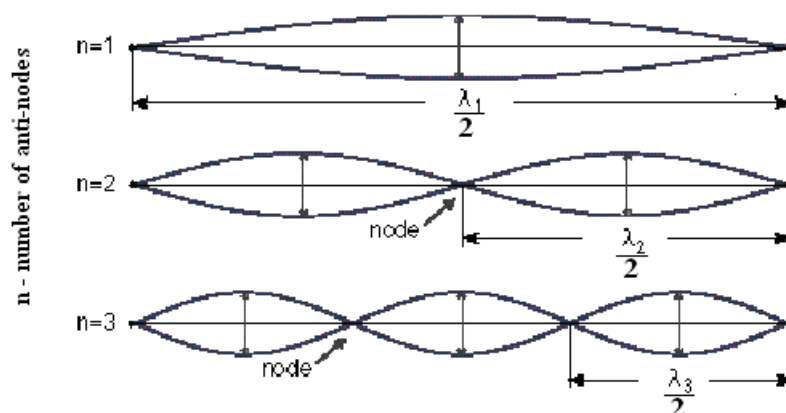


Figure 10.1: Three lowest characteristic frequencies of a string (with  $n = 1, 2$  and 3 maxima).

not have a chance to grow. A string has *many* characteristic frequencies and the string's amplitude will grow whenever the driving force has *any* of these characteristic frequencies. If a string is set to vibrate at one of these characteristic frequencies a standing wave is set up on the string. When a standing wave is present, nodes and antinodes will be visible on the string. A node is a location on the string where the string does not move. On the other hand, an antinode is a location that undergoes a vibration with very large amplitude. Figure 10.1 shows the lowest three characteristic frequencies for a given string under constant tension.

The speed,  $c$ , of a transverse wave in a string depends on the string's mass per unit length  $\rho$  and the tension  $T$  ( $\rho$  is the Greek letter rho and is frequently used to represent mass density). By setting the tension with the pulley system shown in Figure 10.2 and by measuring the mass density, one can determine the speed of the transverse wave by

$$c = \sqrt{\frac{T}{\rho_s}} \quad (10.4)$$

The fractional uncertainty in the stretched string density is

$$\frac{\delta\rho_s}{\rho_s} = \left( \frac{\delta l_s}{l_s} + \frac{\delta l_0}{l_0} \right) \quad (10.5)$$

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Here,  $l_0$  and  $l_s$  are the respective lengths of the string when it is unstretched and stretched. Both of these quantities are required to calculate  $\rho$ . The uncertainty of the unstretched string  $\delta l_0$  includes both the uncertainty in reading the meter stick and the uncertainty associated with aligning the unstretched string with the meter stick (the string needs to be in a straight line and at the same time not stretched). Similarly, the uncertainty of the length of the stretched string  $\delta l_s$  includes both the uncertainty associated with reading the meter stick and the uncertainty of aligning the meter stick with the string. In this case, the string changes directions as it passes over the pulley as shown in Figure 10.3. You should make reasonable estimates of both of these uncertainties.

The fractional uncertainty in the speed  $c$  found by equation 10.4 is given by:

$$\frac{\delta c}{c} = \frac{1}{2} \frac{\delta \rho_s}{\rho_s} \quad (10.6)$$

Because the fractional uncertainties in the string's mass and the tension in the string are both small compared to the fractional uncertainties of  $l_0$  and  $l_s$ , they are not included in the fractional uncertainty equation 10.6.

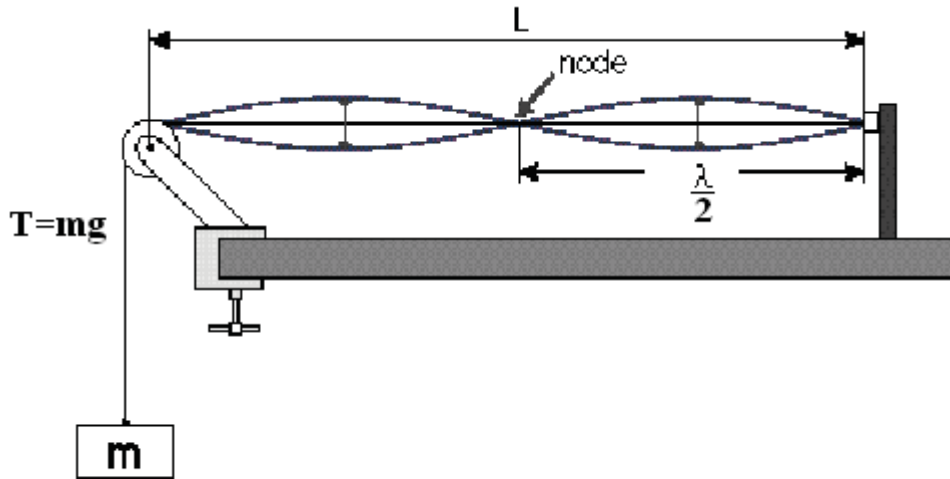


Figure 10.2: Diagram of how the string looks when driven at the second lowest resonant frequency. This configuration has two anti-nodes (points of maximum oscillation).

## 10.5 In today's lab

Today we will investigate the wave speed traveling through a string under tension in 3 ways. First we will find the resonant frequencies of the string for 1–11 antinodes. We will then calculate the wave speed using Equation 10.3 for each resonant frequency and average the values together. Next, we will calculate the wave speed using Equation 10.4. Finally, we will plot our results and use the graph to measure our wave speed. We will then see whether or not all 3 methods are consistent with one another.

## 10.6 Equipment

- Variable Frequency Oscillator
- Pulley and Weight System
- String

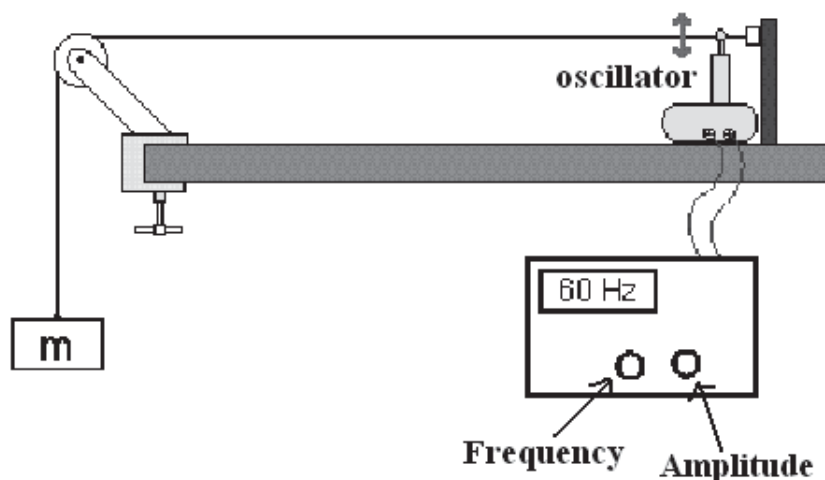


Figure 10.3: Diagram showing the experimental apparatus.

- 2 Meter Stick
- Mass Scale

## 10.7 Procedure

1. Measure the rest length  $l_0$  of the string and record it in your data sheet.
2. Measure the mass of your hanger and mass system and record it in your data sheet.
3. Attach the mass system to the end of your string and hang it over the pulley. Measure the length  $L$  (see Figure 10.2) and record it in your data sheet.  $L$  should be approximately 150 cm.
4. Measure the **total** stretched length of the string with your mass system hanging on the end and record it in your data sheet. Using this value, calculate the stretched string density using the density of the unstretched string,  $\rho_0 = (m_0/l_0) = 0.0375[\text{g/cm}]$ .

5. Using Equation 10.4,  $T = mg$ , and  $\rho_s$ , calculate the wave speed and record it in your data sheet.
6. Position the oscillator near the fixed end of the string and adjust the oscillator to “Hz 1-100”.
7. Starting around 100 Hz, find the first frequency mode where you have 11 antinodes in the string. Be sure to finely adjust the frequency until you get the largest possible peaks. Record this frequency in your data sheet.
8. Gradually decrease the frequency until you find the next lowest integer of antinodes ( $n = 10$  in this case) and record the frequency in your data sheet.
9. Repeat step 8 for  $n = 9$  to  $n = 1$  antinodes. Then using equation. 10.1 to find  $\lambda_n$ .
10. Using the formula  $c_n = \lambda_n f_n$ , calculate the wave speed for each resonant frequency in Excel and calculate the mean value of this speed. You may use the formula “=AVERAGE(E23:E33)” for this calculation.
11. In Excel, calculate the standard deviation and standard deviation of the mean for your resonant frequencies as well. For the standard deviation, you may use the formula “=STDEV(E23:E33)”. The standard deviation of the mean,  $s_m = s/\sqrt{N}$  will be your uncertainty for this wave speed calculation ( $N = \text{Number of Trials}$ ).
12. Make a plot of  $f_n$  vs.  $n$  in KaleidaGraph and include a best fit line. You do not need error bars for this plot.
13. Answer question # 2 and fill in the respective cell in Excel before you print your Excel sheets.

## 10.8 Checklist

1. Excel Sheets
2. Plot of  $f_n$  vs.  $n$
3. Questions







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3. Is the speed of the wave measured from your graph consistent with the mean value of your eleven  $c = \lambda_n f_n$  calculations?

4. Is the speed of the wave measured from your graph consistent with the value you obtained using Equation 10.4?