Homework Assignment 1 due Friday January 23.

1. Enumerate all symmetric, orthonormal basis states for N = 3 identical bosons. Write the basis wave functions $\Phi_{\{n\}}(x_1\ x_2\ x_3)$ in terms of the single-particle wave functions $\psi_{Ei}(\mathbf{x})$ where i = 1 2 3 … ∞ and {n} = the list of occupation numbers. { Note: $\mathbf{x} = (x,y,z)$ }

2. Consider N identical spin 0 particles. The first quantized Hamiltonian is

$$H = \frac{-\hbar^2}{2m} \sum_{k=1}^{N} \nabla_k^2 + \frac{1}{2} \sum_{k,l}{}' V(x_k, x_l)$$

Use the eigenstates of ∇^2 for the single particle states: the quantum numbers are $E = (p_x, p_y, p_z)$; the single particle wave functions are $\psi_E(\mathbf{x}) = 1/\mathrm{sqrt}(V) \exp\{i\mathbf{p}.\mathbf{x}/\hbar\}$; $\mathbf{p} = (2\pi\hbar/L)\ \mathbf{n}$ where the components of $\mathbf{n}$ are integers; $V=L^3$.

Also, assume the interaction potential is $V(x_k , x_l) = \underline{V}(\mathbf{x}_k - \mathbf{x}_l)$.

The second quantized Hamiltonian is

$$\hat{H} = \sum_{\vec{p}} \frac{p^2}{2m} b_{\vec{p}}^{\dagger} b_{\vec{p}} + \hat{V}$$

Derive the formula for **V**, in terms of the Fourier transform of $\underline{V}(\mathbf{r})$.

3. Prove that **N** and **H** commute, using the field theoretic formulas (2.9) and (2.4). Prove it for both bosons and fermions.

4. Derive the formula for the Fermi energy E_F for N spin-½ particles confined in a volume V at T = 0.