

## Radioactivity III: Measurement of Half Life.

### Introduction

This experiment will again use the Geiger counter, this time to measure radiation intensity as a function of time and thereby to measure the half-life of an isotope. This is briefly described in the "Introduction to Radiation" handout under "Half-life".

### Assigned problems: Solve Before Coming to Lab!

Along with preparing an introduction for the report on this experiment, also solve problems 1 and 2 below. Show your reasoning clearly.

- P1.** Silver 108 ( $^{108}\text{Ag}$ ) has a half-life of 2.4 min. How long will it take for a source of  $^{108}\text{Ag}$  to decay to 1/8 of its original counting rate?
- P2.** If the source originally consisted of 2000 atoms, what would be the initial disintegration rate ( $dN/dt$ )?

### Procedure

Although this is the third experiment with essentially the same equipment, there is one fundamental difference. This time **you are to make up the experimental procedure**. Therefore, **before you come to the lab spend some time in writing down a detailed prescription of what you will do in order to measure the half-life of each of the two isotopes**. The only information that you will be given is that the half-life of the isotopes is between 20 seconds and 3 minutes, and that you will be given two samples (both are of the same material) which you can irradiate. Given this information and given the isotope at  $t = 0$  what will you do? To aid your measurement, a program called "Half\_life" is available to you. Look it up in the 'New Nuclear' folder on the desktop. Write down what measurements you will want to make and to what accuracy. Things to bear in mind include: the operating point, background, when to start measuring, time intervals for measuring the rate, time intervals between measurements, how will you determine the half life from the data, etc.

We will use two silver (Ag) isotopes as sources, which will be activated by a neutron source. Neutron activation is described in Appendix A. However, we are essentially interested in the end product of this activation, which is one component with a short lifetime and one with a longer lifetime. Thus many of the details described in the appendix are not of crucial importance to this experiment.

Here are a couple of things to keep in mind: both silver coins have a mixture of two naturally occurring isotopes. By neutron activation, a few atoms of each of these isotopes adds a neutron and becomes a new radioactively unstable isotope. The two different isotopes have different (fairly short) lifetimes, so the counts you see are decays coming from a mixture of these two unstable isotopes. Your job is to measure the lifetime of each of the two isotopes by tracing out the decay rate as a function of time. This analysis will

resemble the analysis you did last week to find the different absorption lengths of beta and gamma radiation, even though they came from the same Cs source.

**Question 1:** For measuring the half-life of  $\sim 2.4$  min (as one of the decay channel), you may elect to measure the count rate at regularly spaced intervals of about 10 – 15 seconds. Is there a way to use the counter to accomplish this function?

**Question 2:** Can you determine the two lifetimes from your measurements or must you be content with a measurement of the average? (Hint: think about your experiences with the absorption experiment.)

**Question 3:** If you were to fit the data using the general fit feature of Kaleidagraph using a function, could you determine the decay lifetimes for the two isotopes that way? Do you need to include a constant rate in your fitting function to take the background rate into account?

### Experiment

Insert the silver sample (a coin) into one of the plastic tubes accessible from the side of the "Neutron Howitzer" and slide the cylinder into one of the test holes in the moderator for 10 minutes. In two half-lives (5 mins or so) of  $^{108}\text{Ag}$ , the activity of this nucleus will reach about 3/4 of its peak value; activating longer will get closer to the peak value. The activity of  $^{110}\text{Ag}$  will reach essentially its peak value.

**Question 4:** As soon as you take the silver out of the howitzer, the activities of the two isotopes will begin to decrease. Which isotope begins to decrease more rapidly? Is it important to get the silver in front of your Geiger counter quickly?

**Question 5:** You can analyze each run separately, but the number of counts per timing interval will be small. If you were to use several silver pieces at once, would the count rate be larger? Would it be possible to combine data from different activations in order to get a higher statistical accuracy for the measurements at later times?

**Question 6:** You have two silver pieces available. What is the best way to use the second silver piece? Should you irradiate both at the same time? How long should you devote to each measurement in order to determine the longer decay time properly? How long should you spend on irradiating, as compared to measuring?

**Question 7:** What is the best way to plot your data to see the exponential decay most clearly?

**Question 8:** Can you determine the original ratio of the activities of  $^{108}\text{Ag}$  and  $^{110}\text{Ag}$ ? If you can, do it.

**Question 9:** The Geiger counter is insensitive for a time of 150 microseconds after each count. What is the probability of missing a count, at the highest count rate you measured? Is this experimental “dead time” likely to have any important impact on your measurements? Which lifetime measurement (the shorter or the longer) would be more likely to be impacted, and why?

## Appendix A

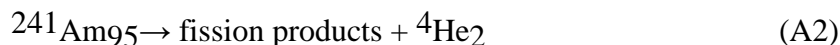
### NEUTRON ACTIVATION

#### Introduction

The process of neutron activation involves the production of a radioactive source as a result of the absorption of neutrons. Since a radioactive source emits characteristic alpha, beta, or gamma rays, one may, from an analysis of the radiation, determine the source. Thus, neutron activation may be used for determining the composition of a sample. Activation is more commonly used, however, in searching for trace amounts of certain elements that strongly absorb neutrons and result in the production of highly radioactive nuclei. As we shall see, if the initial neutron flux, the activation cross section, the life time and the decay rate are all known, one may determine the concentration of trace nuclei. Neutron activation is used, for example, in analyzing for trace amounts of gunpowder.

#### Americium-Beryllium Neutron Source

A commonly available source of neutrons is prepared by mixing  $^{241}\text{Am}$  with finely ground beryllium powder. Americium-241 decays by spontaneous fission with the emission of an alpha particle (the nucleus of helium). Some fraction of the alpha particles is absorbed by the  $^9\text{Be}$  nuclei, resulting in the production of an excited state of  $^{13}\text{C}$ . Although  $^{13}\text{C}$  is normally stable, the excited state of  $^{13}\text{C}$  may decay to  $^{12}\text{C}$  with the emission of a neutron. The nuclear reactions that we have been describing may be written as



The asterisk (\*) symbol in Equation A1 indicates that the  $^{13}\text{C}$  is formed in an excited state. The subscript gives the atomic number and the superscript the atomic mass. Both mass and charge must balance on the two sides of the equation.

A source with an activity of 1 Ci (Curie) has the same activity as 1 g of radium which is entirely  $^{226}\text{Ra}$ . The half-life is 1,622 years so that the activity of 1 g must be:

$$1 / 226.0254 \times 6.023 \times 10^{23} \times (0.69135 / (1622 \times 3.156 \times 10^7)) = 3.666 \times 10^{10} \text{ s}^{-1} \quad (\text{A4})$$

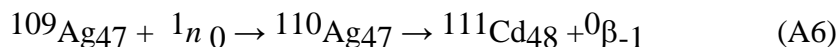
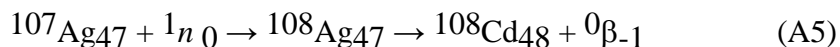
where  $1/226.0254$  is the fraction of a mole corresponding to 1 g;  $6.023 \times 10^{23}$  is the number of nuclei in a mole;  $3.156 \times 10^7$  is the number of seconds in a year;  $0.69315$  is  $\ln 2$  which, when divided by the half-life, gives the decay rate per nucleus. Now, the half-life of  $^{241}\text{Am}$  is 432 years or about 1/4 times as long as that of  $^{226}\text{Ra}$ . Since the atomic masses are about the same, this means that we shall need 1/4 g of americium to yield 1 Ci. One curie of americium in turn produces  $1.5 \times 10^6$  neutrons/sec. Thus, only 1 neutron is produced for every 20,000 alpha particles.

The americium-beryllium source is surrounded by a lead shield, which absorbs any alpha particles not captured by the beryllium metal. The lead also absorbs most of the emitted gamma rays. The neutrons are absorbed very little by the lead, however, and pass through the lead shield. The probability that a neutron is captured by a nucleus depends on the energy of the neutron, generally being larger the lower the neutron energy. In order to reduce the neutron energy as much as possible, the source is surrounded by either paraffin or water. As a result of inelastic collisions with protons, the neutrons slow down until they reach a mean energy of  $4 \times 10^{-21}$  J, which is the mean thermal energy at room temperature. (At this energy the most probable neutron velocity is 2200 m/sec.) Slowing the neutrons to thermal energies is called moderation. Neutrons in paraffin are moderated in a distance of about 4 cm.

In order to provide adequate moderation and shielding, a 1-Ci source should be placed at the center of a moderator about 2 ft. high and 1 1/2 ft. in diameter (just about the size of a garbage can). The sample to be activated is dropped into a tube at the same depth as the source.

### Activation of Silver

Naturally occurring silver is composed of two isotopes,  $^{107}\text{Ag}$ , which is 51.82% abundant, and  $^{109}\text{Ag}$ , which is 48.18% abundant. The extent to which a nucleus interacts with an incident particle may be described in terms of a capture cross section. That is, an incident particle coming within that area surrounding the nucleus will be captured. The thermal-neutron capture cross section of  $^{107}\text{Ag}$  has been measured to be 40 b (a barn is  $10^{-24}$  square centimeters);  $^{109}\text{Ag}$  has a thermal-neutron capture cross section of 82 b. When  $^{107}\text{Ag}$  captures a neutron, it is converted to  $^{108}\text{Ag}$  which decays principally to  $^{108}\text{Cd}$  by the emission of a beta particle. About 2% of the  $^{108}\text{Ag}$  nuclei decay to  $^{108}\text{Pd}$  by the emission of a positron. Silver-109 is converted to  $^{110}\text{Ag}$  by neutron capture. Silver 110 decays to  $^{111}\text{Cd}$  by the emission of a beta particle. These reactions are summarized below:



In addition to the reactions described above, there is a small probability that isomeric states of  $^{108}\text{Ag}$  and  $^{111}\text{Ag}$  are formed. These are higher energy nuclear states that are relatively stable. They may also decay by beta emission, but the lifetimes are much longer

so that they contribute very little to the observed activity. In the following discussion we shall ignore all but the reactions indicated in Equations. (A5) and (A6).

### Calculation of Activity

How long should the silver foil be activated in order to become reasonably active? Does this time depend on the strength of the neutron source? We can answer these questions from a solution of the equation describing the rate of production of radioactive nuclei. In what follows we consider a single nuclear species. Our results may later be combined to describe the production of several nuclear species. Let  $n$  be the number of source nuclei in the foil,  $\sigma$  the thermal-neutron capture cross section, and  $\Phi$  the neutron flux in particles per unit area per second. Then if  $N$  is the number of radioactive nuclei, the rate of production of these nuclei is given by:

$$dN / dt = n \Phi \sigma \quad (\text{A7})$$

But these nuclei decay to the ground state in a mean time  $\tau$  at a rate given by:

$$dN / dt = - N / \tau \quad (\text{A8})$$

In order to describe both processes, we must add Equations. (A7) and (A8) to obtain:

$$dN / dt = n \Phi \sigma - N / \tau \quad (\text{A9})$$

The solution of Eq. (A9), assuming that  $N = 0$  at  $t = 0$  is given by:

$$N = n\Phi\sigma\tau(1 - e^{-t/\tau}) \quad (\text{A10})$$

Note that the number of radioactive nuclei produced does not increase indefinitely, but levels off at a value  $n \Phi \sigma \tau$ . In a mean time  $\tau$ , we achieve  $1 - 1/e = 63.2\%$  of this value.

Let us now ask the following question: If we irradiate the foil for a time  $T$  and then remove it from the moderator, what is the expression for the activity  $A = -dN/dt$  following the removal of the foil? At time  $T$  we have from Eq. (A10)

$$N_0 = n\Phi\sigma\tau(1 - e^{-T/\tau}) \quad (\text{A11})$$

With the sample removed from the moderator, the rate of change of  $N$  is given by Eq. (A8), which may be solved to obtain

$$N = N_0 e^{-t/\tau} \quad (\text{A12})$$

The activity is given by

$$A = -dN/dt = \frac{N_0}{\tau} e^{-t/\tau} \quad (\text{A13})$$

Substituting from Eq. (A11), we have finally:

$$A = n\Phi\sigma(1 - e^{-T/\tau})e^{-t/\tau} \quad (\text{A14})$$

Note that from Eq. (A14), as long as T is much longer than  $\tau$ , the initial activity is independent of  $\tau$ .

Finally, if we have two nuclear species corresponding to  $n_1$  and  $n_2$ , we may write the total activity as:

$$A = n_1\Phi\sigma_1(1 - e^{-T/\tau_1})e^{-t/\tau_1} + n_2\Phi\sigma_2(1 - e^{-T/\tau_2})e^{-t/\tau_2} \quad (\text{A15})$$