The Compton Effect

Introduction

Light is made up of (many) photons. "Photon" is the name given to the particle-like aspect of light's behavior (in addition to the wave-like behavior you have previously measured). In this experiment we will study two specific aspects of the interaction of photons with electrons. The first of these is the Compton effect, named after Arthur Holly Compton who received the Nobel Prize for physics in 1927 for its discovery. The other effect deals with the radiation emitted when a tightly bound electron from a heavy element is kicked out by a photon. This gives rise to "characteristic" X-rays that can be used to identify the element.

This experiment uses material from the Introduction to Nuclear Radiation.

Theory of the Compton Effect

Kinematics of the Compton Effect

The Compton effect is based on treating light as consisting of particles of a given energy related to the frequency of the light wave. In this context, the particle of light is given the name "photon". An energetic photon with energy of 0.1 MeV (million electron volts) or larger is also often referred to as a gamma ray. An MeV is an energy unit, equal to the kinetic energy an electron would gain by being accelerated through a voltage difference of 1 MV (10^6 volts). Photons whose energy is in the range of 0.1 to 100 keV are usually referred to as X-rays (1 keV = 10^{-3} of 1 MeV).

If a photon with energy E_0 strikes a stationary electron, as in Figure 1, then the energy of the scattered photon, E, depends on the scattering angle, Θ, that it makes with the direction of the incident photon according to the following equation:

$$\text{Cos } \Theta = 1 - m_e c^2 \left[\frac{1}{E} - \frac{1}{E_0}\right] \qquad (1)$$

where m_e is the mass of the electron and $m_e c^2$ = 511 keV = .511 MeV. In real matter, electrons are not stationary, but if the initial kinetic energy of the electron is small compared to the energy of the incoming photon, Eq. 1 will describe the situation well. This will be the case for gamma rays (E > 10^5 eV) scattering off outer electrons of atoms (typical kinetic energy of a few eV).
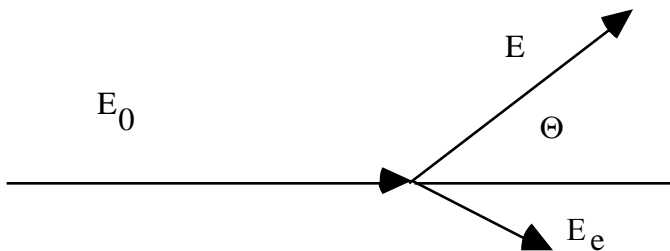

Fig. 1: Schematic diagram of Compton Effect kinematics.

The derivation of this equation is based on applying special relativity and kinematics to the photon as a quantum of light, but in the form we use, requires little or no reference to the wave nature of light! It's all about energy of the electron and photon, and the angle of the outgoing photon compared to the initial direction.

The total energy of the electron E_e is the sum of its kinetic energy T_e and its rest energy $m_e c^2$, i.e. $E_e = T_e + m_e c^2$. The total **energy of the recoiling electron can be computed from energy conservation** in the reaction and is given by:

$$E_e = E_0 + m_e c^2 - E \qquad (2)$$

or equivalently:

$$T_e = E_0 - E \qquad (3)$$

Clearly the electron energy achieves its maximum value in this scattering where the photon has its minimum energy. The lowest energy for the scattered photon results when it is back-scattered, *i.e*, it emerges at 180 degrees with respect to its original direction, in which case Eq. 1 shows that the incident energy *Eo* and *Emin,* the minimum scattered photon energies are related as:

$$\frac{1}{E_{min}} - \frac{1}{E_0} = \frac{2}{m_e c^2} \qquad (4)$$

The Compton Edge

These formulas do not tell us the relative probability of the possible scattering angles (and, equivalently, the relative probability of various recoiling electron energies). In Figure 2 we plot the energy dependence.
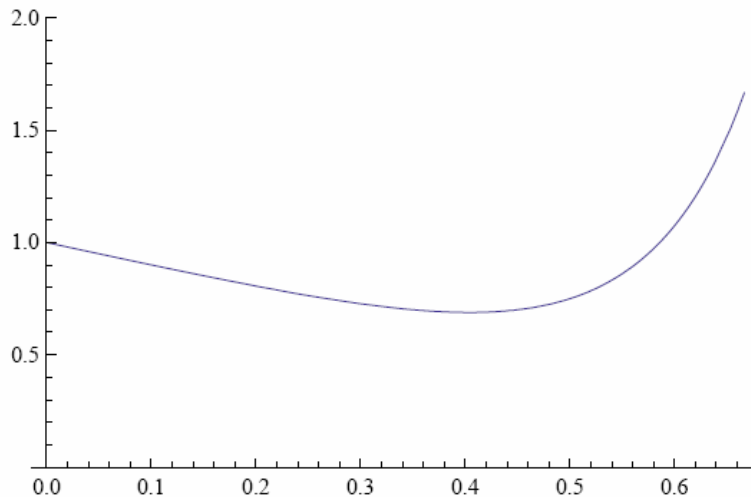

Fig. 2: The relative probability of finding an electron with scaled kinetic energy $t = T_e / m_e c^2 = (E_e / m_e c^2 - 1)$. This varies somewhat with Eo; Eo = .511 MeV for this plot.

The important thing to take away from this plot is the rise in the probability with increasing kinetic energy up to the kinematic limit, where it abruptly falls to zero. In our experiment we will be looking for this shape, referred to as the Compton edge.

We have chosen to plot this for an incident photon with energy equal to the rest mass of an electron, E_o = .511 MeV = $m_e c^2$, but if we had chosen a different incident energy, the shape would be similar, just with a different end point. A more detailed explanation of how this plot was calculated is given in Appendix 3.

Characteristic X-Ray spectra

When electrons or photons scatter from atoms, they sometimes impart sufficient energy to atomic electrons to free them from their bound states. If this happens in a multi-electron atom, then a hole is created which is rapidly filled by an electrons cascading down from higher levels emitting the lost potential energy in the form of photons. When the level filled is the innermost atomic level (n=1, or K shell), then X-rays are produced which uniquely identify the element. These are called characteristic K (or K-shell) X-rays. This can happen in our experiment in Cs → Ba decays if the Ba decay product ejects an n=1 electrons, so the X-ray is characteristic of Ba. Alternatively, the energetic Cs or Co gammas may hit atoms in the detector or elsewhere and produce characteristic X-rays associated with the object they hit. These characteristic X-rays may form a peak in the observed spectrum if enough of them have their entire energy absorbed by the NaI detector.

The K X-ray energy varies with the atomic number (Z) of the substance as $(Z - 1)^2$ where the subtractive constant arises due to the shielding effect of the other inner shell electron. The energies of these X-rays can be substantial e.g. for Pb they are about 80 keV. We can make a rough calculation of this quantity if we recall that the ionization potential of hydrogen (Z = 1) is about 13.6 eV which multiplied by $(82 - 1)^2$ gives a number of the right order of magnitude. These X-rays help identify the substance because they are measuring something related to the atomic number Z: in essence, the energy level of the most tightly bound electron.

The Experiment

The Apparatus

The apparatus in this experiment consists of a NaI (Tl) crystal attached to a photo multiplier tube. The crystal is surrounded by a heavy lead ring, whose purpose is to shield it by absorbing stray radiation not arriving from directly below the crystal. While the Geiger counter is useful in indicating the presence of ionizing radiation, it cannot give us any information about its energy, it has low efficiency for counting gamma rays and it has somewhat poor time response. A class of counters that overcome these difficulties are scintillation counters in which the incident radiation creates photons in proportion to the energy it loses and then these photons are amplified with a photomultiplier and converted into a voltage pulse whose amplitude is proportional to the energy deposited. One of the best of this kind of counters is an inorganic crystal of NaI doped with Tl. Some of its properties are summarized in the table below:

Sodium Iodide Scintillator

Density (g/cm^2)	3.67
Radiation length (cm)	2.59
Moliere Radius (cm)	4.5
dE/dx (MeV/cm) for MIP	4.8
Nucl. Int. Length (cm)	41.4
Decay time (ns)	250
Peak emission λ (nm)	410
Refractive Index	1.85
Light Output (photons/MeV)	40,000

The Radiation length and Moliere Radius are length scales associated with electromagnetic radiation. Electrons and photons deposit their energy in the NaI detector over a finite distance, not just at a single point. For high energy electrons (E >> a few MeV), for example, the Radiation length (longitudinal, *i.e.*, along the incident direction) is the distance in which an electron loses all but 1/e of its energy and the Moliere radius is the transverse radius containing about 90% of the energy. In this lab our sources have a few MeV or less in which case the radiation length and Moliere radius are even shorter, assuring us that the particles (electrons or photons) deposit their full energy in our detector. So **the detector can capture all the energy—if the particle hits near enough to the center of the detector.**

The decay time of a pulse is the time it takes to decay to 1/e of its original amplitude. Two pulses will not be confused if they come a few decay times (5 or 6) separated from one another.

The light created by energy deposition in the crystal is funneled by plastic light guides to the photocathode of a photomultiplier tube where it undergoes amplification on the way to being turned into a voltage pulse. Since the light collection process is not perfectly efficient (a typical number is 10%) and since a typical photocathode efficiency for converting photons into electrons is 25%, the 40,000 photons per MeV created in the crystal appear as about 1,000 electrons/MeV after the first stage of the photomultiplier tube. If the energy resolution of the detector were dominated by counting statistics only then we would expect a fractional resolution of 1/√1000 or about 3% at 1 MeV. In practice, other effects tend to make this number somewhat larger.

Using the NaI detector

The photomultiplier tube amplifies the initial signal by creating cascades of new electrons at several stages in proportion to the number of incident electrons. Typical gains in PM tubes can be 10^6 or more. The gain of the tube is proportional to the high voltage applied and thus we must take care that the voltage is constant during a measurement.

After amplification in the photomultiplier, the signal undergoes some shaping and is fed to an analog-to-digital (ADC) converter residing in the PC. There it is turned into a number proportional to its amplitude, suitable for computer display and manipulation.

NaI can measure the energy deposition due to electrons or photons (which may be gamma rays or lower-energy X-rays). This is a more sophisticated detector than the Geiger counter. Instead of just a count (on or off), the output of the NaI counter is a voltage pulse *proportional to the energy deposited in the counter*, which is fed into a Multi Channel Analyzer (MCA) box read by the PC. Thus you will record a spectrum of energies, where the output recorded by the MCA is calibrated to match the amount of energy deposited in the detector. The output is a histogram of counts vs "channel number" (= histogram bin number). An instruction manual comes with each device and computer (PC). You should take a part of the first laboratory session to become familiar with the operation of the detector and the PC with the MCA card. Learn how to record and erase spectra, how to store spectra on your disk and how to subtract background spectra from spectra containing interesting characteristics. You should also learn how to make hard copies of your plots for inclusion into your formal write-up. Below you will find basic instructions for using the MCA; in Appendix 2 you will find more detailed information about the MCA.

Calibration

Calibration is the process of turning the horizontal scale of the histogram ("channel number") into an energy scale in keV, by using the facts that a) the channel number is proportional to energy and b) certain features in the histogram occur at specific energies. We will rely on four lines as standards: (1) the photo-peak of the 661.6 keV gamma ray, which is the highest energy peak in the ^{137}Cs spectrum, (2) a smaller X-ray peak at 32 keV which is the characteristic Barium K X-ray emitted by the ^{137}Cs source, (3) the photo-peak of the 1.33 MeV gamma ray of the ^{60}Co source, and (4) the photo-peak of the 1.17 MeV gamma ray of the ^{60}Co source. The "photo-peaks" are caused by photons of specific energy being fully absorbed by the detector. Details of the Cs and Co spectra can be found in Appendix 1.

First use the ^{137}Cs source. Put the source near the top of the source holder. Use the AutoCalibrate feature to find an appropriate high voltage and amplifier gain. The result will place the photopeak at 662 keV (AutoCalibrate expects a Cs source!).

Next add the ^{60}Co source and take data with *both* sources side by side. You will probably find that the Co peaks (and 1300 keV) are not on scale. Go to Settings | Amp/HV/.. and lower the fine gain until the Co peaks are on scale (should find the Cs peak a bit below middle of x axis).   To establish a new calibration (since you have changed the fine gain), you can utilize the above four known energy scales. First, select 'Uncalibrate' from the menu, and then perform 2-point or 3-point calibration procedure. You need to retrieve the channel numbers from the spectra that correspond to the known energy scales, which are input parameters (in pair) for performing the point calibration. A good calibration covers a broad range. Be sure to *check that all the four known energy values are accurately reflected in the spectra after the calibration!* From now on carefully refrain from changing the high voltage or gain. After calibration, the program gives energy corresponding to the cursor position instead of channel number. Check the energies by putting the cursor on your peaks. Save.
If you find these instructions too sketchy, refer to Appendix 2.

You will need to repeat this process the second week of the lab before you go on to do the remainder of your measurements. If you have time left, start your measurements (read on!).

Measurements

General Overview
We will not measure the angular Compton scattering distribution given by the Klein-Nishina formula. That would need a very powerful source of gamma rays. The safe handling of these sources would not be practical in this laboratory setting. We will, however, measure the end point for Compton scattering by measuring the energy of the photon that is back-scattered from a stationary electron. The photon is back-scattered when its final direction is roughly 180 degrees from its incoming direction (see Equation 1). The stationary electron from which it scattered might be in the detector, or in other nearby materials—electrons are in all matter. We will also measure the energy of the outgoing electron in the case that the incoming photon is back-scattered. We will do these measurements using first the ^{137}Cs source and then the ^{60}Co source, recalling that the former emits one photon and the latter two, at different energies.

In separate measurements we will measure the characteristic K X-rays of Pb and an "unknown" metal and use the energy of the characteristic spectra to identify the unknown.

The Compton Plateau
Measure the spectrum of the Cs source. Put it on a shelf just below the Nal detector. Choose a time sufficiently long so that statistics are not a problem. Save and print your spectrum
Question 1) Is printing in linear or log better? Why?
Repeat for Co source.   Do you need to perform background subtraction? If so, see detailed directions in Appendix 2.

On the spectra, label (as appropriate) the ^{137}Cs and ^{60}Co photo-peaks, the Barium X-ray, the Compton plateaus, and the Compton edge(s). Write the energy corresponding to each of these features next to the feature on the plot. Use the spectrum software to help with the energy measurements. Estimate your uncertainty in the peak position by trying to re-measure the peaks with the software. The Compton plateau should show up in your spectrum as a region of higher counting rate due to either electrons or photons in or near the most probable energy configurations, according to the theory of Compton scattering (remember that the detector measures photons or electrons equally well).

Q2) Calculate the expected energy of each Compton edge from Compton Effect kinematics.
Q3) Use the calculations to identify the Compton edges on the plots and label them according to the photo-peak from which they originate. Are they consistent?

The Backscattered Photon
Put the ^{60}Co source in the lowest position in the holder. Elevate your NaI(Tl) detector assembly above the table to reduce the scattering material directly under the source. Take a 15 minute (Live time) measurement in this configuration and store it as background.   Put six thick aluminum plates under the source holder (but on top of the blocks) and repeat the 15 minute measurement. Using the strip function of the MCA program, subtract the stored spectrum from the new one to see the spectrum of photons scattered from the aluminum.
        Plot the difference spectrum.
Q4) Measure, identify and explain the new source of gamma line(s) How does this line (or lines) relate to the measurements you made on the Compton Plateau?
Q5) Calculate a predicted energy for this peak and compare to your measurement.
Q6) Can you demonstrate that energy is conserved in the Compton scattering process, using your measured values (and uncertainties)?
Q7) Why is it important to put the source in the lowest position for this measurement?

The K X-rays of Pb
This measurement is performed in a similar detector configuration to the measurement (identification) of the backscattered photon, but now our goal is to concentrate on identifying a characteristic X-ray (much lower energy) a substance. Note that back-scattering itself comes from (nearly) free electrons, which are in all atoms, so don't really help identify a substance.

Count and store a background run. Repeat the previous measurement with the lead plate under the detector instead of Al. After background subtraction, identify a photon line near 80 keV.
Q8) Note its energy and provide an explanation for its existence.
EC) Why would a characteristic X-ray from Al be harder to find than one from Pb?

Identification of Unknown Metal
        Count and store a background run. Place the "unknown" metal sample under your counter and count for the same period of time.
Q9) Note the appearance of an X-ray line in the subtracted spectrum.
Q10) From the measurement of its energy, try to identify the "unknown" element. A good resource is the web site

   http://Hyperphysics.phy-astr.gsu.edu/hbase/Tables/kxray.html

which derives its data from a comprehensive database maintained by NIST, the National Institute of Standards and Technology. The web site above will also list X-ray energy values for other elements such as Al, Ba, and

Pb. With our apparatus, we won't be able to distinguish between the two lines listed (corresponding to two closely spaced X-ray lines), since they are less than 1-2 keV apart for most elements.

The Report

Your report should contain a brief description of the experiment. It should also contain an *orderly* exposition of your measurements including computer printouts and graphs where appropriate. And of course, a summary table. Show examples of needed calculations and state clearly your conclusions: what did you measure? Did it agree with expectations?
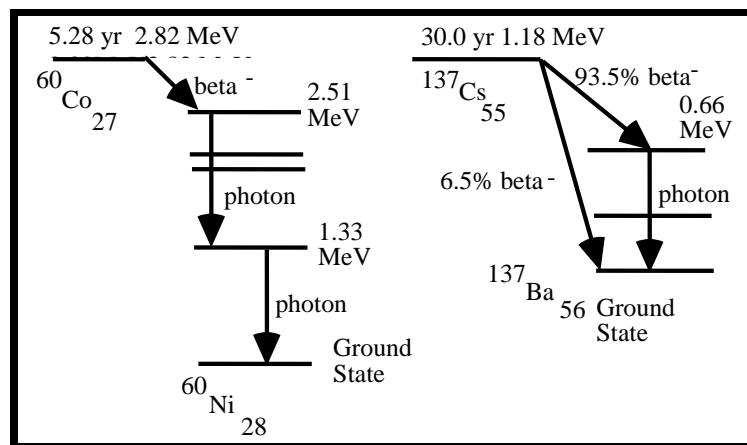

Extra credit:

Derive formula (6) from Eq 1-3.
Derive (explaining as you go) formula (1); you'll need relativistic particle kinematics.
There are a number of possible Extra Credit measurements as well; ask your TA for suggestions.

Appendix 1: Decay schemes for ^{60}Co and ^{137}Cs

The isotope notation $^{137}Cs_{55}$ indicates an isotope of Cesium (element 55, with 55 protons) which has 137-55=82 neutrons. Vertical arrows represent simple transitions from one nuclear energy level to another, which result in emission of a photon whose energy is equal to the difference in energy levels.   The sloped arrows represent a beta decay which results in an electron and a neutrino being emitted. The beta electron does not have a fixed energy but rather a spectrum which has a maximum value; a typical value might be about half the maximum value.

The Cs sources we use have thin windows (for easier transmission of betas), while the Co sources do not have a beta window. Also the Co beta energy is much less than for Cs. Some details of the decays are given below:

^{60}**Co:** This isotope has a half life of 5.28 years and beta decays to the excited states of the stable nucleus of ^{60}Ni. In 99.9% of the cases the ^{60}Ni (2.51 MeV 4th excited state) is formed, with a maximum beta energy of 0.31 MeV. The excited state subsequently decays in less than 10^{-12} sec to the ground state via the emission of 2 photons, one with energy 1.17 MeV and one with 1.33 MeV.

^{137}**Cs:** This isotope beta decays with a half-life of 30.0 years, with 93.5% of the decays creating ^{137}Ba in its 2nd excited state at 0.66 MeV and 6.5% creating ^{137}Ba in the ground state. The maximum beta energy is 0.52 MeV for the decay to the excited state, and 1.18 MeV for the decay directly to the ground state. The excited state has a half-life of 2.55 min. and decays 90% of the time through the emission of a 0.66 MeV photon and 10% of the time with the emission of an atomic conversion electron of a similar energy, equal to 0.66 MeV minus the binding energy. So 84% (= 0.935 * 0.90) of all beta decays of this isotope produce a photon in the final state.

Appendix 2: Multichannel Analyzer Detailed Instructions

Basic commands for software:

Turn on analyzer box's power switch **before** clicking on the little spectrum icon on the PC.

> There may be issues with starting up the MCA after computers have been down for a while. Some computers claim to not find any such USB hardware, or that the USB is off, when it's not. Sometimes just trying again does the trick, or logging out and back in. Sometimes it requires a full reboot *with MCA power off*; once you were logged back in, turn the power on, and try again. At worst you may have to agree to install software: say Accept, Continue Anyway, etc through at least 2 cycles of install. Remember, you certainly need power on the box *before*clicking on the MCA icon.

You start in Live mode (see title bar).

Settings | Energy Calibration | Autocalibrate   for Cs   to do initial setup. Print and it will preserve settings.

Eraser Icon: deletes spectrum

File | Print prints spectrum, along with additional settings

Now put *both* Cs and Co sources in top drawer.

Click Go and keep going till you like it, then Stop

File | Save   to save spectrum

   (or Save Setup   and later Load Setup to snag the settings you used later)

Timed spectrum: Settings | Presets | Live Time |then pick some big number like 1000 sec

Region of Interest   Settings | ROI | Set ROI

>   Drag cursor from lower to upper region, fairly symmetrical around peak
>   Gives you total counts in peak, and calculates Centroid (peak) & full width at half-maximum
>   Can also do Settings | Presets |Integral Counts   to run until desired counts in ROI

Ba X-ray: it may appear cut off or hard to see. If so, try the following: on the bottom of the x axis there are two little triangles. These are ULD and LLD (upper and lower discriminator level). The default LLD is 6%; lower it by clicking and dragging all the way to the left (0%).

Calibration

Don't be bothered by the report that the detector is at 500V after Autocalibrate: there is no strong reason for the NaI voltage to be the same as a GM voltage! The important issue is whether it puts the Cs peak at 662 like it should.

Next do 2-point or 3-point calibration with Co and Cs sources side by side. Autocalibrate doesn't work because it will put the Co gammas off scale (> 1024) after having put the Cs at channel 662.

>   Pick keV for units then click on peaks you choose, giving energy in keV for each peak

If both Co peaks aren't on scale, or 1300 keV isn't on the x axis, go to Settings | Amp/HV/.. and lower the fine gain until the Co peaks are on scale.

Be sure to check all four calibration points to see that they have good energies. If the energy at channel 0 is substantially different than 0 (more than ± 10 keV), the calibration is dubious. This will almost certainly force you to pick Ba X-ray as one of the calibration points. The peak of the Pb X-ray is at the K alpha line. K alpha is a close-spaced doublet, usually the strongest of the K X-rays, from 2p to ground.

Save your spectra! Then you can go back and remeasure! But print in **live mode** to capture settings. Just print from the MCA program—no need to export to Kgraph or Excel.

Background subtraction

1)   The software has the smarts to scale background by live time.

2)   Procedure: save real and bkg spectrum via File | Save

File | Open (you are now in file mode—see Title bar; by default, same size as original screen!)

>   Open spectrum file
>   Then Strip Background | Load Background | open background file
> | Strip   subtracts it.   You can also display side by side.

For whatever reason, the term commonly used in Nuclear Physics for background subtraction is Stripping the Spectrum.

Appendix3: Some Details of the Compton Effect

The Klein-Nishina Angular Distribution Formula
     While Equations 1-3 tell us how to compute the energies of the scattered photon and electron in terms of the photon's angle, they do not tell us anything about the likelihood of finding a scattered photon at one angle relative to another. For this we must analyze the scattering process in terms of the interactions of electrons and photons.

     The electron-photon interaction in the Compton effect can be fully explained within the context of our theory of Quantum Electrodynamics or QED for short. QED is well beyond the scope of this course and we shall simply quote some results. We are interested particularly in the angular dependence of the scattering or the differential cross-section and the total cross-section both as a function of the energy of the incident photon. If you haven't seen the term "cross section" before, for now you can just think of it as a quantity proportional to the relative probability of an interaction with a particular value of the variables.

     First we write the differential cross-section, also known as the Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{\frac{1}{2}r_o^2[1+\cos^2\Theta]}{[1+2\varepsilon\sin^2\Theta/2]^2}\left\{1 + \frac{4\varepsilon^2\sin^4\Theta/2}{[1+\cos^2\Theta][1+2\varepsilon\sin^2\Theta/2]}\right\} \qquad (5)$$

where $\varepsilon = E_0/m_ec^2$ and r_0 is the "classical radius of the electron" defined as e^2/m_ec^2 and equal to about 2.8 x 10^{-13} cm. The formula gives the probability of scattering a photon (integrated over φ) into the solid angle element $d\Omega = 2\pi |d(\cos\Theta)| = 2\pi|\sin\Theta| d\Theta$ when the incident energy is E_0. We illustrate this angular dependence in Figure 3 for one particular photon energy.
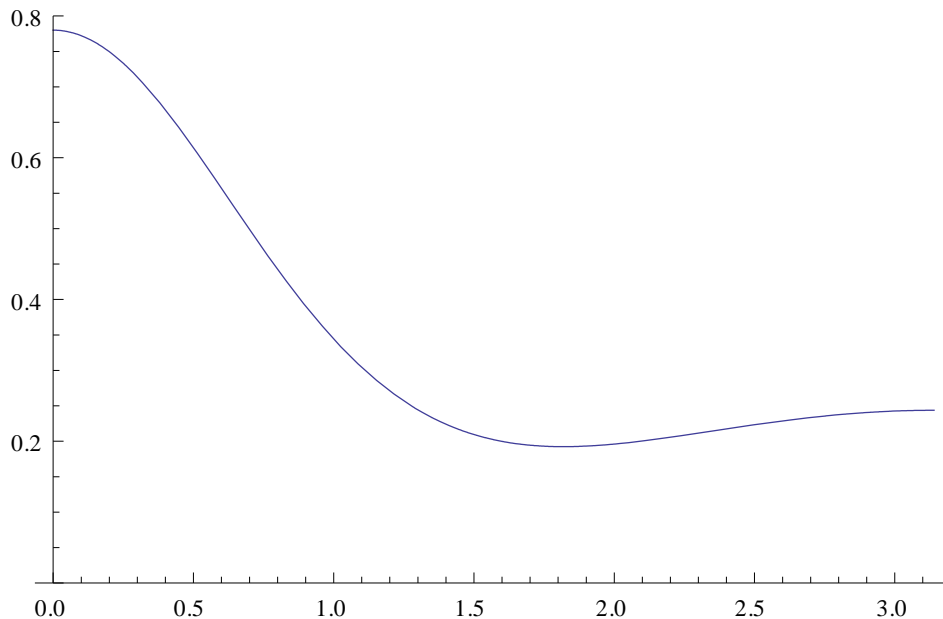

Fig. 3: Differential Cross-section of Compton scattering vs. angle (radians)

     Note that the most likely scattering is in the forward direction ($\theta=0$) and that the probability of scattering backward (θ near π) is relatively constant with angle. This shape varies mildly with the initial photon energy E; here we chose $E_o = m_ec^2 = .511$ MeV.

     Since we will be measuring energy, it is of interest to rewrite this to give the probability of measuring electrons with a given kinetic energy $T = E_e - m_ec^2$. We can get the expression plotted in figure 2 by

substituting for the angle Θ in Eq. (5) via Equations (2) and (3), and then using the solid angle definition given below Eq. (5), and applying the relationship:

$$\frac{d\sigma}{dT_e} = \frac{d\sigma}{d\Omega} \cdot \frac{d\Omega}{dT_e} = \frac{d\sigma}{d\Omega} \cdot \frac{2\pi\, m_e c^2}{(E_o - T_e)^2} = \frac{d\sigma}{d\Omega} \cdot \frac{2\pi}{m_e c^2 (\varepsilon - t)^2} \tag{6}$$

The final form on the right uses the definitions $\varepsilon = E_0/m_e c^2$ and $t = T_e / m_e c^2$.

Energy dependence

     The Klein-Nishina formula can be integrated over photon scattering angle (or electron recoil energy) to yield the total cross-section, which displays the dependence on the incident energy (shown in Fig 4) for the Compton process:

$$\sigma = 2\pi \cdot r_o^2 \left\{ \frac{1+\varepsilon}{\varepsilon^2} \left[\frac{2+2\varepsilon}{1+2\varepsilon} - \frac{ln(1+2\varepsilon)}{\varepsilon} \right] + \frac{ln(1+2\varepsilon)}{2\varepsilon} - \frac{1+3\varepsilon}{(1+2\varepsilon)^2} \right\} \tag{7}$$
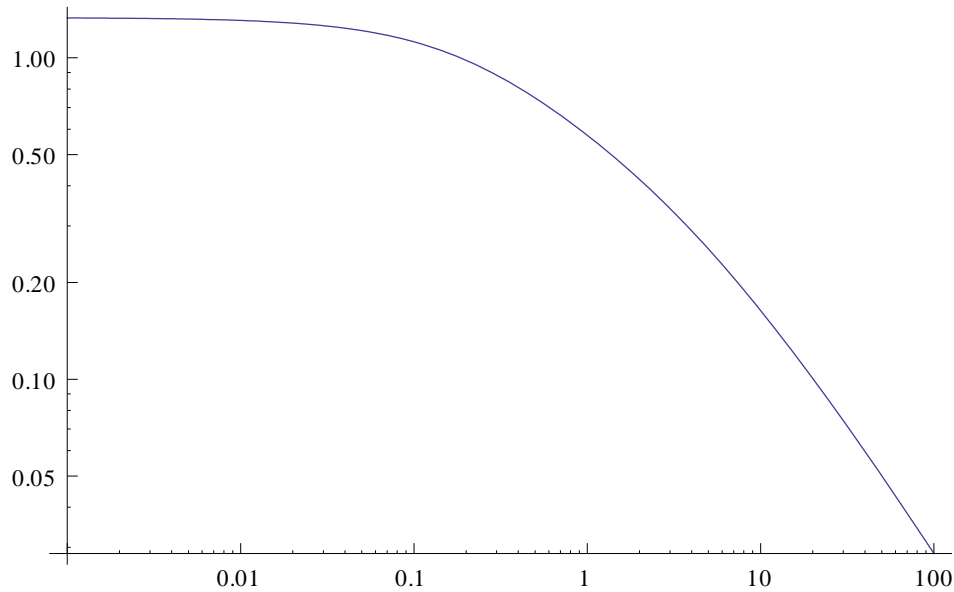

Fig. 4: Energy dependence of Compton scattering, vs. $x = \varepsilon = Eo/ m_e c^2$.

The Compton process is weakly energy dependent up until about .1 MeV, when it begins to decrease significantly; other processes become more important at higher energies.