PHY410 Homework Set 4

1. [10 pts] Kittel-Kroemer, problem 3-6.

2. [5 pts] Kittel-Kroemer, problem 3-8.

3. [10 pts] The following pertains to a gas of photons in different number of dimensions.

   (a) Compute the total number of photons within a macroscopic cavity of volume V maintained at temperature τ.

   (b) Show that for the gas of photons satisfies an equation of state $P\,V = \alpha\,N\,\tau$ and determine the corresponding numerical coefficient α.

   (c) Consider next a narrow transmission line of length L, within which the electromagnetic waves satisfy the one-dimensional wave equation $v^2\,\partial_x^2\,E = \partial_t^2\,E$, where E is an electric field component. Find the heat capacity of the photons for that line, when it is in thermal equilibrium at temperature τ. The enumeration of independent modes proceeds in the usual way for one dimension: take the solutions as standing waves with zero amplitude at each end of the line, just as in the case of a one-dimensional Schrödinger equation.

4. [5 pts] Consider now the case of a single photon mode at frequency ω within a cavity held at temperature τ. Demonstrate that the entropy for that mode can be expressed in terms of the average photon number $\langle s \rangle$, as $\sigma = \langle s+1 \rangle\,\log\langle s+1 \rangle - \langle s \rangle\,\log\langle s \rangle$. It is convenient to start from the partition function.

5. [5 pts] Kittel-Kroemer, problem 4-3.