Chapter 3 : GREEN'S FUNCTIONS AND FIELD THEORY (FERMIONS)

Review

9. DIAGRAMMATIC ANALYSIS OF PERTURBATION THEORY

9a. Feynman diagrams in coordinate space

9b. Feynman diagrams in momentum space

9c. Dyson's equations

9d. Goldstone's theorem

9a. Feynman diagrams in coordinate space

$$iG_{\alpha\beta}(x,y) = \sum_{\nu=0}^{\infty}\left(\frac{-i}{\hbar}\right)^{\nu}\frac{1}{\nu!}\int_{-\infty}^{\infty}dt_1\cdots dt_\nu$$

$$\frac{\langle\Phi_0|T[\hat{H}_1(t_1)\ldots\hat{H}_1(t_\nu)\hat{\psi}_\alpha(x)\hat{\psi}_\beta^{+}(y)|\Phi_0\rangle}{\langle\Phi_0|\hat{S}|\Phi_0\rangle}$$

$$= i\tilde{G}_{\alpha\beta}(x,y)/\langle\Phi_0|\hat{S}|\Phi_0\rangle$$

where

$$\hat{H}_1(t) = \psi^{+}_{\lambda_1'}(x_1)\psi^{+}_{\lambda_2'}(x_2)\frac{1}{2}V(\vec{x}_1\vec{x}_2)_{(\lambda_1'\lambda_2'\lambda_1\lambda_2)}\psi_{\lambda_2}(x_2)\psi_{\lambda_1}(x_1)\,d^3x_1\sum_{\lambda_1\lambda_1'}d^3x_2\sum_{\lambda_2\lambda_2'}$$

$$(t = t_{k_1} = t_{k_2})$$

In first order perturbation theory

$$i\tilde{G}_{\alpha\beta}(x,y) = iG^{0}_{\alpha\beta}(xy) + \left(\frac{-i}{\hbar}\right)\sum_{\lambda_1\lambda_1'}\sum_{\lambda_2\lambda_2'}\int d^4x_1\,d^4x_2\,\frac{1}{2}V(\vec{x}_1\vec{x}_2)_{(\lambda_1'\lambda_2'\lambda_1\lambda_2)}\,\delta(t_1-t_2)$$

$$\langle\Phi_0|T[\psi^{+}_{\lambda_1'}(x_1)\psi^{+}_{\lambda_2'}(x_2)\psi_{\lambda_2}(x_2)\psi_{\lambda_1}(x_1)\psi_\alpha(x)\psi^{+}_\beta(y)|\Phi_0\rangle$$

use Wick's theorem to calculate this

= sum of complete contractions; recall that only $c(\psi,\psi^+)$ is nonzero

$\therefore\ \exists$ 6 terms = A B C D E F

$$\langle \Phi_0 | T[\psi^+_{\lambda_1'}(x_1)\,\psi^+_{\lambda_2'}(x_2)\,\psi_{\lambda_2}(x_2)\,\psi_{\lambda_1}(x_1)\,\psi_\alpha(x)\,\psi^+_\beta(y)] | \Phi_0 \rangle$$

2 like this = A, B

2 like this = C, D

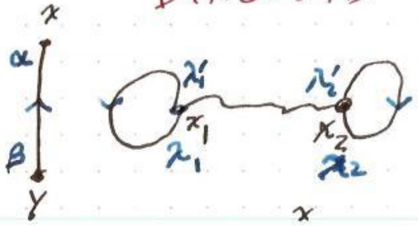
2 like this = E, F

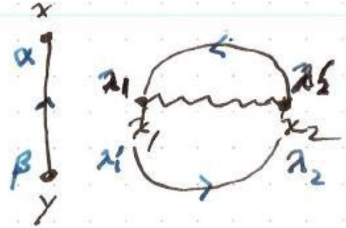
FEYNMAN DIAGRAMS

Terms A and B

$$= iG^0_{\alpha\beta}(xy)\left[\, iG^0_{\lambda_1\lambda_1'}(x_1,x_1)\; iG^0_{\lambda_2\lambda_2'}(x_2x_2) \right. \quad (A)$$

$$\left. -\, i\,G^0_{\lambda_2\lambda_1'}(x_2x_1)\; iG^0_{\lambda_1\lambda_2'}(x_1x_2)\right] \quad (B)$$





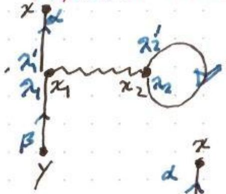
These are *disconnected* graphs;
the fermion lines with propagation from (yβ) to (xα)
are not connected to the interactions.
They will be canceled by terms in the denominator.

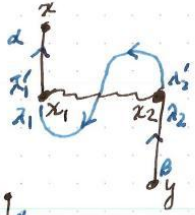
FEYNMAN DIAGRAMS

Terms C and D

$$= iG^0_{\alpha\lambda_1'}(x x_1)\Big[\cancel{\pm}\, iG^0_{\lambda_1\beta}(x_1 y)\; iG^0_{\lambda_2\lambda_2'}(x_2,x_2) \quad (C)$$

$$+\; iG^0_{\lambda_2\beta}(x_2 y)\; iG^0_{\lambda_1\lambda_2'}(x_1 x_2)\Big] \quad (D)$$



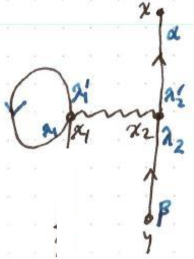


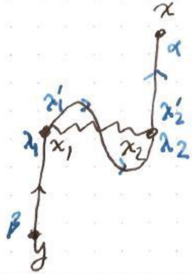
These are connected graphs.

Terms E and F

$$= iG^0_{\alpha\lambda_2'}(x x_2)\Big[\; iG^0_{\lambda_2\beta}(x_2 y)\; iG^0_{\lambda_1\lambda_1'}(x_1 x_1) \quad E)$$

$$-\; iG^0_{\lambda_1\beta}(x_1 y)\; iG^0_{\lambda_2\lambda_1'}(x_2 x_1)\Big] \quad F)$$





Note that C and E differ only by the exchange of x_1 and x_2. But we integrate over x_1 and x_2, and $V(x_1,x_2) = V(x_2,x_1)$. Thus C = E. Keep one diagram and cancel the factor of ½.

Similarly, D = F.

Factorization and Cancellation of disconnected diagrams

$$iG_{\alpha\beta}(xy) = i\tilde{G}_{\alpha\beta}(xy) \Big/ \langle \Phi_0 | U(\infty,-\infty) | \Phi_0 \rangle$$

$$= \frac{(\text{connected graphs}) \times (\text{vacuum graphs})}{(\text{vacuum graphs})}$$

(vacuum graphs = Feynman diagrams with ~~no~~ external lines)

= connected graphs of $\tilde{G}$

$$iG_{\alpha\beta}(xy) = \sum_{\nu=0}^{\infty}\left(\frac{-i}{\hbar}\right)^{\nu}\frac{1}{\nu!}\int_{-\infty}^{\infty} dt_1 \ldots dt_\nu$$

$$\left.\langle \Phi_0 | T[\hat{H}_1(t_1)\ldots\hat{H}_1(t_\nu)\,\hat{\psi}_\alpha(x)\,\hat{\psi}^+_\beta(y)] | \Phi_0\rangle\right|_{\text{Connected}}$$

⊕ $\nu!$ topologically equivalent graphs (exchanges of integrations variables) $\Rightarrow$ cancel the $\frac{1}{\nu!}$ and calculate only one topologically distinct case.

Feynman rules in coordinate space

To calculate $G_{\alpha\beta}(xy)$ …

R1. Draw all topologically distinct connected graphs with one entering line and one exiting line.

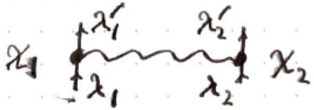
R2. Vertices -- spacetime position; integrate d^4x for all internal vertices.

$$x = (\vec{x}, t)$$

R3. Fermion lines -- free propagator function

$$= G^0_{\alpha\beta}(x,y)$$

R4. Interaction lines -- 2-body potential energy



$$= V_{\lambda_1'\lambda_2',\lambda_1\lambda_2}(\vec{x}_1, \vec{x}_2)\,\delta(t_1 - t_2)$$

R5. Perturbation order factor = $(i/\hbar)^n$

$$(-i)(-i/\hbar)^n (i)^{2n+1} = \left(\frac{i}{\hbar}\right)^n$$

R6. Overall sign = ± ;

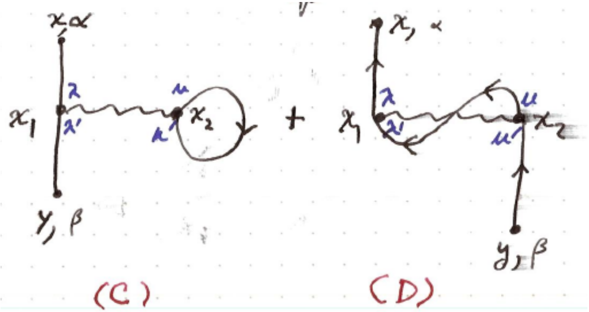
probably need to go back to Wick's theorem to figure it out.

9b. Feynman diagrams in momentum space

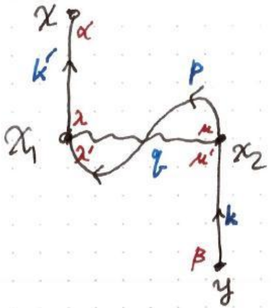
Now let's go back to calculate the 1-particle Green's function.

$G_{\alpha\beta}(x,y)$ where $x = (\mathbf{x},t_x)$ and $y = (\mathbf{y},t_y)$

$G_{\alpha\beta}(x,y) = G^0_{\alpha\beta}(x,y) + G^{(1)}_{\alpha\beta}(x,y)$

$G^{(1)}_{\alpha\beta}(x,y) =$ 

D: the irreducible diagram



$$D = \frac{i}{\hbar}\int d^4x_1\, d^4x_2\; G^0_{\alpha\lambda}(x\, x_1)\, U^{(x_1,x_2)}_{\lambda\lambda'\mu\mu'}\, G^0_{\lambda'\mu}(x_1 x_2)\, G^0_{\mu'\beta}(x_2 y)$$

Now introduce Fourier transforms

$$G^0_{\alpha\beta}(x\, y) = \int \frac{d^4k}{(2\pi)^4}\, e^{ik\cdot(x-y)}\, \bar{G}^0_{\alpha\beta}(k)$$

$$k = (\vec{k}, \omega)$$

$$k\cdot(x-y) = \vec{k}\cdot(\vec{x}-\vec{y}) - \omega(t_x - t_y)$$

$$D = \frac{i}{\hbar}\int d^4x_1\, d^4x_2 \int \frac{d^4k'}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\; \bar{G}^0_{\alpha\lambda}(k')\, \bar{U}_{\lambda\lambda'\mu\mu'}(q)\, \bar{G}^0_{\lambda'\mu}(p)\, \bar{G}^0_{\mu'\beta}(k)\; e^{ik'\cdot(x-x_1)}\, e^{iq\cdot(x_1-x_2)}\, e^{ip\cdot(x_1-x_2)}\, e^{ik\cdot(x_2-y)}$$

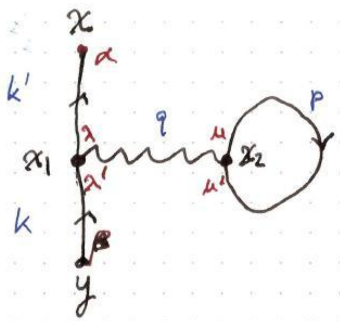
$$\left.\begin{array}{l} \int d^4x_1 \Rightarrow (2\pi)^4\, \delta^4(-k'+q+p) \\ \int d^4x_2 \Rightarrow (2\pi)^4\, \delta^4(-q-p+k) \end{array}\right\}\; q = k-p \text{ and } k' = k$$

$$D = \int \frac{d^4k}{(2\pi)^4}\, e^{ik\cdot(x-y)}\, \bar{G}^{(1D)}_{\alpha\beta}(k)$$

where

$$\bar{G}^{(1D)}_{\alpha\beta}(k) = \frac{i}{\hbar}\, \bar{G}^0_{\alpha\lambda}(k)\left[\int \frac{d^4p}{(2\pi)^4}\, \bar{U}_{\lambda\lambda'\mu\mu'}(k-p)\, \bar{G}^0_{\lambda'\mu}(p)\right] \bar{G}^0_{\mu'\beta}(k) \qquad (1D)$$

C: the tadpole diagram



$$C = \frac{-i}{\hbar}\int d^4x_1\, d^4x_2\, G^0_{\alpha\lambda}(x x_1)\, U_{\lambda\lambda'\mu\mu'}(x_1 x_2)\, G^0_{\lambda'\beta}(x_1 y)\, G^0_{\mu\mu'}(x_2 x_2)$$

$$= \frac{-i}{\hbar}\int d^4x_1\, d^4x_2 \int \frac{d^4k'}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\, \bar{G}^0_{\alpha\lambda}(k')\, \bar{U}_{\lambda\lambda'\mu\mu'}(q)\, \bar{G}^0_{\lambda'\beta}(k)\, \bar{G}^0_{\mu\mu'}(p)\, e^{ik'\cdot(x-x_1)}\, e^{iq\cdot(x_1-x_2)}\, e^{ik\cdot(x_1-y)}\, e^{ip\cdot(x_2-x_2)}$$

$$\left.\begin{array}{l} d^4x_1 \Rightarrow (2\pi)^4\, \delta^4(-k'+q+k) \\ d^4x_2 \Rightarrow (2\pi)^4\, \delta^4(-q) \end{array}\right\} \quad q=0 \text{ and } k'=k$$

$$C = \int \frac{d^4k}{(2\pi)^4}\, e^{ik\cdot(x-y)}\, \bar{G}^{(1C)}_{\alpha\beta}(k)$$

$$\bar{G}^{(1C)}_{\alpha\beta}(k) = \frac{-i}{\hbar}\, \bar{G}^0_{\alpha\lambda}(k)\left[\bar{U}_{\lambda\lambda'\mu\mu'}(0)\int \frac{d^4p}{(2\pi)^4}\, \bar{G}^0_{\mu\mu'}(p)\right] \bar{G}^0_{\lambda'\beta}(k) \qquad (1C)$$

Compare (1C) and (1D) to Eq. 9.15.

$$\bar{G}^{(1D)}_{\alpha\beta}(k) = \frac{i}{\hbar}\bar{G}^{0}_{\alpha\lambda}(k)\left[\int\frac{d^4p}{(2\pi)^4}\bar{U}_{\lambda\lambda'\mu\mu'}(k-p)\;\bar{G}^{0}_{\lambda'\mu}(p)\right]\bar{G}^{0}_{\mu'\beta}(k) \quad (1D)$$

$$\bar{G}^{(1C)}_{\alpha\beta}(k) = \frac{-i}{\hbar}\bar{G}^{0}_{\alpha\lambda}(k)\left[\bar{U}_{\lambda\lambda'\mu\mu'}(0)\int\frac{d^4p}{(2\pi)^4}\bar{G}^{0}_{\mu\mu'}(p)\right]\bar{G}^{0}_{\lambda'\beta}(k) \quad (1C)$$

<u>Spin sums</u>

Recall $G^0_{\alpha\beta}(xy) = \delta_{\alpha\beta}\, G^0(xy)$

Also $\bar{G}^0_{\alpha\beta}(k) = \delta_{\alpha\beta}\,\bar{G}^0(k)$

For case (1D),

$\delta_{\alpha\lambda}\,\bar{U}_{\lambda\lambda'\mu\mu'}\,\delta_{\lambda'\mu}\,\delta_{\mu'\beta} = \bar{U}_{\alpha\mu\mu\beta}$

For case (1C),

$\delta_{\alpha\lambda}\,\bar{U}_{\lambda\lambda'\mu\mu'}\,\delta_{\mu\mu'}\,\delta_{\lambda'\beta} = \bar{U}_{\alpha\beta\mu\mu}$

$$\boxed{\begin{aligned}\bar{G}^{(1)}_{\alpha\beta}(k) = \frac{i}{\hbar}\left[\bar{G}^0(k)\right]^2 \;\cdot \\ \times\left[\int\frac{d^4p}{(2\pi)^4}\bar{U}_{\alpha\mu\mu\beta}(k-p)\,\bar{G}^0(p) - \int\frac{d^4p}{(2\pi)^4}\bar{G}^0(p)\,\bar{U}_{\alpha\beta\mu\mu}(0)\right]\end{aligned}}$$

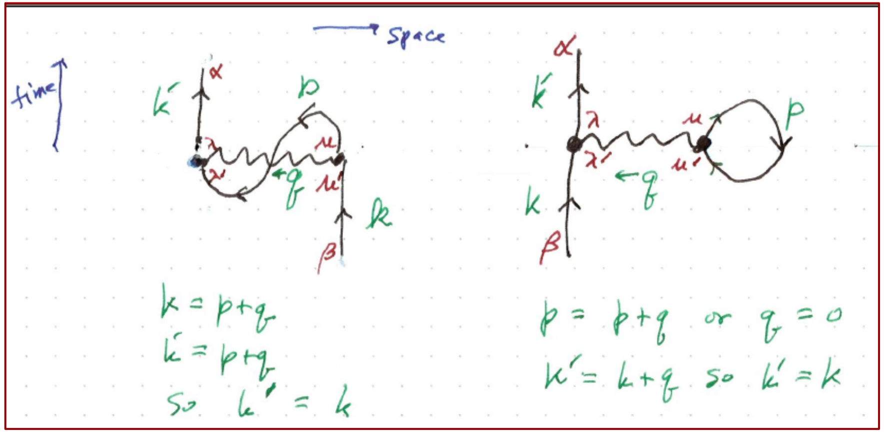
Compare to Eq. (9.15).

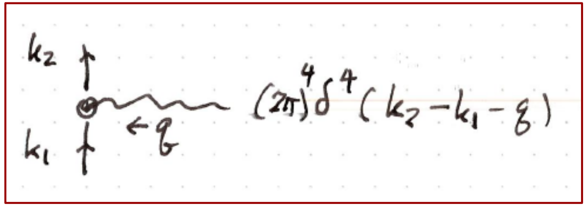
Feynman rules in momentum space

To calculate $G_{\alpha\beta}(k)$ ...

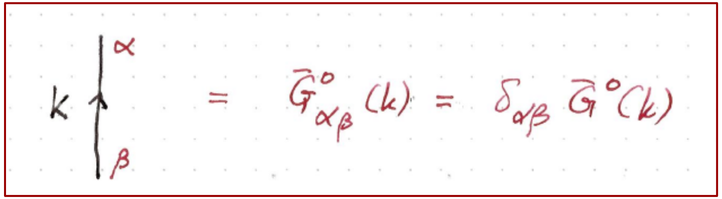
R1'. Draw all topologically distinct connected graphs with one entering line and one exiting line.

R2'. Vertices -- momentum/energy transformations; *momentum and energy are conserved at each vertex.*

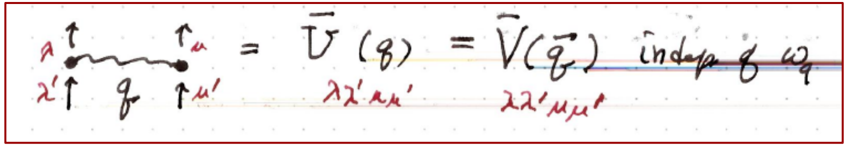




R3'. Fermion lines -- free propagator function



R4'. Interaction lines -- 2-body potential energy



R5'. Perturbation order factor = $(i/\hbar)^n$

R6'. Overall sign = ± ;

probably need to go back to Wick's theorem to figure it out.

9c. Dyson's equations

First order perturbation theory won't be an adequate approximation, if there are important interactions.

(In most problems of physical interest there are important interactions.)

But with the help of the diagrammatic analysis we can do better: we can add together whole classes of diagrams, to all orders in perturbation theory.

It is not an exact solution because we are still neglecting some higher order diagrams. But since we are including whole classes of higher order diagrams it should improve on fixed order perturbation theory.

Similar methods have been developed for relativistic Q.F.T. ; e.g., "renormalization group"; or , "resummations".

We'll consider two examples of Dyson's equations:

(1) Self energy insertions; these apply to the 1-particle Green's function

(2) Polarization insertions; these apply to the 2-particle Green's function.

… to be continued

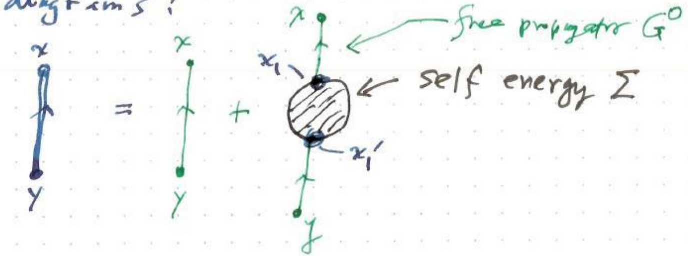
Self-energy insertions

The 1-particle Green's function has 2 external vertices

$$G_{\alpha\beta}(xy) = -i\langle\Psi_0|T[\psi_\alpha(x)\psi^+_\beta(y)]|\Psi_0\rangle$$

it describes the propagation of a particle from y to x in the interacting theory.

The perturbation theory expansion in diagrams:



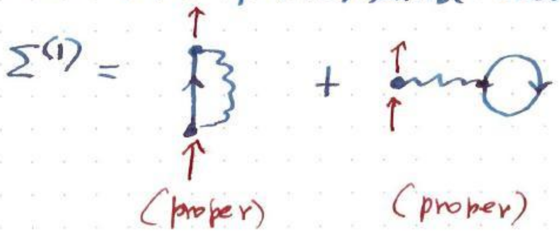
$$G_{\alpha\beta}(xy) = G^0_{\alpha\beta}(xy) + \int d^4x_1\, d^4x_1'\; G^0_{\alpha\lambda}(x x_1)\, \Sigma_{\lambda\mu}(x_1 x_1')\, G^0_{\mu\beta}(x_1' y)$$

A self energy insertion is a graph with two open vertices.

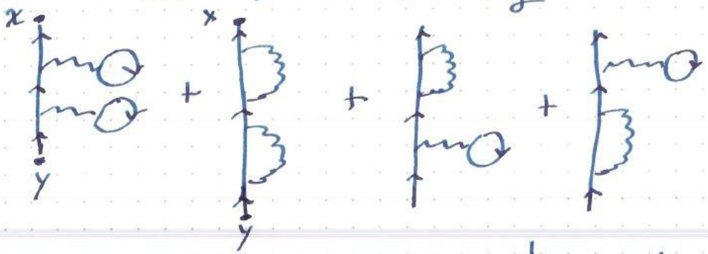
It represents the effect of interacting on the propagation.

Define a <u>proper</u> self energy insertion = a self energy insertion that cannot be separated into 2 parts by cutting one internal particle line.

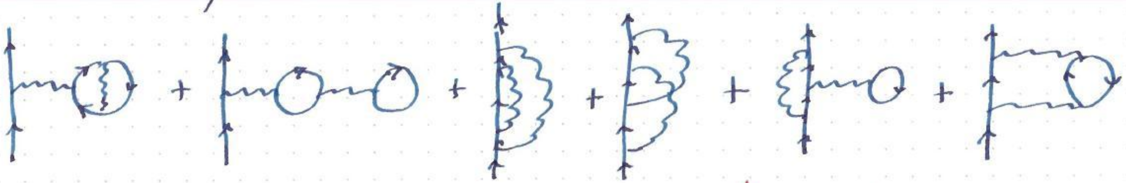
§ First order perturbation theory



Second order perturbation theory

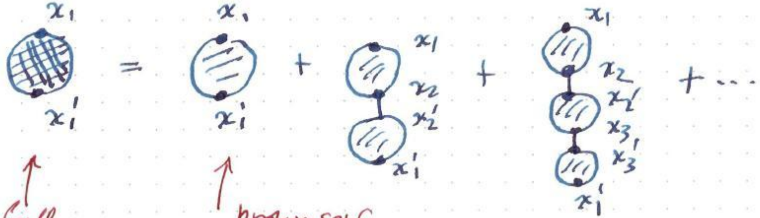


← improper S.E. insertions
(4)



↑ proper S.E. insertions
(6)

Let Σ^* = the sum of all <u>proper</u> S.E. insertions.



full self energy

proper self energy

$$\Sigma = \Sigma^* + \Sigma^* G^0 \Sigma^* + \Sigma^* G^0 \Sigma^* G^0 \Sigma^* + \dots$$

$$G(xy) = G^0(xy) + \int d^4x_1\, d^4x_1'\, G^0(xx_1)\, \Sigma^*(x_1 x_1')\, G^0(x_1' y)$$

$$+ \int d^4x_1\, d^4x_1'\, d^4x_2\, d^4x_2'$$

$$G^0(xx_1)\, \Sigma^*(x_1x_1')\, G^0(x_1'x_2)\, \Sigma^*(x_2x_2')\, G^0(x_2' y)$$

$$+ \cdots$$

$$\underset{\alpha\beta}{G(xy)} = \underset{\alpha\beta}{G^0(xy)} + \int d^4x_1\, d^4x_1'\, \underset{\alpha\lambda}{G^0(xx_1)}\, \underset{\lambda\mu}{\Sigma^*(x_1x_1')}\, \underset{\mu\beta}{G(x_1' y)}$$

For a translation invariant system, make the transformation to momentum/energy space

$$G_{\alpha\beta}(xy) = \int \frac{d^4k}{(2\pi)^4}\, e^{ik\cdot(x-y)}\, \bar{G}_{\alpha\beta}(k)$$

$$\Sigma^*_{\lambda\mu}(xx') = \int \frac{d^4k}{(2\pi)^4}\, e^{ik\cdot(x-x')}\, \bar{\Sigma}^*_{\lambda\mu}(k)$$

etc.

$$\Rightarrow \quad \bar{G}_{\alpha\beta}(k) = \bar{G}^0_{\alpha\beta}(k) + \bar{G}^0_{\alpha\lambda}(k)\, \bar{\Sigma}^*_{\lambda\mu}(k)\, \bar{G}_{\mu\beta}(k)$$

and this is just a matrix equation in the spin indices.

Wed Feb 4

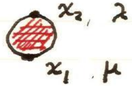
Dyson's Equations

(1) The Self-energy

$$G_{\alpha\beta}(xy) = -i \langle \Psi_0 | T[\psi_\alpha(x)\, \psi^\dagger_\beta(y)] | \Psi_0 \rangle$$

=  = the complete propagator

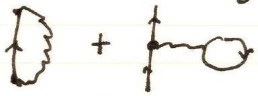
= the sum of Feynman diagrams with one incoming fermion and one outgoing fermion

=  where  = the sum of self energy insertions

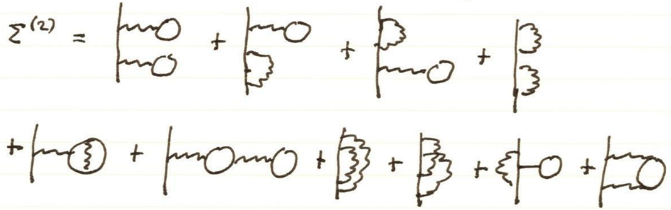
$$= G^0_{\alpha\beta}(xy) + \int d^4x_1\, d^4x_2\, G^0_{\alpha\lambda}(xx_2)\, \Sigma_{\lambda\mu}(x_2x_1)\, G^0_{\mu\beta}(x_1y)$$

($\sum_\lambda \sum_\mu$ implied)

First order perturbation theory

$\Sigma^{(1)}$ = 

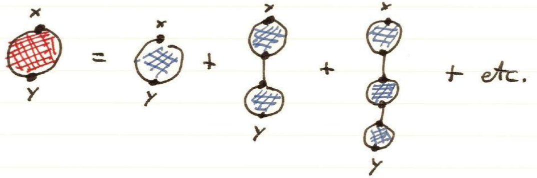
Second order perturbation theory,

$\Sigma^{(2)}$ = 

= 4 improper s.E. insertions

+ 6 proper s.E. insertions.

Let Σ^* = the sum of all proper S.E. insertions = 



$$\Sigma = \Sigma^* + \Sigma^* G^0 \Sigma^* + \Sigma^* G^0 \Sigma^* G^0 \Sigma^* + \cdots$$

$$= \Sigma^* \{ 1 + G^0\Sigma^* + G^0\Sigma^* G^0\Sigma^* + \cdots \}$$

Now

$$G = G^0 + G^0 \Sigma G^0$$

$$= G^0 + G^0 \Sigma^* \underbrace{\{1 + G^0\Sigma^* + G^0\Sigma^* G^0\Sigma^* + \cdots\} G^0}_{G}$$

$$G = G^0 + G^0 \Sigma^* G$$

That is, in coordinate space

$$G_{\alpha\beta}(xy) = G^0_{\alpha\beta}(xy) + \int G^0_{\alpha\lambda}(xx_2)\, \Sigma^*_{\lambda\mu}(x_2x_1)\, G_{\mu\beta}(x_1y)$$

Or, in momentum space

$$\bar{G}_{\alpha\beta}(k) = \bar{G}^0_{\alpha\beta}(k) + \bar{G}^0_{\alpha\lambda}(k)\, \bar{\Sigma}^*_{\lambda\mu}(k)\, G_{\mu\beta}(k)$$

($\sum_\lambda \sum_\mu$ is implied)

We have $\bar{G}^0_{\alpha\beta}(k) = \delta_{\alpha\beta}\, \bar{G}^0(k)$

Suppose also $\bar{\Sigma}^*_{\lambda\mu}(k) = \delta_{\lambda\mu}\, \bar{\Sigma}^*(k)$

Then also $\bar{G}_{\alpha\beta}(k) = \delta_{\alpha\beta}\, \bar{G}(k)$

$$\delta_{\alpha\lambda}\,\delta_{\lambda\mu}\,\delta_{\mu\beta} = \delta_{\alpha\beta}$$

where

$$\bar{G}(k) = \bar{G}^0(k) + \bar{G}^0(k)\, \bar{\Sigma}^*(k)\, \bar{G}(k)$$

Solve it $\Rightarrow$

$$\bar{G}(k) = \frac{\bar{G}^0(k)}{1 - \bar{G}^0(k)\bar{\Sigma}^*(k)}$$

or $$\bar{G}(k) = \frac{1}{[\bar{G}^0(k)]^{-1} - \bar{\Sigma}^*(k)}$$

Now recall the free Green's function

$$\bar{G}^0(k) = \frac{\theta(k-k_F)}{\omega-\omega_k+i\eta} + \frac{\theta(k_F-k)}{\omega-\omega_k-i\eta}$$

η = a positive infinitesimal

So

$$[\bar{G}^0(k)]^{-1} = \theta(k-k_F)(\omega-\omega_k+i\eta) + \theta(k_F-k)(\omega-\omega_k-i\eta)$$

$$= \omega-\omega_k + i\eta\,\epsilon(k-k_F)$$

$\epsilon(k) = \begin{cases} +1 \text{ if } k>0 \\ -1 \text{ if } k<0 \end{cases}$

another step function

$$G_{\alpha\beta}(k) = \frac{\delta_{\alpha\beta}}{\omega-\omega_k-\Sigma^*(k) \quad \underbrace{+i\eta\epsilon}}$$

may not be necessary; if Σ^* has an imaginary part

Notation:

$k = (\vec{k}, \omega)$

and $\omega_{\vec{k}} = \frac{\epsilon_k^0}{\hbar} = \frac{\hbar^2 k^2/2m}{\hbar}$

The Lehmann representation

(sec 7)

The singularities of $G_{\alpha\beta}(k)$ in the complex ω plane occur at the excitation energies (/ħ) of the interacting system. The damping factor of an excited state is the imaginary part of the pole position.

"Quasi particles" = single particle excitations

$$G(\vec{k},t) \approx -ia\, e^{-i\epsilon_k t/\hbar}\, e^{-\gamma_k t}$$

(eq. 7.79)

$$\epsilon_k = \epsilon_k^0 + \mathrm{Re}\,\hbar\,\Sigma^*(\vec{k}, \epsilon_k/\hbar)$$

2nd term = "self" energy

$$\gamma_k = \frac{\mathrm{Im}\,\Sigma^*(\vec{k}, \epsilon_k/\hbar)}{1 - \partial\,\mathrm{Re}\,\Sigma^*/\partial\omega \text{ at } \omega = \epsilon_k/\hbar}$$

Problem 3.14

$\boxed{\text{The poles of the Green's function}}$ $G(\vec{k},\omega)$

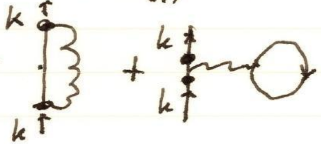
$$G_{\alpha\beta}(\vec{k},\omega) = \frac{1}{\omega - \epsilon_k^0/\hbar - \Sigma^*(k,\omega)}\,\delta_{\alpha\beta}$$

$$\epsilon_k^0 = \hbar^2 k^2/2m$$

In first order perturbation theory,

$$\epsilon_k^{(1)} = \epsilon_k^0 + \hbar\,\Sigma^*_{(1)}(\vec{k})$$

$\hbar\,\Sigma^*_{(1)}(\vec{k}) =$



$$= n V_0(0) - \frac{1}{(2\pi)^3}\int d^3k' \left[V_0(\vec{k}-\vec{k}') + 3V_1(\vec{k}-\vec{k}') \right]$$

constant energy shift from the tadpole diagram

real self energy; no damping in 1st order

F.W. problem 3.12

Calculate the second order contribution to the proper self energy (6 Feynman diagrams).

Then

$$\frac{E^{(2)}}{V} = \frac{2m}{\hbar^2}\int \frac{d^3k\, d^3p\, d^3l\, d^3n}{(2\pi)^{12}}$$

$$\times (2\pi)^3\, \delta^3(\vec{k}+\vec{p}-\vec{l}-\vec{n})$$

$$\times \theta(k_F-p)\,\theta(k_F-k)\,\theta(n-k_F)\,\theta(l-k_F)$$

$$\times \frac{2V_0^2(l-k) - V_0(l-k)\,V_0(p-l)}{p^2+k^2-l^2-n^2+i\eta}$$

F.W. problem 3.14

$$\epsilon_k = \epsilon_k^0 + \mathrm{Re}\, \hbar\Sigma^*(\vec{k},\omega_k)$$

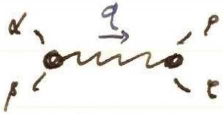
$$\gamma_k = \frac{\mathrm{Im}\,\Sigma^*(\vec{k},\omega_k)}{1 - \left[\partial(\mathrm{Re}\,\Sigma^*)/\partial\omega\right]_{\omega=\omega_k}}$$

Polarization insertions

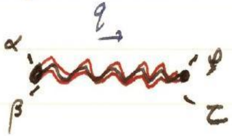
Dyson's Equations

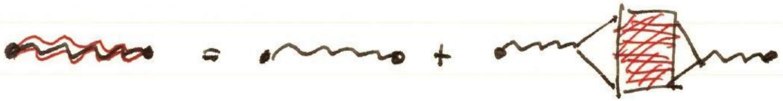
(2) The polarization insertion. $\Pi_{\mu\nu\eta\lambda}(q)$

To sum an infinite set of diagrams, we can define a "perturbed interaction"

The interaction = 

$= U_{\alpha\beta\rho\tau}(q)$

Replace that by 



$$\boldsymbol{U}(q) = U_0(q) + U_0(q)\,\boldsymbol{\Pi}(q)\,U_0(q)$$

Define proper polarization insertions Π^*

$$U(q) = U_0(q) + U_0\,\Pi^*\,U(q)$$

similar to the treatment of self energy insertions.

9d. Goldstone's theorem

We probably won't use this.

FW point out that "Goldstone diagrams" are more like old fashioned perturbation theory than Feynman-Dyson diagrams.

One Feynman-Dyson diagram is equal to the sum of several Goldstone diagrams.

So the Feynman-Dyson methods are more powerful in principle.

Nevertheless, FW sometimes use Goldstone diagrams later in the book. We'll skip this for now and come back to it if necessary.

So now we'll proceed to Chapter 4.