Review...

Chapter 4: Feynman Rules for correlation functions in coordinate space

(1) Draw all topologically distinct connected diagrams.

(2) Vertex $= e\gamma^\mu$

(3) Fermion propagators

$$S_F(x-y) = \psi(x)\bar{\psi}(y)$$

(4) Photon propagators

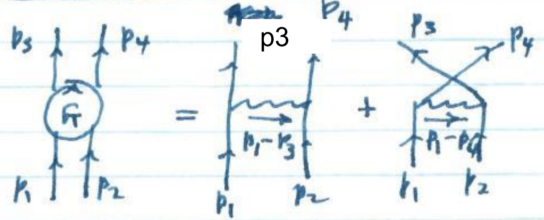
$$D_{\mu\nu}(\xi_1-\xi_2) = A_\mu(\xi_1)A_\nu(\xi_2)$$

(5) Integral over vertex positions

$\int d^4\xi_1$ etc

Chapter 4: Feynman Rules for correlation functions in momentum space

Momentum Space



(1) Draw topologically distinct diagrams.

(2) Vertex $= e\gamma^\mu$; 4-momentum is conserved

(3) $$\hat{S}_F(p) = \frac{i(\not{p}+m)}{p^2-m^2+i\epsilon}$$

(4) $$\hat{D}_{\mu\nu}(k) = \frac{i g_{\mu\nu}}{k^2+i\epsilon}$$ "Feynman gauge"

(5) Overall 4-momentum conservation gives a factor $(2\pi)^4\delta^4(P_f - P_i)$.

(6) $\int \frac{d^4q}{(2\pi)^4}$ over loop momenta.

But now we need Feynman Rules for
<u>transition amplitudes</u> .

First, *how are transition amplitudes related to correlation functions?*

Section 4.5 : Cross Sections and S-Matrix Amplitudes

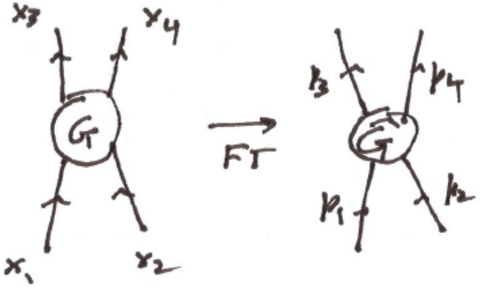
Section 4.7 : Feynman Rules for Fermions

Section 4.8 : Feynman Rules for QED

$G(x_1 x_2 x_3 x_4)$

$$= \langle 0 | T\, \psi(x_3)\,\psi(x_4)\, H_1 H_1\, \bar\psi(x_1)\, \bar\psi(x_2)\, |0\rangle$$

$$T_{fi} = \langle p_3 p_4 | T\, H_1 H_2 | p_1 p_2 \rangle$$



↑ 4 factors of $\hat S_F(p_\ell)$

Correlation function $\hat G(p_1\, p_2\, p_3\, p_4)$

$\longrightarrow$ transition amplitude T_{fi}.

Using PS conventions

$$\psi_i(\vec x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \Big[a(ps)\, u(ps)\, e^{-ip\cdot x} + b^\dagger(ps)\, v(ps)\, e^{ip\cdot x} \Big]$$

They define

$$|\vec p s\rangle = \sqrt{2E_p}\; a^\dagger(ps)\, |0\rangle$$

So the normalization of states is

$$\langle \vec p r | q s \rangle = 2E_p\, (2\pi)^3\, \delta^3(\vec p - \vec q)\, \delta_{rs}$$

(or $V\, \delta_{Kr}(\vec p, \vec q)$ for finite volume V)

Now, in G we have

$$\psi(\xi)\,\bar\psi(x_1)\,|0\rangle\ ;$$

but ~~Now~~, in T we have

$$\psi_i(\xi)\,|\vec p_1 s_1\rangle$$

$$= \int \frac{d^3p'}{(2\pi)^3}\frac{1}{\sqrt{2E_{p'}}}\sum_{s'} a(p's')\,u(p's')\,e^{-ip'\cdot\xi}\ \sqrt{2E_{p}}\ a^\dagger(p\,s)\,|0\rangle$$

$$\Rightarrow (2\pi)^3\,\delta^3(p-p')\,\delta_{ss'}$$

$$= e^{-ip\cdot\xi}\,u(p,s)\,|0\rangle$$

Eq. (4.114)

$\therefore$ Replace the factors $\hat S_F(p_\ell)$ by $u(p_\ell)$ $\bar u(p_\ell)$ etc as appropriate.

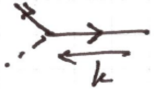
Similarly $\langle ps|\,\bar\psi_i(\xi)$

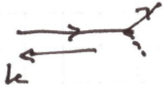
$$= e^{ip\cdot\xi}\,\bar u(p,s)\,\langle 0|$$

So external fermion legs have these Feynman rules:

$\underbrace{\psi\ |p,s\rangle}$ =  $= u(p,s)$ incident particle

$\langle ps|\,\bar\psi$ =  $= \bar u(ps)$ outgoing particle

$\bar\psi\,|ks\rangle$ =  $= \bar v(ks)$ incident antiparticle

$\langle ks|\,\psi$ =  $= v(ks)$ outgoing antiparticle

Matrix Element → Cross Section
$T_{fi} \rightarrow \sigma$

$$S_{fi} = \langle p_3\, p_4 \,|\, U(T,-T) \,|\, p_1 p_2 \rangle$$

$$= \delta_{fi} + i T_{fi} \quad \text{(Eq. 4.72)}$$

But $|\vec{p}s\rangle = \sqrt{2E_p}\; a^\dagger(ps)\,|0\rangle$

With this normalization,

$$dP = \sum_f \frac{|T_{fi}|^2}{\prod_f 2E_{p_f} \cdot \prod_i 2E_{p_i}}$$

$$\left[\frac{\langle f | U | i \rangle}{\sqrt{\langle f|f\rangle}\sqrt{\langle i|i\rangle}} \right]^2 = \frac{|T_{fi}|^2}{2E_3\, 2E_4 \cdot 2E_1\, 2E_2}$$

Now, T_{fi} will have the form:

$$T_{fi} = \mathcal{M}\; (2\pi)^4\, \delta^4(p_3 + p_4 - p_1 - p_2)$$

(Eq. 4.73).

But we can't square a Dirac delta function!

P&S use wave packets.

Or, use a finite volume V with normalized plane waves.

Keeping V finite $\Rightarrow$

replace $(2\pi)^3\, \delta^3(p_f - p_i) = V \delta_{Kr}(p_f, p_i)$.

Keeping T fixed

$$2\pi\,{}^{``}\delta{}^{"}(E_f - E_i) = \int_{-T}^{T} e^{i(E_f - E_i)t}\, dt$$

$$= \frac{2\sin(\Delta E \cdot T)}{\Delta E}$$

Then

$$[2\pi\,{}^{``}\delta{}^{"}]^2 = \frac{4\sin^2(\Delta E \cdot T)}{(\Delta E)^2}$$

$$\xrightarrow[T\to\infty]{} 4\pi T\,\delta(\Delta E)$$

Now we have well defined

$$|T_{fi}|^2 = |\mathcal{M}|^2\, V^2\, \delta_{Kr}(\vec{P}_f, \vec{P}_i)\; 4\pi T\,\delta(\Delta E)$$

$$= |\mathcal{M}|^2\, V T\, 2(2\pi)^4\, \delta^4(P_f - P_i)$$

Cross section = rate / inc. flux

$$d\sigma = \frac{dP/2T}{1/V \cdot v_{rel}}$$

$$= \frac{V}{2T\, v_{rel}} |\mathcal{M}|^2\, V T\, 2(2\pi)^4\, \delta^4(P_f - P_i)$$

or

$$\sigma = \sum_{p_3} \sum_{p_4} V^2 \frac{|\mathcal{M}|^2}{v_{rel}} \frac{(2\pi)^4\, \delta^4(P_f - P_i)}{\Pi_{l=1,4}(2E_{pl}V)}$$

$$\sigma = \int \frac{d^3p_3\, d^3p_4}{(2\pi)^3\,(2\pi)^3} \frac{|\mathcal{M}|^2\, 2(2\pi)^4\, \delta^4(P_f - P_i)}{\prod_l (2E_{pl})\; v_{rel}}$$

$$= \frac{1}{2E_1\, 2E_2\, v_{rel}} \int \frac{d^3p_3}{(2\pi)^3\, 2E_3} \int \frac{d^3p_4}{(2\pi)^3\, 2E_4}\; |\mathcal{M}|^2\, (2\pi)^4\, \delta^4(P_f - P_i)$$

(Eq. 4.79)