Experiment 6

The Pendulum

6.1 Objectives

- Investigate the functional dependence of the period (τ)[1] of a pendulum on its length (L), the mass of its bob (m), and the starting angle (θ_0).

- Use a pendulum to measure g, the acceleration due to gravity.

6.2 Introduction

Everyday we experience things moving in a periodic manner. For example, when you go to a park, you can see children playing on a swingset. As they move back and forth, they are undergoing periodic motion, much like that of a pendulum. Pendula are great tools for measuring time intervals accurately, but they also can be used to measure the acceleration due to gravity, g. Today, we will investigate how a pendulum works, what affects its period, and measure g.

6.3 Key Concepts

You can find a summary on-line at Hyperphysics.[2] Look for keywords: angular acceleration, pendulum.

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[1]The Greek letter tau (τ) is often used to denote a time period or time interval.
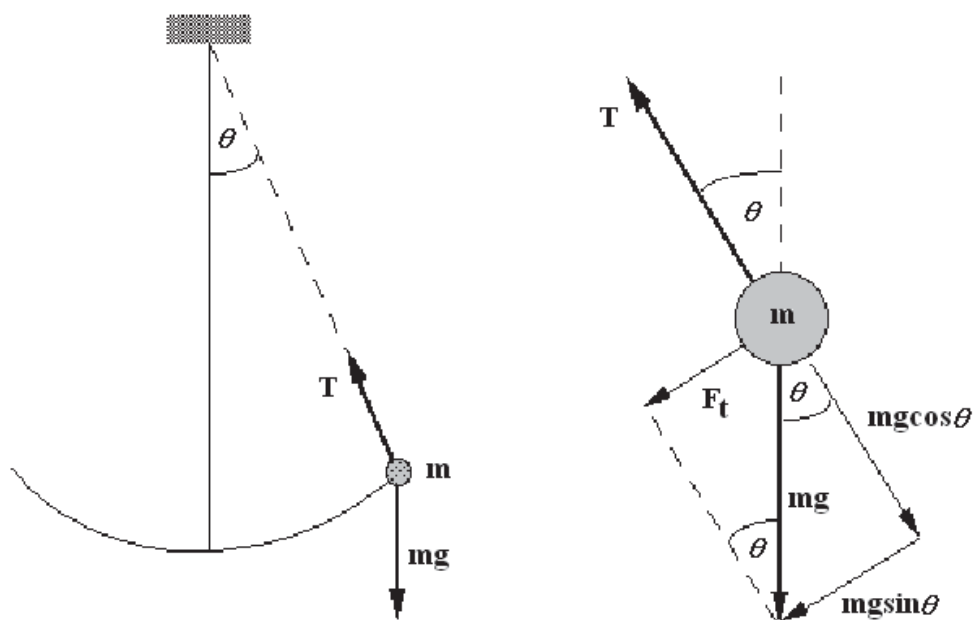[2]http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

Figure 6.1: Force diagram of a pendulum

6.4   Theory

Consider the pendulum shown in Fig. 6.1 swinging back and forth. When thinking about the motion of a pendulum note that:

1. The motion is part of a circle so angular acceleration (α) is a useful variable.

2. The angular acceleration will not be a constant throughout the motion.

The weight at the end of the string is called the "bob" of the pendulum. The acceleration of the bob **tangent**, a_t, to the arc "drawn" by the pendulum as it swings is determined by the force tangent, F_t, to the arc. Since the tension in the string, T, always acts along the **radius**, it does not contribute to F_t. Decomposing the gravitational force, mg, into components perpendicular to and parallel to the string, as shown in Fig. 6.1, one finds that:

$$F_t = mg\sin\theta$$

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where m is the mass of the bob, g is the acceleration due to gravity and θ is the angle of the pendulum measured from vertical (straight up and down), also called displacement angle. This force must also obey Newton's 2nd Law, $F = ma$, where a if the linear acceleration, in this case a_t, giving:

$$F_t = ma_t = mg\sin\theta$$

Notice that the mass of the bob, m, cancels and the acceleration tangent to the circle, a_t, is given by:

$$a_t = g\sin\theta$$

As noted earlier, angular acceleration (α) is a more useful variable to describe the motion of the pendulum. Using the definition of angular acceleration, $\alpha = \frac{a_t}{r}$ where r is the radius of the circle, the angular acceleration, α, of the pendulum in Fig. 6.1 is:

$$\alpha = -\frac{a_t}{r} = -\frac{g}{L}\sin\theta$$

where L is the length of the pendulum. Thus, as was mentioned earlier, *the angular acceleration, α, is not a constant* but varies depending on the displacement angle, θ, of the pendulum.

A pendulum is a resonant system with a single resonant frequency. For small angles ($\theta < 0.5$ radians[3]), angular accelerations can be shown (with a little calculus) to lead to an oscillation of the angle θ by:

$$\theta = \theta_0 \cos\left(\frac{2\pi t}{\tau}\right)$$

where θ_0 is the angle at time $t = 0$ (when we release the pendulum), and τ is the period of the motion.

The period, τ, is the time it takes to complete *one full cycle* of the motion. For small angles ($\theta < 30$ degrees), the period of a simple pendulum[4] can be approximated by:

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[3]A radian is an angle measure based upon the circumference of a circle $C = 2\pi r$ where r is the radius of the circle. A complete circle ($360°$) is said to have 2π radians. Therefore, a 1/4 circle ($90°$) is $\pi/2$ radians.

[4]A simple pendulum is one that can be considered to be a point mass suspended from a string or rod of negligible mass.

$$\tau = 2\pi\sqrt{\frac{L}{g}} \quad \text{or} \quad \tau = \frac{2\pi}{\sqrt{g}}\sqrt{L} \tag{6.1}$$

Note that the period, τ, of a pendulum *depends only on its length, L, and the acceleration due to gravity, g.* The period is independent of the mass of the bob, m, and the starting angle, θ_0.

Equation 6.1 has the same form as the equation of a straight line, $y = mx + b$, with a y-intercept of zero (i.e. $b = 0$). In today's lab you will make a plot of τ vs. $\sqrt{L}$ so the period corresponds to y and $\sqrt{L}$ corresponds to x. The slope, m, is just what is left (i.e. $m = \frac{2\pi}{\sqrt{g}}$).

6.5   In today's lab

Today you will change various parameters of the pendulum and see how they affect its period, τ. You will change the

1. Mass of the bob, m

2. Starting angle, θ_0

3. Length, L

of the pendulum independently of each other and measure the period, τ. You will then plot the results, notice which parameters affect the period of the pendulum and see if you can accurately measure the acceleration due to gravity, g, using a pendulum.

6.6   Equipment

- Various masses

- String

- Photogate

- Protractor

- Meter Stick

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6.7 Procedure

1. Measure the mass of 6 of the point masses provided. Use the heaviest masses in order to get the most accurate data.

2. Measure the length, L_0, you will use for the trials where you vary the mass m and starting angle θ_0 of the pendulum and record it in Excel for L in the top and middle tables.

3. Put the photogate on the PEND setting. The PEND setting uses the photogate in the following way: when the light beam is interrupted the first time the timer is started, it keeps going when the beam is interrupted a second time but stops on the third interruption of the beam. This measures the time taken for one complete oscillation of the pendulum, i.e. the period τ.

4. For the first set of measurements use a starting angle of $\theta_0 = 15°$ from vertical and record it in Excel.

5. Place a mass on the end of your pendulum and record it in Excel.

6. Move the pendulum to $\theta_0 = 15°$ and release it, allowing it to complete one full oscillation (i.e. one period τ). Record the time shown on the photogate in Excel in the column labeled Tau.

7. Repeat steps 5 and 6 five more times using a different mass each time.

8. Now choose one of the masses you just used for the middle Excel table where the starting angle is varied. Hang it on the end of the pendulum, and record the mass in appropriate cell. Use the same length L as the previous trials.

9. Keeping the same string length, record the period for 6 trials where you vary your starting angle θ_0. Here you can choose any starting angle as long as θ_0 remains less than $30°$ from the vertical. Make sure that one of your trials is for $\theta_0 = 15°$ and that the other angles vary by at least $5°$. For example you could use $\theta_0 = 5, 10, 15, 20, 25$, and 30 degrees.

10. Keeping the same mass on the end of the string and starting your pendulum at $\theta_0 = 15°$, measure the period for 6 trials where you

change the length L of the pendulum and record it in your data sheet. Use a large range of lengths from 25 - 80 cm, making sure that one of the lengths is the same as was used in the previous Excel tables and there is at least a 5 cm difference between the other lengths.

11. Calculate $\sqrt{L}$ in Excel.

12. Create plots for:

 - Period (τ) vs. m (for fixed θ_0 and L)
 - Period (τ) vs. θ_0 (for fixed m and L)
 - Period (τ) vs. $\sqrt{L}$ (for fixed m and θ_0)

in KaleidaGraph. Use the same scale for the y axis (τ) on all of your plots. You can determine this scale by selecting the smallest and largest values out of **all** of your trials and using those as your minimum and maximum values respectively for your axis limits. Be sure to fit each plot with a best fit line. There are no error bars on these plots.

6.8 Checklist

1. Excel spreadsheet (you do NOT need to turn in a formula view)

2. Plot of τ vs. m with observations and best fit line

3. Plot of τ vs. θ_0 with observations and best fit line

4. Plot of τ vs. $\sqrt{L}$ with observations and best fit line

5. Answers to questions

6.9 Questions

1. Comment on how the period of the pendulum depends on each of the three parameters: 1) mass of the bob m, 2) starting angle θ_0 and 3) length $\sqrt{L}$. Justify your answers. (Hint: The period is independent of a parameter if the slope of your best fit line is consistent with zero.)

2. Is the angular acceleration, α, constant throughout your pendulum's motion? Justify your answer by calculating α at 2 different angles, e.g. $\theta = 0$ and $15°$. Use the L from your first Excel table and $g = 981$ cm/sec^2.

3. Use the slope of your τ vs. $\sqrt{L}$ graph to calculate g and its uncertainty.

$$\delta g = 2g \frac{\delta(\text{slope})}{\text{slope}}$$

4. Is your value of g consistent with 980.6 ± 0.2 cm/sec^2? If not, give possible reasons why.

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	A	B	C	D	E	F	G
1			**The Pendulum**				
2							
3			Measure		Calculate		
4							
5		τ (period) vs. mass					
6							
7			θ		[degrees]		
8			L		[cm]		
9							
10		Trial	mass	Tau			
11			[grams]	[seconds]			
12		1					
13		2					
14		3					
15		4					
16		5					
17		6					
18							
19		τ vs. angle (θ$_0$)					
20							
21			mass		[grams]		
22			L		[cm]		
23							
24		Trial	θ$_0$	tau			
25			[degrees]	[seconds]			
26		1					
27		2					
28		3					
29		4					
30		5					
31		6					
32							
33		τ vs. length (L)					
34							
35			m		[grams]		
36			θ		[degrees]		
37							
38		Trial	L	sqrt(L)	tau		
39			[cm]	[cm$^{1/2}$]	[seconds]		
40		1					
41		2					
42		3					
43		4					
44		5					
45		6					
46							
47							