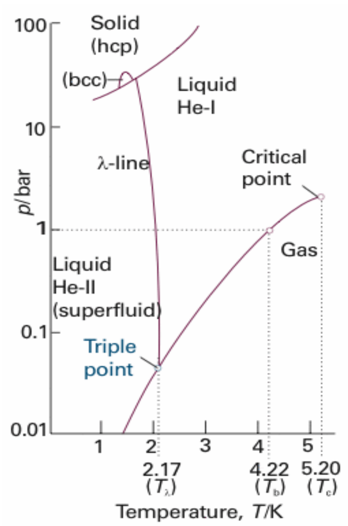
Second Sound in Superfluid Liquid He

In this experiment, the propagation of sound waves in gases and liquids will be studied. You will measure sound speeds at temperatures varying from room temperature (300 K) to temperatures as low as 1.6 K. Of particular interest are the unique properties of liquid helium. He liquefies at 4.2 K under a pressure of 760 Torr (1 bar). The coexistence of liquid and gaseous phases is a line in the P-T (pressure-temperature) plane



with positive slope. Thus, by reducing the pressure over liquid He, one is able to cool the He to temperatures limited only by the speed of the vacuum pump. At P=38 Torr, with T=2.17K, normal liquid He (He I) undergoes a second-order phase transition to a superfluid phase (He II) with seemingly bizarre properties. The transition temperature is often referred to as the "lambda point", since the heat capacity vs. temperature has the shape of a λ symbol, with a maximum at T_λ. Below T_λ, the liquid resembles a Bose-Einstein condensate in which the fluid assumes macroscopic coherence, allowing it to be characterized by single quantum mechanical wavefunction. The flow of heat is described by a classical wave equation, rather than a diffusion equation as is the case for all normal fluids. The wave-like excitation in He II is called "second sound", in contrast to normal "first sound". Both modes exist below T_λ, where the sound speeds u_1 and u_2 are given by the following wave equations:

$$\frac{\partial^2 \rho}{\partial t^2} = u_1 \nabla^2 \rho; \qquad u_1 = \left(\frac{\partial P}{\partial \rho}\right)^{1/2} \qquad \text{First sound}$$

$$\frac{\partial^2 S}{\partial t^2} = u_2 \nabla^2 S; \qquad u_2 = \left(\frac{\rho_s}{\rho_n}\frac{TS^2}{C_p}\right)^{1/2} \qquad \text{Second sound}$$

In the above equations, the total density of the helium $\rho = \rho_s + \rho_n$ is the sum of the superfluid (s) and normal (n) densities, P is pressure, S is entropy, C_p is the heat capacity at constant pressure, and u_i is the sound speed. The separation of density into superfluid and normal components is a fiction which aids theoretical description and does not imply the separation into two distinct phases. In this laboratory experiment, you will be able to measure u_1 and u_2 as a function of temperature. Since $\rho_s \to 0$ as $T \to T_\lambda$ from below, u_2 also will go to zero, essentially as $\rho_s^{1/2}(T)$.[See Greywall and Ahlers (1973)].

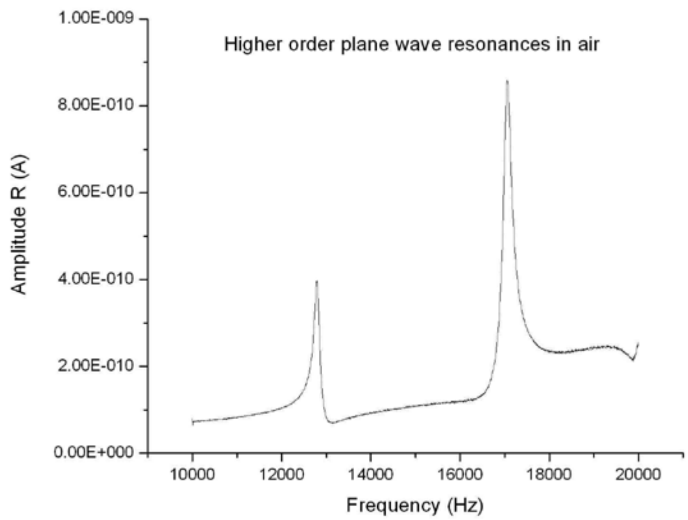
Measurement methods

Transduction

To measure sound speeds, you will use a brass cylindrical cavity. Sound waves are generated and detected by identical capacitive transducers located at the two ends of the cavity. The application of an electric field between the rigid end of the cylinder and the grounded, metalized surface of a thin (15 μm) polymer membrane exerts an electrostatic force on the membrane or diaphragm. The diaphragm exerts a force on the fluid in the cavity which gives rise to a density oscillation or sound wave. The membrane is made from Nuclepore, a porous polycarbonate polymer with a high density of randomly placed 1 μm pores. For normal gases and liquids, their viscosities are sufficiently high that no flow occurs through the pores over a oscillation period: the membrane is effectively impenetrable. However, when the cavity is filled with superfluid helium, the superfluid component has little flow resistance or viscosity so it passes easily through the pores whereas the normal fluid is blocked. The two components then counterflow in such a way as to maintain constant total density, the characteristic behavior of second sound. Thus, the porous membrane is a highly efficient generator of 2^{nd} sound waves.

Cavity

The cavity resonator has the following dimensions at 300K: length L = 4.0 cm and radius a = 0.5 cm. When driven by an oscillator at a frequency $\omega = 2\pi f$ such that an integral number n of half-wavelengths, $n\lambda/2$ equals L, a standing wave resonance occurs leading to a greatly enhanced acoustic amplitude i.e., the system exhibits a standing wave resonance. By sweeping the oscillator over a wide frequency region, several resonances can be observed. For plane waves, the dispersion relation is given by $u = f\lambda$. By measuring the frequency difference Δf between adjacent resonances, you can easily calculate the sound speed from u = 2LΔf. (You should derive this relationship before the end of the first session.) Note that other modes are also possible in which the wave has a radial amplitude dependence. These are known as Bessel-function modes.



Detection

A major aspect of this laboratory experiment is learning how to use a phase-sensitive, or lock-in, detector (PSD). This is a fairly common method for enhancing the signal-to-noise ratio (S/N) of low-level electrical signals. With PSD, you are able to discriminate against random noise as well as interference by detecting only signals that have the same phase (and frequency) as a reference oscillator. Further noise reduction is accomplished by narrowing the bandwidth of the output signal. The method has similarities to homodyne detection (as used in the NMR experiment). An excellent description is given in the SRS 830 manual. With this instrument, you are able to measure simultaneously the signal at the output transducer which is in-phase (0 deg phase difference) and in-quadrature (90 deg phase difference) with the reference oscillator, the Agilent sweep oscillator. This feature is useful here because the acoustic cavity behaves like a driven harmonic oscillator. Near a resonance you are able to record the “real” (X), or in-phase part of the response as well as the “imaginary” (Y) or quadrature part of the response. You can also acquire the absolute value $R = (X^2 + Y^2)^{1/2}$. Among other features, it allows you to see the phase-shift that occurs near resonance and the fact that the quadrature component is zero at resonance. [You should review the features of the driven harmonic oscillator before starting the experiment.]

A final comment about the transducers, which are also referred to as capacitive microphones. From Coulomb's law for a parallel plate capacitor, we know that the force F per unit area A between the plates is $F/A = \frac{\varepsilon_0 E^2}{2}$, where E is the electric field between the plates. If E is produced by an oscillator at frequency ω with $E_{ac} = E \sin \omega t$, each half cycle of the wave produces the same force since F is quadratic in E, leading to a doubling of the transducer oscillation frequency and considerable distortion. One solution to this problem is to apply a large dc bias E_b to the transducer, with $E_b >> E_{ac}$. Now, $F \propto (E_b + E_{ac})^2 \propto E_b{}^2 + 2E_b E_{ac} + E_{ac}{}^2$. The first term is just a constant and the last term is small so that the dominant forcing term is linear in the bias and drive voltages. Therefore, the microphone generates and detects a force at the oscillator frequency. [You should calculate the ratio of the 1st and 2nd harmonic force terms; you can also measure that the microphone response is linear in E_b and E_{ac}. The PSD is also very effective in filtering the second harmonic.]

In the experiment, E_b is provided by a switched battery box. You should not exceed 67 V bias voltage. Always turn off the bias voltage when changing or unplugging cables; you should switch off the box when you leave. Also, note that the circuit contains blocking capacitors that protect the oscillator and detector.

To carry out this experiment you should spend the first two days becoming familiar with the apparatus: the acoustic resonator, the SR830 PSD and Agilent swept oscillator; the thermometer and its calibration; and the gas flow system. In the third week you should be ready to transfer liquid helium and look for second sound. Your instructor will assist you in learning the low temperature techniques. The final session should be used to acquire high-quality data for second sound speed as a function of temperature.

Warning: The only time you should pump on the inner dewar with the cavity in place is when it is filled with liquid He. An abrupt change of pressure will rupture the Nuclepore membrane.

Suggested sequence

Part 1. Familiarize yourself with the electronics and software. Instructions for using SecondSound.vi and LamdaFluke.vi (sic) are on-line. Carry out the exercises for the SR 830 in the handout. Calculate the speed of sound for the air-filled cavity at 300 K (room temperature). The Agilent sweeper should be set to maximum output voltage and high-impedance output.

Part 2. See if you can detect the 2^{nd} harmonic signal generated by the transducer (with or without bias). At higher frequencies look for Bessel-function modes of the cavity. Calculate these resonances and compare with experiment. Flow He gas into the dewar at 300 K as you measure the resonance frequency. Explain what happens in terms of the ideal gas law. How long does it take to exchange He for air in the resonator (there are two 1 mm holes in the body of the cavity)? Take 2-3 deep gulps of gaseous He and talk to your lab partner. What has happened? (Your larynx is a tunable acoustic resonator).

Part 3. Set up and understand temperature measurement using the Cernox resistor and the P-T coexistence line. Just before breaking for lunch, transfer liquid nitrogen to the outer chamber of the dewar while keeping gaseous He in the inner dewar. Record the temperatures during cooldown until you return. In the evening, check all electronics before transferring liquid He. Measure the speed of sound in He I at 4.2 K. Start cooling by pumping slowly on the He bath. (What happens to your signal as you approach T_λ?) You should reach 1.6 K with the pumping valve fully open. Look for 2^{nd} sound resonances. Calculate u_2 to confirm that you are in the superfluid phase. Allow the helium to warm slowly by throttling the by-pass valves to increase the vapor pressure. Sweep through two adjacent resonances to obtain u_2 (T) as you slowly approach T_λ .

Part 4. Repeat part 3. See how close you can get to T_λ . Since the 2^{nd} sound speed decreases as T_λ is approached, the frequency separation between resonances will decrease; they may overlap eventually.