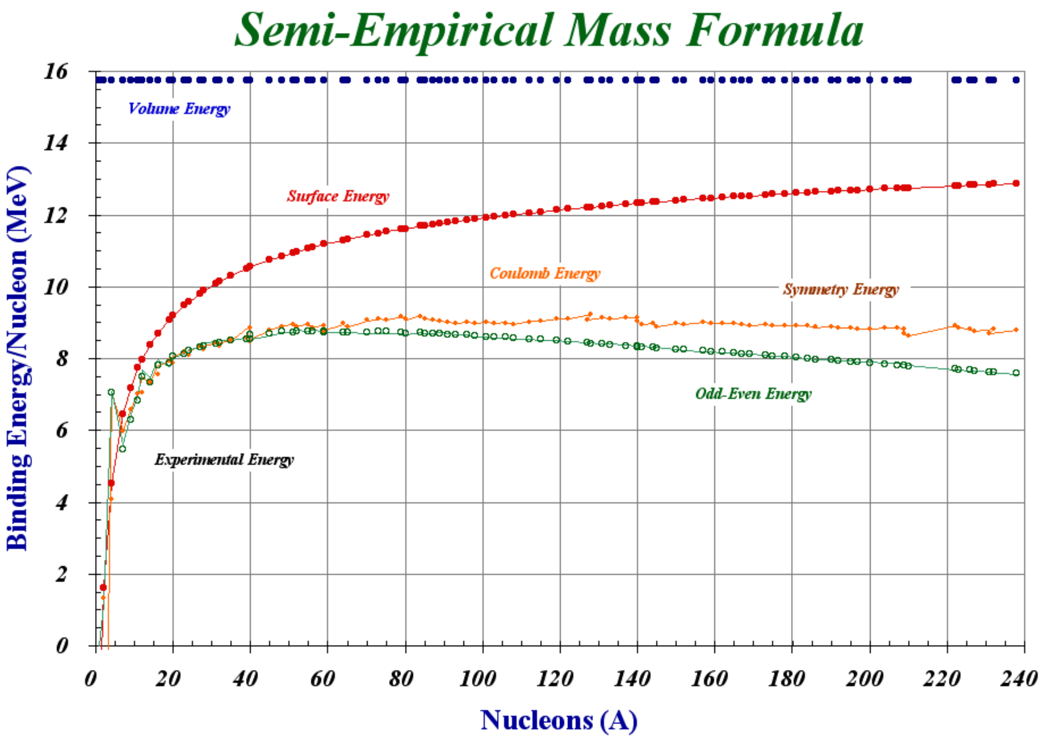
Chapter 11 : Nuclear Matter

ǂ There is no such material in reality. "Nuclear matter" is a theoretical model that tests our understanding of the atomic nucleus.

ǂ The goal is to start with the empirical nucleon-nucleon force, and then calculate the nucleon binding energy in isotopes with large A.

ǂ What precision is expected? After all, the nucleon is not an elementary particle.



This graph compares data to the semi-empirical mass formula. The theory of nuclear matter has nothing to say about the surface energy or the Coulomb energy. So the question is, can it explain the volume energy ?

38. Nuclear forces -- a review

The textbook was written around 1970.

One of the authors is a renowned nuclear theorist.

Has our understanding of the NN interaction changed since 1970?

Yes and no.

In 1970 the quark model was just a hypothesis, and quantum chromodynamics (QCD) was unknown.

Today we know that to treat the nucleon as an elementary particle is only an approximation, and not a very good one in some cases. (That was already suspected in 1970.)

- meson fields; quarks and gluons

On the other hand, the excitation energy required to see inside the nucleon is of order 1 GeV, while nuclear binding energies are only of order 10 MeV. So treating a nucleon as a “particle” without internal dynamics might be OK at the level of a few percent accuracy.

- ★ The semi-empirical mass formula is quite accurate.
- ★ Single-particle models for nuclear structure, like the shell model, are pretty accurate.

Properties of the nucleon-nucleon interaction

F&W describe some properties of the low-energy nucleon-nucleon (strong) force.

/1/ It is attractive at "large" distance.

- existence of the deuteron bound state (spin 1 ; isospin 0 ; binding energy = 2.2 MeV);
- evidence from pp scattering $\Rightarrow$ attractive

/2/ It has short range.

- low energy np scattering is s-wave; $l_{max} < 1$ implies r ~ a few fm

/3/ It is spin-dependent.

- σ_{np} = 0.75 triplet + 0.25 singlet and σ_{np} > predicted by the theory of the deuteron

/4/ It is noncentral.

- the deuteron has a quadrupole moment so it must be a mixture of l=0 and l=2; then the NN interactions must have a tensor component.

/5/ It is charge independent.

- this is the isospin symmetry of the *strong* force; of course the electromagnetic force is not isospin symmetric.

/6/ The exchange character

- High energy np scattering (600 MeV lab energy) has $f(\pi-\theta) \approx f(\theta)$; odd l values are small;
- "exchange potential" $V_{Majorana}(\mathbf{r}) = P_M V(\mathbf{r})$ and "Serber potential" $V = V(x)(1+P_M)/2$. (P_M = exchange operator.)

/7/ There is a hard core (i.e., repulsion) at "short" distance.

- the s-wave phase shift for pp scattering becomes negative at 200 Mev;
- $r_c \sim 0.4$ fm.

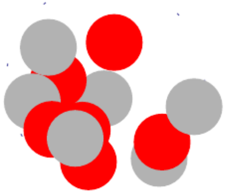
/8/ The spin-orbit force is only relevant at high energies.

- polarization of scattered nucleons implies $\exists$ a spin-orbit force, but not strong at low energies.

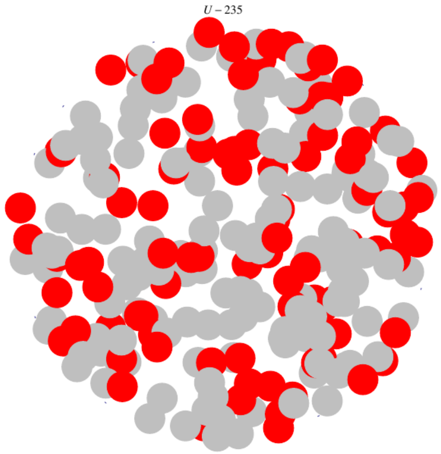
Two nucleons

Deuteron

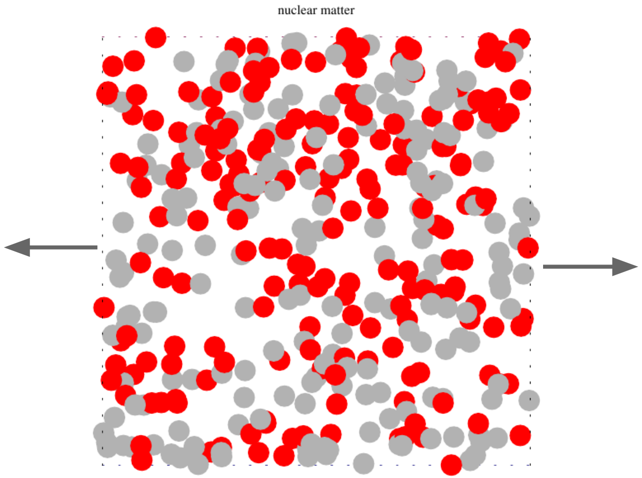
C-12



U-238



Nuclear Matter



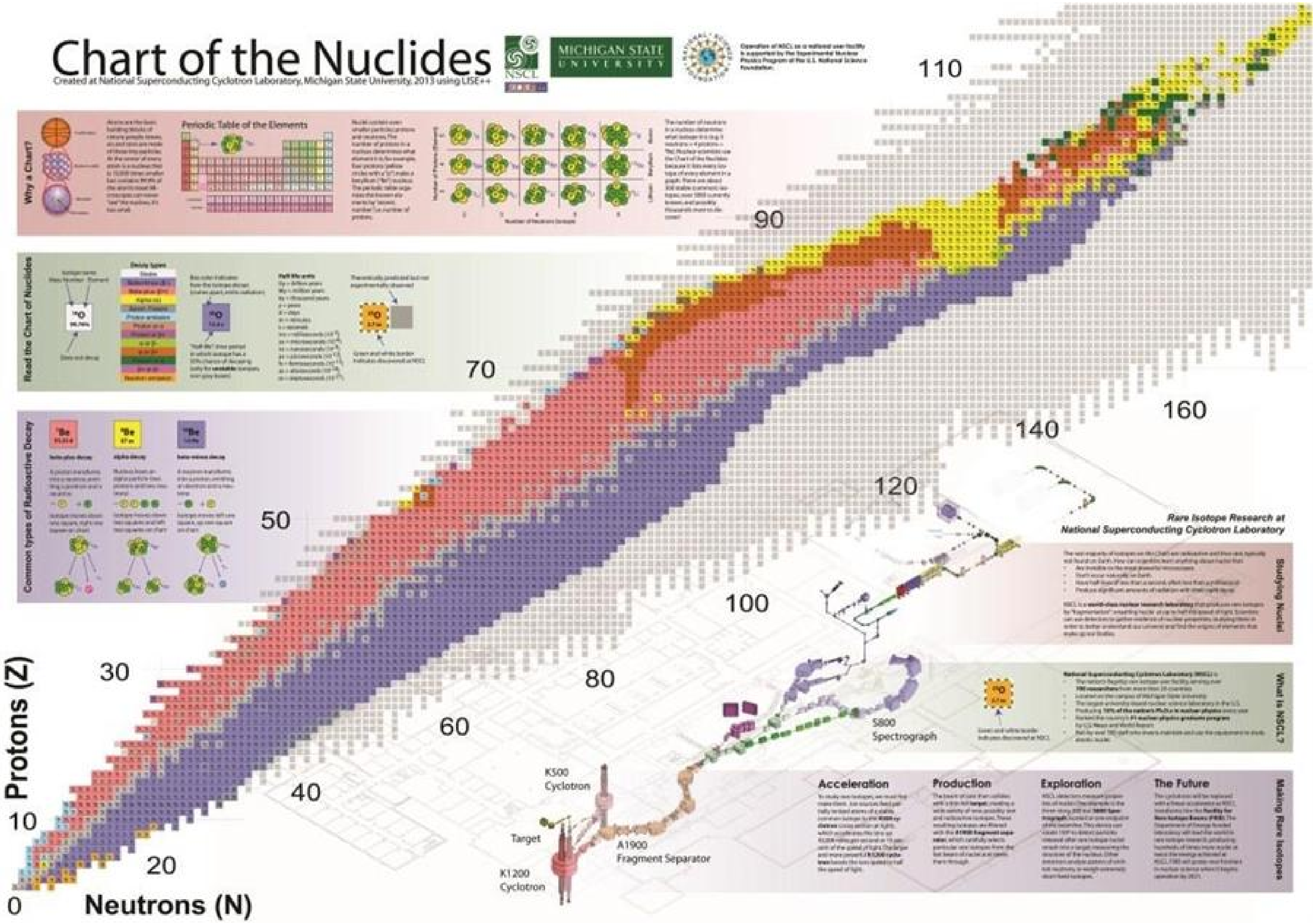


Chart of the Nuclides
Created at National Superconducting Cyclotron Laboratory, Michigan State University, 2013 using LISE++
NSCL
MICHIGAN STATE UNIVERSITY
Why a Chart?
Periodic Table of the Elements
Read the Chart of Nuclides
Common types of Radioactive Decay
Protons (Z)
Neutrons (N)
0
10
20
30
40
50
60
70
80
90
100
110
120
140
160
Rare Isotope Research at National Superconducting Cyclotron Laboratory
Studying Nuclei
What is NSCL?
Making Rare Isotopes
Acceleration
Production
Exploration
The Future
K500 Cyclotron
Target
K1200 Cyclotron
A1900 Fragment Separator
S800 Spectrograph

Table of the Isotopes

- ★ The stable isotopes have N ≈ Z for small Z.
- ★ The stable isotopes have N > Z for large Z.
- ★ There is a valley of stability.
- ★ Weak interactions (beta-decay) limit the number of isotopes for a given value of Z.

The semi empirical mass formula

The ground state energy of an isotope (Z,A)
{note: A = Z + N}
is well described by the formula

$$E(Z,A) = E_1 + E_2 + E_3 + E_4 + E_5$$

where E_1 … E_5 have simple dependences on A and Z, which are understandable in terms of a simple ideas.

E_1 = the volume energy : $E_1 = - a_1 A$

E_2 = the surface energy : $E_2 = + a_2 A^{2/3}$

E_3 = the Coulomb energy : $E_3 = + a_3 Z^2 / A^{1/3}$

E_4 = the symmetry energy :

$$E_4 = - a_4 (A-2Z)^2 / A$$

E_5 = the pairing energy : $E_5 = \lambda a_5 A^{-3/4}$

where λ = {+1, 0, -1} for {OO, OE, EE}.

$E(Z,A) = E_1 + E_2 + E_3 + E_4 + E_5$

$E_1 = - a_1 A$; $E_2 = + a_2 A^{2/3}$

$E_3 = + a_3 Z^2 / A^{1/3}$; $E_4 = - a_4 (A-2Z)^2$

$/A$

$E_5 = \lambda a_5 A^{-3/4}$

Parameter values given by FW

a1 = 15.75 MeV ; a2 = 17.8 MeV

a3 = 0.710 MeV ; a4 = 23.7 MeV

a5 = 3.4 MeV .

Exercise: Explain the Z and A dependences in terms of simple ideas.

How big is a nucleus?

There are exceptional cases, like “halo nuclei” (unknown when this book was written).

Also, not all isotopes are spherical; some are prolate, others oblate.

But as a general rule, for isotopes in the “valley of stability” we have radius

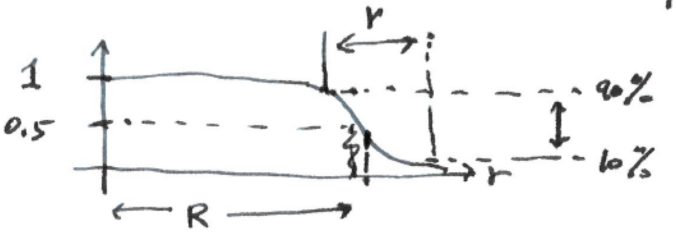
$$R = r_0 \ A^{1/3} .$$

This explains the “volume energy” of the SEMF. Nucleons are just packed in with approximately constant density; so the binding energy is proportional to the number of particles, and hence to the volume,

$$4/3 \ \pi \ R^3 = 4/3 \ \pi \ r_0^{\ 3} \ A.$$

A little more about the size of a nucleus. From electron scattering (Hofstadter's experiments)

Figure 39.1 : $e\rho(r)$ = charge density



may be parametrized by

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

Quantitatively ....

① $A\rho_0/Z$ is $\simeq$ constant, i.e., independent of Z, A

② $R \simeq r_0 A^{1/3}$ with $r_0 \simeq 1.07$ fm

So, the mean particle density is

$$\frac{A}{V} = \frac{A}{\frac{4}{3}\pi R^3} = \frac{A}{\frac{4}{3}\pi r_0^3 A} = \frac{3}{4\pi r_0^3}$$

$$= 0.195 \text{ fm}^{-3}$$

③ The mean distance between particles $= \ell$ defined by $\rho_0 = 1/\ell^3$.

$$\ell = (\rho_0)^{-1/3} \simeq (0.195 \text{ fm}^{-3})^{-1/3} = 1.73 \text{ fm.}$$

{Compare the charge radius of a proton, $r_p \simeq 0.7$ fm}

④ Surface thickness $t = 2a \ln 9$;

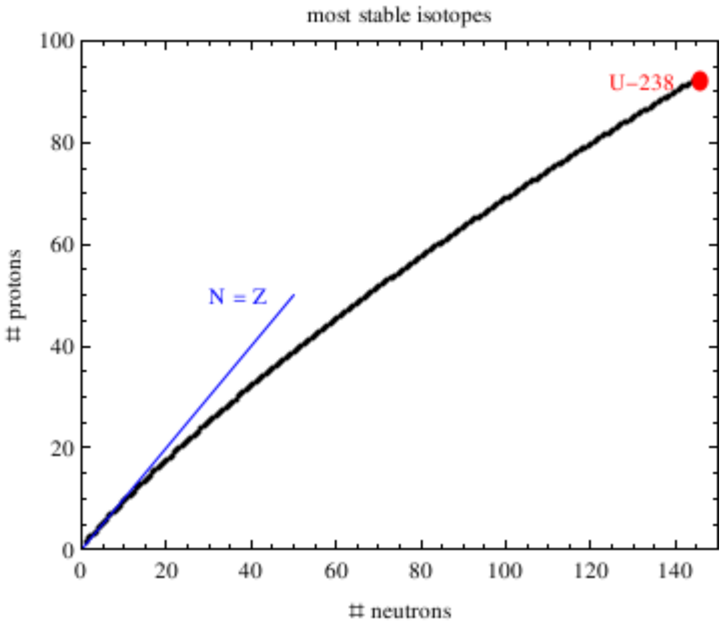
experiments $\Rightarrow$ $t \simeq 2.4$ fm for $A > 24$.

The most stable isotopes, as a function of A, as predicted by the SEMF

$$E = -a_1 A + a_2 A^{2/3} + a_3 Z^2 A^{-1/3} + a_4 (A - 2Z)^2 A^{-1} + \lambda a_5 A^{-3/4}$$

$$\frac{\partial E}{\partial Z} = 0 = 2a_3 Z A^{-1/3} - 4a_4 (A - 2Z) A^{-1}$$

$$\therefore Z^* = A \left(2 + \frac{a_3 A^{2/3}}{2a_4} \right)^{-1}$$



39. NUCLEAR MATTER

Now, what is the definition of "nuclear matter"?

- An imaginary material with strong forces but no electricity;
- *symmetric* nuclear matter means $N = Z$;
- in the limit $A \to \infty$, $V \to \infty$ with constant A/V (= particle density);
- an accurate model for the nucleon-nucleon force.

The theory of nuclear matter should explain a_1 (the volume energy) and r_0 (the density) of the ground state.

I.e., the theory should explain why...

$$E/A \approx -15.7 \text{ MeV}$$

and

$$r_0 \approx 1.07 \text{ fm.}$$

NEGLECTING THE STRONG FORCE ...

As a first step, approximate nuclear matter as an ideal Fermi gas.