

3. [10 pts] The following pertains to a gas of photons in different number of dimensions.
   
   (a) Compute the total number of photons within a macroscopic cavity of volume $V$ maintained at temperature $\tau$.
   
   (b) Show that for the gas of photons satisfies an equation of state $PV = \alpha N \tau$ and determine the corresponding numerical coefficient $\alpha$.
   
   (c) Consider next a narrow transmission line of length $L$, within which the electromagnetic waves satisfy the one-dimensional wave equation $v^2 \frac{\partial^2 E}{\partial t^2} = \frac{\partial^2 E}{\partial x^2}$, where $E$ is an electric field component. Find the heat capacity of the photons for that line, when it is in thermal equilibrium at temperature $\tau$. The enumeration of independent modes proceeds in the usual way for one dimension: take the solutions as standing waves with zero amplitude at each end of the line, just as in the case of a one-dimensional Schrödinger equation.

4. [5 pts] Consider now the case of a single photon mode at frequency $\omega$ within a cavity held at temperature $\tau$. Demonstrate that the entropy for that mode can be expressed in terms of the average photon number $\langle s \rangle$, as $\sigma = \langle s + 1 \rangle \log \langle s + 1 \rangle - \langle s \rangle \log \langle s \rangle$. It is convenient to start from the partition function.