Experiment 10

The Spring: Hooke's Law and Oscillations

10.1 Objectives

- Investigate how a spring behaves when it is stretched under the influence of an external force. To verify that this behavior is accurately described by Hooke's Law.
- Measure the spring constant (k) in two independent ways.

10.2 Introduction

Springs appear to be very simple tools we use everyday for multiple purposes. We have springs in our cars to make the ride less bumpy. We have springs in our pens to help keep our pockets/backpacks ink free. It turns out that there is a lot of physics involved in this simple tool. Springs can be used as harmonic oscillators and also as tools for applying a force to something. Today we will learn about the physics involved in a spring, and why the spring is such an interesting creation.

10.3 Key Concepts

You can find a summary on-line at Hyperphysics.[1] Look for keywords: Hooke's Law, oscillation

10.4 Theory

Hooke's Law

An ideal spring is remarkable in the sense that it is a system where the generated force is **linearly dependent** on how far it is stretched. Hooke's law describes this behavior, and we would like to verify this in lab today. In order to extend a spring by an amount Δx from its previous position, one needs a force F which is determined by $F = k\Delta x$. **Hooke's Law** states that:

$$F_S = -k\Delta x \tag{10.1}$$

Here k is the **spring constant**, which is a quality particular to each spring, and Δx is the distance the spring is stretched or compressed. The force F_S is a **restorative force** and its direction is opposite (hence the minus sign) to the direction of the spring's displacement Δx.

To verify Hooke's Law, we must show that the spring force F_S and the distance the spring is stretched Δx are proportional to each other (that just means linearly dependant on each other), and that the constant of proportionality is $-k$.

In our case the external force is provided by attaching a mass, m, to the end of the spring. The mass will of course be acted upon by gravity, so the force exerted downward on the spring will be $F_g = mg$ (see Fig. 10.1). Consider the forces exerted on the attached mass. The force of gravity (mg) is pointing downward. The force exerted by the spring ($-k\Delta x$) is pulling upwards. When the mass is attached to the spring, the spring will stretch until it reaches the point *where the two forces are equal but pointing in opposite directions*:

$$F_S - F_g = 0 \quad \text{or} \quad -k\Delta x = mg \tag{10.2}$$

This point where the forces balance each other out is known as the **equilibrium point**. The spring + mass system can stay at the equilibrium

[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

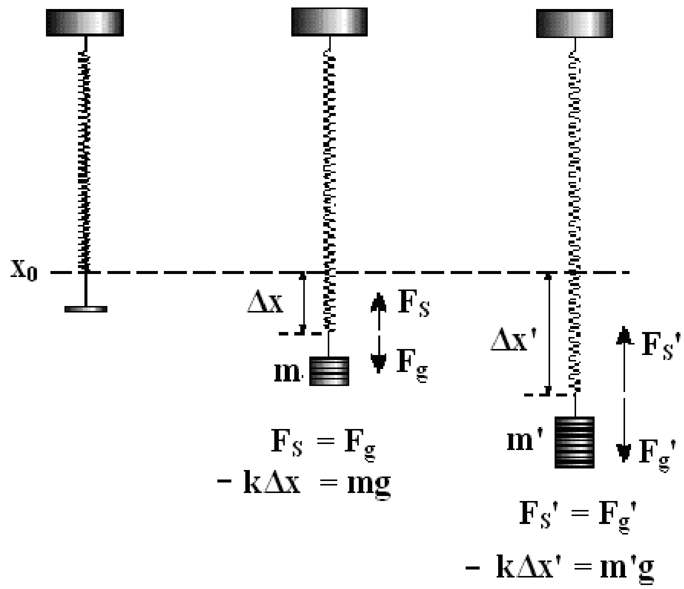


Figure 10.1: Force diagram of a spring in equilibrium with various hanging masses.

point indefinitely as long as no additional external forces are exerted on it. The relationship in Eq. 10.2 allows us to determine the spring constant k when m, g, and Δx are known or can be measured. This is the first way that k will determined today.

Oscillation

The position where the mass is at rest is called the equilibrium position ($x = x_0$). As we now know, the downward force due to gravity $F_g = mg$ and the force due to the spring pulling upward $F_S = -k\Delta x$ cancel each other. This is shown in the first part of Fig. 10.2. However, if the string is stretched *beyond its equilibrium point* by pulling it down and then releasing it, the mass will accelerate upward ($a > 0$), because the force due to the spring is *larger* than gravity pulling down. The mass will then pass through the equilibrium point and continue to move upward. Once above the equilibrium

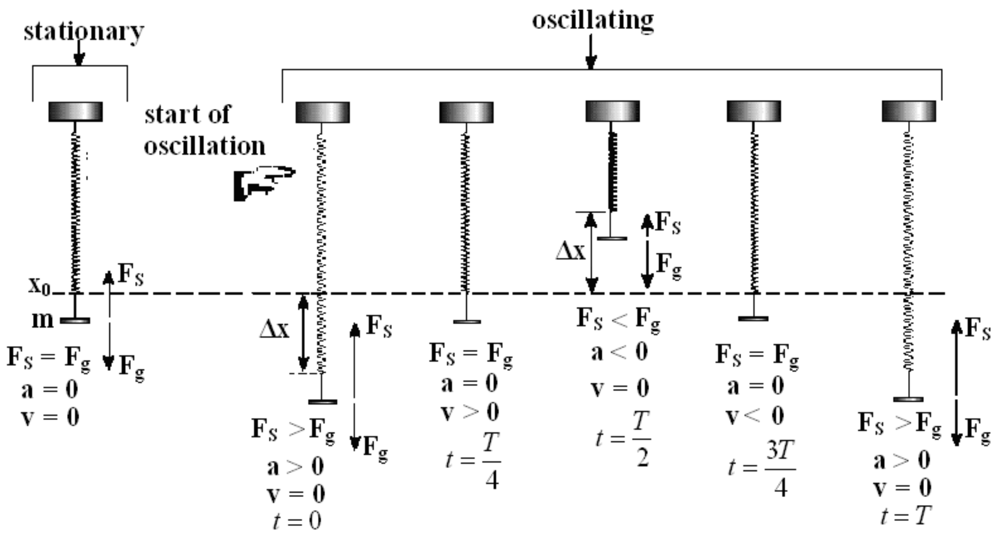


Figure 10.2: One cycle or period (τ) of an oscillation of a spring. Note that in the figure T is used instead of τ to indicate period and t is used as the length of time since the start of the oscillation. For example, the spring is at its maximum compression at time equal to half a period ($t = T/2$).

position, the motion will slow because the net force acting on the mass is now downward (i.e. the downward force due to gravity is constant while the upwardly directed spring force is getting smaller). The mass and spring will stop and then its downward acceleration will cause it to move back down again. The result of this is that the mass will oscillate around the equilibrium position. These steps and the forces (F), accelerations (a), and velocities (v) are illustrated in Fig. 10.2 for a complete cycle of an oscillation. The oscillation will proceed with a characteristic period, τ, which is determined by the spring constant, k, and the total attached mass, m. This period is the time it takes for the spring to complete one oscillation, or the time necessary to return to the point where the cycle starts repeating (the points where x, v, and a are the same). One complete cycle is shown in Fig. 10.2 and the time (t) of each position is indicated in terms of the period τ. The period, τ, of an oscillating spring is given by:

$$\tau = 2\pi\sqrt{\frac{m}{k}} \tag{10.3}$$

where k is the spring constant and m is the hanging mass, assuming the ideal case where the spring itself is massless. (For this lab the spring cannot be treated as massless so you will add $\frac{1}{3}$ of its weight to the hanging mass when calculating m used in Eq. 10.3.)

In order to determine the spring constant, k, from the period of oscillation, τ, it is convenient to square both sides of Eq. 10.3, giving:

$$\tau^2 = \frac{4\pi^2}{k}m \tag{10.4}$$

This equation has the same form as the equation of a line, $y = mx + b$, with a y-intercept of zero ($b = 0$). When plotting τ^2 vs. m the slope is related to the spring constant by:

$$slope = \frac{4\pi^2}{k} \tag{10.5}$$

So the spring constant can be determined by measuring the period of oscillation for different hanging masses. This is the second way that k will be determined today.

10.5 In today's lab

Today you will measure the spring constant (k) of a given spring in two ways. First, you will gradually add mass (m) to the spring and measure its displacement (Δx) when in equilibrium; then using Hooke's law and Eq. 10.2 you will plot F_S vs. Δx to find the spring constant. Second, you will measure the spring's period (τ) of oscillation for various hanging masses; then plot τ^2 vs. m and use Eq. 10.5 to find the spring constant in a different way. You will check whether the two values of k are consistent and if your spring obeyed Hooke's Law.

10.6 Equipment

- Spring
- Photogate
- Masses
- Hanger

10.7 Procedure

DO NOT LEAVE MASSES HANGING ON THE SPRING!

Part I: Hooke's Law

1. Measure the rest length (nothing on the end) of the spring and record it in your data sheet. (Don't remove it from the stand, just hold a meter stick up to it.)
2. Calculate the mass of the spring using the given spring density and the rest length of the spring. Record this value in your data sheet.
3. Record the mass of the hanger, $m_H = 50.0$ g, in your data sheet.
4. Attach the **empty** hanger to the bottom of the spring and measure the height X_0 of the end of the spring from the table. Make sure to put the zero end of the meter stick **on the table**. Choose a reasonable uncertainty for X_0.
5. Increase the **total mass** on the end of the spring to 120 g (this includes the mass of the hanger). Measure the height X of the spring and record it in your data sheet.
6. Increase the mass by 10 g increments, making sure to measure and record the height at each step, until you reach 220 g.
7. Calculate $\Delta m = m - m_H$, $\Delta X = X - X_0$, and $\delta(X - X_0) = 2\delta X_0$ for each trial.
8. Calculate the force of gravity pulling on the spring, $F_S = \Delta m g$, for each trial. We are using Δm, the amount of mass that was added to the hanger, because we measured the distance the spring stretched (ΔX) from the starting point of X_0, which was the height of the spring with the hanger on it.
9. Graph F_S vs. ΔX in KaleidaGraph. Include horizontal error bars and a best fit line.
10. Use your graph to verify Hooke's Law: $F_S = -k\Delta X$. The slope and its uncertainty are related to the spring constant k.

Part II: Period of Oscillation

1. Turn the photogate on and set it to the `PEND` setting.

2. Start with a total mass of 120 g on the end of the spring and measure the period of oscillation τ by causing the masses to oscillate through the photogate. **Do not stretch the spring more than 5 cm** when starting the oscillation (2 cm is enough) and pull straight down so the spring isn't swinging while oscillating. Repeat in 20 g intervals up to 220 g. You can adjust the height of the photogate and the height of the spring to align the equilibrium position with the photogate. Make sure the red light on the photogate flashes each time the mass passes through the gate.

3. You need to take into account the mass of the spring (as this is not an ideal case and the spring can't be considered massless) when calculating the total mass m felt by the spring in Eq. 10.4. To do that add a third of the spring's mass (which you calculated at the top of the Excel spreadsheet) to the hanging mass using the formula $m = m_H + \Delta m + \frac{\text{spring mass}}{3}$ in Excel. (Note that this is a different m than you used in Part 1.)

4. Calculate τ^2 in Excel for each trial.

5. Make a plot of τ^2 vs. m in KaleidaGraph. Be sure to include a best fit line on this plot.

6. Use Eq. 10.5 and the slope from your graph to calculate the spring constant k. The uncertainty δk is found using:

$$\frac{\delta k}{k} = \frac{\delta \text{slope}}{\text{slope}} \tag{10.6}$$

10.8 Checklist

1. Excel spreadsheet and formula view

2. Plot of F_S vs. ΔX with error bars.

3. Plot of τ^2 vs. m

4. Answers to questions

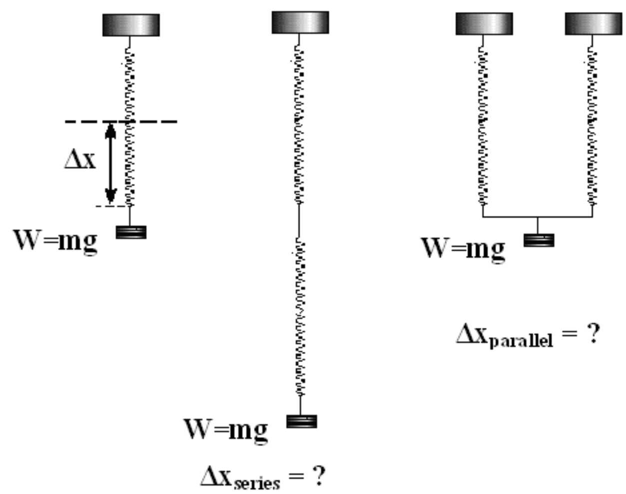
10.9 Questions

1. From Part I: Is your data consistent with Hooke's Law? Discuss why or why not.

2. From Part I: Using your graph, what is the spring constant and its uncertainty?

3. From Part II: Calculate the spring constant and its uncertainty using the information obtained from your graph of τ^2 vs. m. Use Eq. 10.4 for δk.

4. You obtained the spring constant in two independent ways. Discuss the consistency of your two measurements of the spring constant. If they are not consistent, give possible reasons why they are not.



5. When a mass m is attached to a spring it exerts a force $W = mg$ on the spring and the length of the spring is changed by Δx. If the single spring is replaced with a) two identical springs in series, what happens to Δx_{series} compared to the case of a single spring? b) If the single spring is replaced by two identical springs in parallel, what happens to $\Delta x_{\text{parallel}}$ compared to the case of a single spring? See figure above. Assume all springs are identical, i.e. have the same spring constant k, length, mass, etc. Answer questions a) and b) by stating if Δx increases, decreases or remains unchanged and compare it to the single spring case, i.e. what are Δx_{series} and $\Delta x_{\text{parallel}}$ in terms of Δx for the single spring case? **Hint:** Draw a force diagram of the system remembering that the net force on the mass must be zero when it is in equilibrium.