# Radioactivity I:  Counting and Poisson Statistics

Suggested Reading: Ch. 4, 5 and 11 in Taylor.

### Introduction

The objective of this part of Radioactivity experiment is to learn how to conduct counting experiment and to learn about counting statistics. Specifically, the Poisson distribution, which is universal for counting, follows a normal distribution – in its limiting case of high count rate – with the standard deviation equal to the square root of mean.

### Part 1: Learning to Count

The principal device that you will use to measure radiation is the Geiger-Mueller tube, often just called a Geiger tube or GM tube. Your experimental set-up consists of a Geiger tube with its High Voltage (HV) power supply, some signal conditioning electronics and a scaler with digital readout, all shown in Figure 1. The scaler displays the number of counts (= particles registered in the tube) in a given time interval. The principle of the Geiger tube operation is described in the "Introduction to Nuclear Radiation". Please read this introduction before you start this experiment. <u>There should be no HV on the Geiger tube yet at this point.</u>
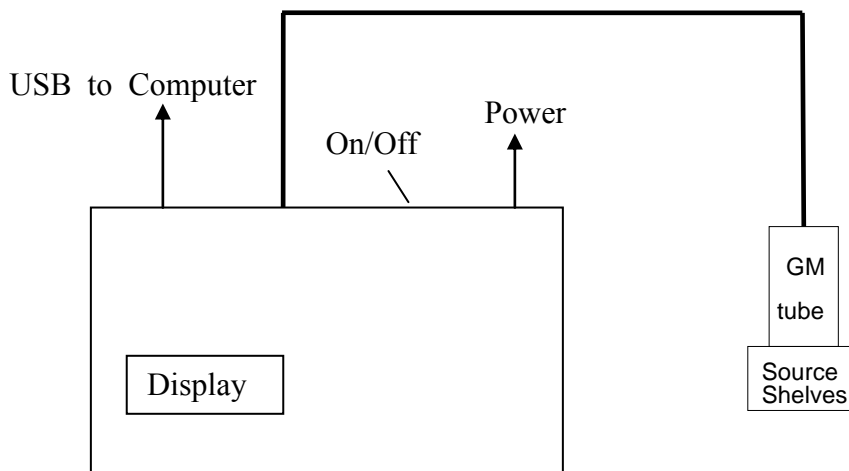


Fig. 1: Counter setup for radioactivity experiments

The hardware registering GM counts can be controlled either by a computer program or by the front panel.  We will use the computer program first, then proceed to the front panel.  First, be sure the electronics pack is on.  There should be cables running to the GM tube and a USB cable running to the computer.  Then, if the display on the front of the electronics is blank, turn on the power switch on the back.

### Part 2: The Plateau

Check out a $^{60}$Co source by following the procedures your TA has explained to you. Your initials indicate you are responsible for this source until you return it. Put your Cobalt ($^{60}$Co) source on the middle "shelf" below the counter. If you put it too close the

count rate will be too high and you will not see the flattish spot called the "plateau" (see Figure 2 in Appendix A); too far away and it takes too long. *In general the highest counting rate should not exceed 1000 counts/second.*

You are now ready to take a curve of count rate vs. GM High Voltage (HV), and look for the plateau region.

**You want to run the tube at a voltage on the "plateau" because it's sensitive to radiation there, but the rate doesn't depend *too* much on the exact voltage.**

*Computer Control.* Go to "New Nuclear" folder on the desktop, and click on the GM_Tube program. For Comm Port, hit "refresh" and select the highest numbered COM entry (e.g. COM4). For "Start Volts" select 600, for "End Volts" select 1200, and for Step select 50. This will scan in HV from 600 to 1200 V in steps of 50V. Then select 10 for the Time interval: you will count 10 sec at each voltage setting. When you are ready, you can click Run. The time it will take to finish the run will be 10 x (1200 – 600)/50 = 120 s = 2 minutes or so. You can watch the HV and counts either in the program, or on the front panel of the electronics. There will likely be no counts until the counter is over 800 V or so. Once the tube starts registering particles you have reached a point on the response curve where it starts to be active. If things get confused, you can always close the GM_Tube window and click the icon again.

*WARNING:* In case the tube starts discharging, which is indicated by an extremely rapid increase in the count rate or even audible sounds from the tube, you should turn off the power switch on the back of the electronics unit immediately. You should wait for the system to rest and then don't go to that high a voltage next time! A less violent method would be to use the Manual Control directions (Part 3) to stop counting and then reset the HV, but that takes longer.

The program will draw a crude graph of the count vs HV curve. When it has finished, it will ask you to save a file. **Save it on the U drive** in your section's folder, under the name of this experiment; give the file a name descriptive of the parameters you chose, such as 600_1200_50_10.

You now can open this file either in Excel or Kgraph. Excel is convenient if you want to add a column with error bars for the data based on square roots of the observed counts; then you could save it as a spreadsheet, close, and open in Kgraph.

Here is how to open the file directly in Kgraph:
click U drive icon | browse to your file | drag and drop on Kgraph icon |
check Space (as a delimiter) | un-check Read Titles | OK
After looking at the crude plateau curve and comparing with Fig 2 of Appendix A, now make a better curve by concentrating more narrowly on the plateau region (say 800-1000 V by 20 V steps for 10s, and eventually for 30 s when you've zero'd in further).

We want to operate the tube at a voltage where the counting rate is rather insensitive to small changes in the voltage: the "plateau". The plateau is easiest to find graphically. Very soon after the voltage at which the tube starts counting, the change in counting rate

from one point to the next should become less than 5-10%. From this point continue raising the voltage in 50 Volt steps until the count rate has increased by another 25%, or until you have raised the HV another 200 volts. Repeat your measurements while decreasing the HV to check that they are reproducible.

Make a graph of count rate vs. Voltage and find the counter plateau, which is the region of the curve that is most nearly horizontal. Calculate the statistical uncertainty in the count rate and put the appropriate error bars on your graph. Pick a voltage near the low end of the plateau as your operating point. In principle you need to recheck this operating point each time the counter or scaler is turned on because changes can occur in both the counter characteristics the calibration of the high voltage meters. Record the width of your high voltage plateau in volts.

### Part 3:  Measuring Rates

*Manual Control.*  Now we change to operating the tube from the keypad on the front panel of the electronics box.   The keypad is good for making repeated measurements at fixed intervals.  While the box is taking data, you will see something on the display like
E980 000224
E990 000218
T007 000**3 AC 0021732
HV 0920 RUNNING

Each E entry gives the label of the 10 second interval of counting, and the count during that interval (interval 98 had 224 counts; 99 had 218). The T entry counts seconds and changes rapidly while running, while AC gives the Accumulated counts (21732) since the last start.  The HV line shows the HV the tube is running at, and if you are stopped or running.

To **set the HV**, key in the desired voltage divided by 10 (if you want 920V, hit 9 then 2).
To **start**, push the ENTER key (you can just do that if you already have the correct HV).
To **stop**, push the red "2ND" key.

Set the Geiger tube at its operating point and take 25 readings of the counting rate for 30 sec. each. You can do this by recording the AC every 3 10-sec interval up to interval 75, or by recording each of 75 10-sec intervals and adding in Excel.  Calculate the standard deviation in the normal manner from the deviations from the mean and compare with the Poisson standard deviation prediction based on the mean count. Make a histogram of your results (see §5.1 in Taylor). Does its shape agree with a normal distribution?  Is the width that expected for the Poisson distribution?

*Part 4:* Return the $^{60}$Co source and repeat Part 3 for a $^{137}$Cs source.

*Part 5:* Remove all the sources, but keep all settings the same, so you can measure the background radiation. Measure the background 25 times and calculate the average and the standard deviation of the background rate.

# Radioactivity II: Absorption of Radiation

## *Introduction*

In this experiment you will use the equipment of the previous experiment to learn how radiation intensity is influenced by its passage through matter. **This subject is described in the "Appendix A: Background Materials on Nuclear Radiation" in the section titled "Interactions with matter".** We will use two sources during this lab ($^{137}$Cs and $^{60}$Co) which both emit β's and photons, although in different ratios and with different energies. It will be possible to see the β's from one of the sources, but not from the other. Their decay schemes are described in the Appendix B of this handout. We will measure the quantity (μ/ρ), the mass attenuation coefficient, which characterizes how much the radiation is absorbed by the material. This coefficient is quite different for different types of radiation, as discussed in the "Introduction to Radiation". Alpha particles are absorbed very readily, beta particles less so, and gamma particles least, for a given amount of absorber. We measure the amount of the absorber by ρx, the density of the material times its thickness, which essentially measures the number of nucleons (and the number of atomic electrons) per unit area, which the particle is passing through. This Lab has a list of questions to help plan your experiments. Answer these questions as you go along.

## *Part 1.* **Background**

Recheck the operating point of your Geiger tube and adjacent points, by measuring the plateau using the $^{137}$Cs source. Remember that the maximum counting rate should not exceed 1000 counts/sec. After establishing the operating point, measure the background rate keeping all sources of radiation at least *5* or 6 feet away. Count for a time period sufficiently long to determine the background to *5%* accuracy. Plan spreadsheets and the rest of your measurements while the background is counting.

**Q1.** How would you try to reduce the background?
**Q2**: What is the minimum count necessary to get 5% fractional statistical precision? Hint: see the "Counting Statistics" section of "Introduction to Radiation" from last week.

## *Part 2:* **Absorption**

Start with the Cs source on a shelf towards the top where the counting rate for the bare source does not exceed 1000 counts/sec. But: check that you have left enough space for a full stack of absorbers between the source and the counter! Point the thin β window of the source towards the counter and count the bare source for 1 minute. All counting measurements with both the bare source and with the absorbers in place should have at least 2% statistical precision.

Your box with absorbers contains a variety of lead and polyethylene absorbers of three thicknesses, given in inches and in [g/cm$^2$]. If you are not sure of the values, measure the absorber thickness with a caliper and calculate the thicknesses in g/cm$^2$. For this, you will need the density of lead (11.3 g/cm$^3$) and of polyethylene (0.96 g/cm$^3$). All the thin polyethylene sheets are 0.004 inches thick (one inch = 2.54 cm; you can convert in Excel).
*Important note*: **wash your hands after this lab**, to remove any lead dust from your hands. Another reason to not eat in lab!

For the $^{137}$Cs source with its thin β window pointing towards the counter, measure the count rate for 1 layer, 2 layers, 4 layers and 8 layers of the thin plastic foil absorber (9.6 mg/cm$^2$). Then measure the count rate for plastic absorbers of about 1.6 mm, 3.2 mm, 6 mm and 12 mm in thickness. You can do a significant part of the analysis with only the polyethylene data for Cs.  Next measure the count rate for lead absorbers of *about* 1.8 mm, 3.4 mm, 6.8 mm, 10.2 mm in thickness.

Repeat the latter measurements for the lead absorbers using the $^{60}$Co source. You should not need to measure the absorption of $^{60}$Co radiation with the polyethylene (you can check by verifying that there is no measureable absorption for a thin poly sheet). Subtract the background from each count for both polyethylene and Pb absorbers. Make sure you express all your results in the same units e.g. counts/sec and that your measurements have at least 2% statistical precision.

Using *Kgraph*, plot the counting rate R against x = ρt in [g cm$^{-2}$] of absorber using a logarithmic "y" axis. Here the count rate can be described by $R=R_0 e^{-(\mu/\rho)\cdot(\rho t)}$. ***See Appendix B on Interpretation of Semi-Log Graphs*** *for help with this analysis.* To start, make 3 plots: all data from Cs source, and lead data from Cs and Co. From the lead data, determine μ/ρ, the mass attenuation coefficient, for each source. Compare μ/ρ you obtained from your experiments with the accepted values listed in Appendix B. For the energy of the gamma ray from each source, you can refer to Appendix C.

**Q3**. Can you tell from your curve or otherwise whether you have appreciable alpha particles?
**Q4**. Do you have substantial beta particle emission? How do you know?
**Q5**. Does your data indicate that Cs emits gamma rays? Does Co? Justify your answer.
**Q6**. Is there evidence of gamma rays of several different energies present in Cs? Justify your answer.

In this semi-log plot, the exponential absorption appears as linear, however with multiple decays the curve might be askew.  The *gamma ray part of your polyethylene* data curve is approximately a straight line, fit an exponential to that part of the data (see semi-log graph discussion, below, and look at the reference material from last lab to decide which parts of which curves represent alpha, beta, or gamma absorption).  **Subtract** the values from the fit curve for gamma ray counting rate, determined by this line, from your total curve to get a curve for beta rays only. Now plot beta particle emission only on a 10 or more times expanded [g·cm$^{-2}$] scale and fit for the absorption coefficient for beta particles.

The reason we try to identify the gamma-ray dominated part of the polyethylene curve is that the absorption coefficient for poly is sufficiently different than lead that it can't be used for the subtraction accurately.

If you run out of time in the lab, a short-cut substitute is to draw a straight line by hand on the graph and extrapolate it back to x=0, then subtract the extrapolated value from the smaller-x count rates.   This is not as accurate as the suggested subtraction procedure, and requires you to interpolate the value of the x intersection.

Do not attempt to fit a straight line to the semi-log plot: this is *not* equivalent! Drawing a line on the semi-log plot instead approximates fitting an exponential to the data, as described below.

**Q7.** Are beta rays absorbed exponentially?

**Q8.** If the Cs source emits approximately equal numbers of beta and gamma rays, what is the approximate efficiency of the counter for gamma rays if you assume every beta ray is counted?  (The efficiency is just the fraction of gamma rays counted, if they hit the counter).

# Appendix A
# Background Materials on Nuclear Radiation

*The Introduction and Appendix contain reference material for the next several experiments.*

### Introduction.

The nucleus of an atom is small (diameter: $10^{-4}$ the diameter of the atom), massive (the mass of the atomic electrons is less than $10^{-3}$ of the nuclear mass), and, for most atoms existing in nature, stable. However, most natural nuclei with atomic number greater than 82 (Pb) are unstable, and change through radioactive disintegration (natural radioactivity), eventually becoming Pb. Also for all stable elements, nuclei that have different masses than the naturally existing stable forms (more or fewer neutrons) can be formed by nuclear bombardment. These nuclei decay by radioactive disintegration (artificial radioactivity) into stable nuclei by emitting various types of radiation. Finally, by nuclear bombardment we can create elements not existing naturally. Such atoms include elements 43, 61 and transuranic-elements (so far atomic numbers 93 through 111 have been made and named; and 112-116, and 118 have also been reported in the literature). All of these elements also undergo radioactive disintegration until a stable form is reached.

### Half -life.

Each unstable nucleus has a well defined probability, $\lambda$, of decaying per unit time. If we have many atoms, N, the number per unit time disintegrating is the number of atoms times the probability per unit time for one atom. So the change in N per unit time is:

$$\frac{dN}{dt} = -\lambda N. \qquad (1)$$

The number of nuclei remaining (not decayed) after time t is the integral of this equation:

$$N = N_0 \, exp(-\lambda t), \qquad (2)$$

where $N_O$ is the number of initial nuclei at time t=0. The time $T_{1/2}$ for half of the nuclei to decay is called the half life. It can easily be shown that:

$$T_{\frac{1}{2}} = ln(2) \, /\lambda = 0.693/\lambda . \quad (3)$$

Each unstable nucleus has its own characteristic mean life $\tau = 1/\lambda$ which is also defined as the lifetime of the nucleus.

### Types of Radiation.

We know three kinds of radiation and they are called alpha ($\alpha$), beta ($\beta$), and gamma ($\gamma$) rays. Alpha "rays" are helium nuclei with a net positive charge of 2 electron charges. When a nucleus emits an alpha particle its mass number (A) decreases by 4 units and its atomic number (Z) by 2 units so we have a different element remaining. Beta rays are electrons (negative charge) or positrons (positive charge). When a nucleus emits a beta particle its mass number remains the same but its atomic number increases (negative) or decreases (positive) by one. Gamma rays are photons of high energy, typically on the order of MeV  (note: l MeV = $10^6$ eV; see below[1]). When a nucleus emits a gamma ray,

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[1]1 eV (electron Volt) is the amount of kinetic energy gained or lost by a particle with one
elementary electric charge e (e = 1.6 x $10^{-19}$ Coulombs) if accelerated through a 1 Volt potential

neither its mass number nor its atomic number changes. It typically goes from a higher level energy state to a lower level energy state. In every case the kinetic energy released (mostly to the α, β, or γ) is given by:

$$KE = M_i c^2 - M_f c^2 - M_p c^2 \ ,$$                    (4)

where $M_i$ and $M_f$ are the initial and final nuclear masses respectively and $M_p$ is the rest mass of the emitted particle (zero for gamma rays).

### *Interactions with Matter.*

All nuclear radiation interacts with and eventually is stopped by matter. How the three different kinds of radiation interact with matter will now be discussed in more detail.

### *Alpha Particles.*

These particles, which are emitted with discrete energies of several MeV, interact mainly with the electrons of atoms in matter and only rarely strike a nucleus. Because their mass is large, they are not deflected much by the electrons and proceed more or less straight ahead until they stop at the end of a rather well defined distance or range. Because of their larger mass their velocity is much less than that of beta rays. The low velocity and double charge results in knocking many electrons off atoms (ionizing atoms and forming ions) and a very short range. A few centimeters of air or a piece of paper will stop alpha particles of several million volts of energy (MeV).

### *Beta Particles.*

Beta particles (electrons or positrons), in contrast to alpha particles, are emitted in a given decay with energies from zero up to a definite maximum (which is usually less than alpha particle energies). Except for very high energy beta rays (greater than several MeV) the main slowing-down interaction is with the electrons of the matter through which the radiation passes. Because of their much smaller mass, their velocity is much greater than alpha particles and since their charge is also less, they make far fewer collisions in a given distance and travel much farther in matter. Also because of their smaller mass they are deflected much more easily and do not travel in very straight lines. The continuous distribution of energies combined with the tortuous trajectory means that beta particles do not show a definite range but are absorbed approximately exponentially as a function of distance. The most energetic beta rays are stopped by a book of a few hundred pages rather than one page as with alpha particles.

### *Gamma Rays.*

Since gamma rays (photons of energy hv) are uncharged, they pass through matter with very few interactions. However, with a well defined beam of gamma rays each interaction effectively removes the gamma ray from the beam either by scattering it out of the beam or by absorbing it so that it disappears. For very low energies (less than 0.2 MeV) and especially for high atomic number absorbers such as Pb, the main absorption mechanism is the transfer of all of the gamma ray energy to an inner electron which is

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difference.  So 1 eV = 1.6 x 10$^{-19}$ Joules.

ejected from the atom and absorbed in a short distance. This is the reason lead (Pb) is used to shield x-ray tubes. This phenomenon is called the Photo-electric effect.

For intermediate energies (0.1 to 2 MeV), collisions with outer electrons in which the gamma ray bounces off the electron in a new direction with reduced energy (the electron gets a good deal of energy and is absorbed in a fairly short thickness of material) are the most important. While this scattering effectively removes the gamma rays from a beam, they still continue to exist and experiments must be carefully designed to avoid confusion from scattered gamma rays. This effect is called the Compton Effect.

Starting at 1 MeV and becoming increasingly important at higher energy and for high atomic number materials, are interactions of gamma rays with atoms, in which the gamma ray is converted into a positron-electron pair (Pair Production) and the nucleus serves as a recoil to conserve energy and momentum. The produced electron and positron are absorbed in a manner described under beta particles.

The infrequency of collisions of gamma rays with matter and their removal from a beam in a single interaction results in absorption of an almost pure exponential character with distance for gamma rays of a single energy and also results in great penetration. Typically it takes a shelf of books to reduce a gamma ray beam to a few percent of its initial intensity. Of course because of its greater density and higher atomic number, an inch or two of lead (Pb) will accomplish the same absorption.

### *Absorption of radiation.*

Because of the difference in density between materials, we usually measure the amount of material traversed in $[g \, cm^{-2}]$ rather than the linear thickness traveled by the particle.

$\rho x$ = density $[g \, cm^{-3}]$ x linear thickness $[cm]$ = "thickness" in $[g \, cm^{-2}]$

$\rho x$ = mass of the absorber / area $[g \, cm^{-2}]$

When radiation is exponentially absorbed, the variation in counting rate with thickness is:

$$R(x) = R_o e^{-(\mu/\rho)\rho x} \quad , \tag{5}$$

where $R_O$ is the counting rate at $x = 0$, $R(x)$ is the counting rate after the radiation traverses thickness x, $\mu$ is called the linear absorption coefficient (dimensions $[cm^{-1}]$ if x is in $[cm]$) and $\rho$ is the density of the material in $[g \, cm^{-3}]$. $\mu/\rho(E,Z)$ is a function of the energy (E) of the radiation and the atomic number (Z) of the absorber. It will, therefore, in general be different for different energies and absorbers. For gamma rays a simple exponential decay implies the presence of one and only one energy of gamma rays. If more than one energy is present, Eq. (5) would take the form:

$$R(x) = R_{o1} e^{-(\mu_1/\rho)\rho x} + R_{o2} e^{-(\mu_2/\rho)\rho x} + R_{o3} e^{-(\mu_3/\rho)\rho x} \tag{6}$$

for three different energies and would not give a straight line on a semilog plot.


## Radiation detectors

All radiation detectors make use of the interactions of matter with radiation described previously in which radiation separates electrons from their parent atoms, namely ionizing them. The presence of the radiation therefore is made evident by detecting the ionization. This is done in several ways:

1. The ions make a non-conducting substance briefly conducting and the resultant current through the substance can be measured. This can either be a "steady" current from many particles or a pulse of current due to a single particle.
2. The detector can be so arranged that each ion pair generates many others in an avalanche so that a large pulse from the detector results.
3 . As the radiation passes through material atoms are exited to higher energy levels and the light emitted as the electrons return to their normal positions can be picked up by a photomultiplier tube which gives an amplified pulse at its output.
4. The ionization leaves a trail of ions which can be made visible when a photographic plate is developed. (This is the way radioactivity was originally discovered and is still used for x-ray pictures today.)

### *Geiger Counter.*

One of the detectors that we shall use is the Geiger Counter. It is a metallic cylinder with a small wire down the center and frequently has a thin "window" on one end to minimize absorption of the entering radiation. A schematic drawing is given below.
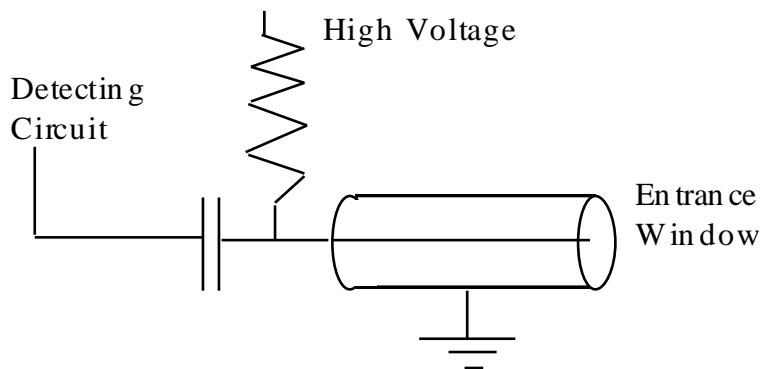


Fig. 1: Schematic of a Geiger-Mueller Tube.

The center wire (anode) is connected through a resistor to the positive side of a high voltage power supply. The cylinder is usually filled with inert gas at a low pressure. Each time when radiation strikes a gas atom, an ion pair is formed in the counter volume, where the electrons move toward the wire. The field near a small wire is very large, the electrons could gain sufficient speed in short distance to knock other electrons off from the gaseous atoms and these electrons knock still others off, leading to an avalanche. Light emitted from the avalanche knocks out electrons in other parts of the tube and the avalanche spreads the whole length of the wire. The potential of the wire is suddenly lowered by the arrival of the avalanche of electrons and this sudden drop in potential is passed through the capacitor (the capacitor keeps the high voltage away from the detecting circuits but lets the pulse through) to the detecting circuitry.

     The counter is left filled with the heavier positive ions which drift slowly out to the outer cylinder which is the cathode. Until most of the tube is clear of positive ions, another avalanche cannot occur and the counter is not sensitive. This dead time is of the order of $10^{-4}$ seconds and can cause an appreciable loss of counts at high counting rates (more than 1000 counts/sec), and at too high counting rates can stop the counter altogether.

One of the most useful characteristics of the Geiger Counter is the fact that though it will not count at all for too low a voltage, above a certain critical voltage the number of pulses per second (counting rate) depends only on the number of particles entering and very little on the voltage for a range of 50 to several hundred volts. This "plateau" of counting rate means that the operating voltage is not very critical. Toward the high end of the plateau spontaneous counts begin to occur and at a little higher voltage the counter breaks into a continuous discharge, which is harmful to it. An ideal response curve of a counter is shown in the figure below. For a real tube the plateau region does not extend so far in voltage. The operating voltage of your counter should be somewhere in the plateau.
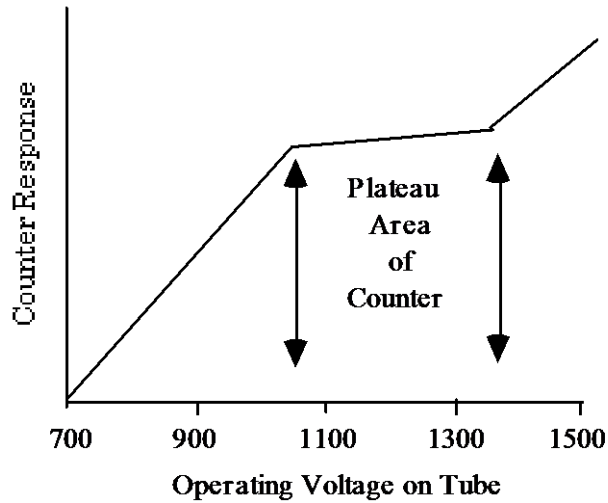


Fig. 2: Response of G-M tube as a function of applied voltage.

A Geiger Counter will count all charged particles which penetrate its window. However, alpha particles have such a short range that they have difficulty in penetrating even a thin window, especially if the source is a few cm away from the window. Gamma rays must strike an electron in the gas or very near the surface of the counter wall in order to produce a countable secondary electron. Because gamma rays travel large distances before making a collision, Geiger Counters have an efficiency of only a very few percent for detecting those which pass through the counter.

**Efficiency of Geiger Counters**

| Radiation | Efficiency |
|-----------|------------|
| $\alpha$ | Hard to get in counter - 100% if get in. |
| $\beta$ | 100% |
| $\gamma$ | a few % |

Background radiation

Will a counter in the presence of no specific source give zero counts? Not at all. Most nuclei of atomic number greater than 82 are naturally radioactive, as are a few lighter ones, such as Potassium 40. Also there are Cosmic Rays coming in from outside the atmosphere. Thus even with no source, a counter will count slowly. This count rate is

called the counter background or background count. It can be reduced by various means, the most common of which is shielding. However, for most of your experiments this background has to be determined and subtracted from your measurements to get the true counting rate.

Counting statistics

Read Chapter 11 of "An Introduction to Error Analysis" by John R. Taylor for additional information on counting statistics and the Poisson distribution. The normal or Gaussian distribution is discussed in chapter 5.

Since each nucleus emits its radiation independently, the radiation from N nuclei will be emitted randomly at an average rate which is proportional to N. Thus in one second 20 nuclei may decay, in the next 24. Because each nucleus is independent the time of arrival of counts in a counter is also completely random.

Thus, if we take n readings of the activity of a source [let the readings be $c_1$, $c_2$, $c_3$, etc. counts in some fixed interval ($\tau$)], the measured average count will be

$$\mu = (c_1 + c_2 + c_3 + \cdots)/n = \Sigma c_i/n \qquad (7)$$

Not all of the $c_i$ are the same. If they are large ($\geq 10$, see §11.2 of Taylor) and n is also large, they will distribute themselves about $\mu$ roughly as shown in the figure. The figure illustrates two normal distributions each with the same average ($\mu$) but with different sigmas or standard deviations ($\sigma$). What is shown here is the probability to measure a certain counting rate and obviously the most probable value to find is the average.
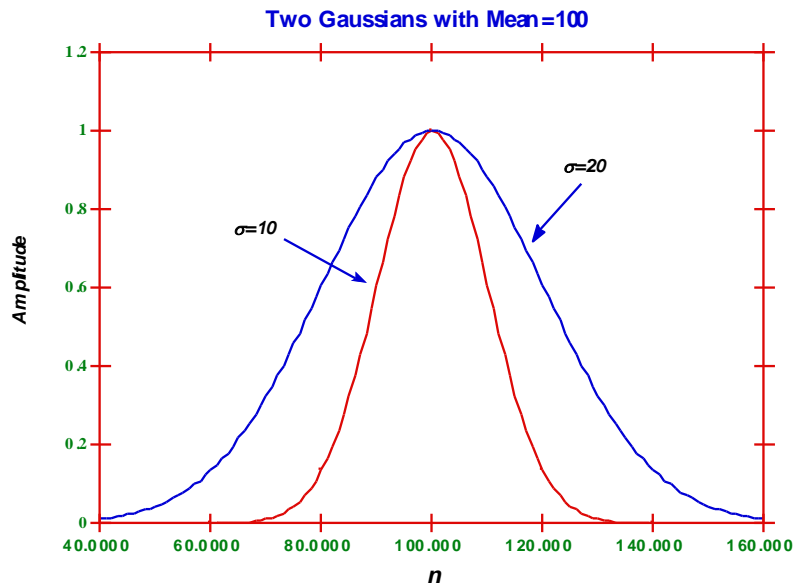


Fig. 3: Normal distributions with different standard deviations.

Normally, $\mu$ occurs more often than any other value but there is considerable spread. A normal distribution is completely defined by the average $\mu$ and the standard deviation or sigma ($\sigma$). If your counting rate follows a normal distribution (which it typically will do for rates >10) then one can say something about the probability of measuring a certain

rate: 68% of all measurements will be between $\mu - \sigma$ and $\mu + \sigma$. This can be readily deduced by integrating the distribution between these values and dividing by the total integral (say from $\mu - 8\sigma$ to $\mu + 8\sigma$) is equal to 1. Sigma ($\sigma$) can be obtained from the measurements in the standard way:

$$\sigma = \sqrt{\left[\frac{\Sigma(c_i - \mu)^2}{n-1}\right]} \qquad (8)$$

While the normal distribution is one of the most frequently occurring statistical curves, in counting experiment the statistics follow Poisson distribution better. However when the count rate is large, the Poisson distribution turns into a form akin to a normal distribution. This feature makes some of the language used for normal distribution, such as confidence level of 68% at 1 $\sigma$ deviation, generally applicable for counting experiment as well. For details, read Ch. 11 in Taylor.

Due to nature of Poisson statistics, the statistical error for a given Poisson count $c_i$ [given in Taylor (p.211)] by:

$$\sigma_i \approx \sqrt{c_i} \; . \qquad (9)$$

Since, in the above distribution $c_1 \approx c_2 \approx c_i \approx \mu$, for any series of counts

$$\sigma \approx \sigma_i \approx \sqrt{\mu} \qquad (10)$$

The <u>standard deviation of the mean</u> $\mu$, which is derived from n readings is:

$$\sigma_m = \sigma/\sqrt{n} = \sqrt{\left[\frac{\Sigma(c_i - \mu)^2}{n \cdot (n-1)}\right]} \qquad (11)$$

Therefore the value of $\sigma_m$ predicted for Poisson statistics is also given by:

$$\sigma_m = \frac{\sqrt{\mu}}{\sqrt{n}} = \sqrt{\left(\frac{\mu}{n}\right)} \qquad (12)$$

The **fractional** standard deviation of the mean $\mu$ is (again assuming Poisson statistics):

$$\frac{\sigma_m}{\mu} = \sqrt{\left(\frac{\mu}{n}\right)} \frac{1}{\mu} = \sqrt{\frac{1}{\mu \cdot n}} = \sqrt{\frac{1}{N_n}} \; , \qquad (13)$$

where $N_n$ is the total number of counts in n measurements. We shall try to verify crudely the Gaussian distribution for our counts and we will always calculate the confidence interval in order to know whether one count rate is significantly different from another at a given level of confidence. This is done by measuring $c_i$ and then calculating the probability for any other measurement by using the tables in Appendix A of Taylor.

Finally, we often need to measure a count rate :

$$R = \mu / \tau,\tag{14}$$

where $\tau$ is the measurement time interval. The uncertainty on R is just $\sigma_m / \tau$.

The fractional uncertainty of R is also given by (13), so long as the uncertainty of the time interval is negligible.

**EC**: Demonstrate this fact, and derive Eq (14).

## Appendix B:
## Interpreting the Semi-log graphs as absorption.

**Exponential Absorption**. In the "Introduction to Radiation" handout under "Interactions with matter", we predicted behavior for the radiation intensity as a function of distance into an absorber:

$$R = R_0 \exp\left[-(\mu/\rho) \cdot (\rho t)\right] \qquad (1)$$

where

    R = no. of counts per unit time after passing through material of thickness t

    $R_0$ = no. of counts per unit time before passing through material

    $(\mu/\rho)$= mass attenuation coefficient.

    $\rho t$ = "thickness" in [g/cm$^2$];

    $\rho$= density of the material [g/cm$^3$].

The mass attenuation coefficient depends strongly on the kind of particle being absorbed, and more weakly on the kind of material doing the absorbing, and the energy of the particle being absorbed. We choose the peculiar combination $\mu/\rho$ because the combination x = $\rho t$ is a good-first approximation description of the absorption effects of a material: a lot of material of low density $\rho$ has about the same effect as a smaller thickness of a much denser material. This happens because $\rho t$ the number of protons per unit area through which the radiation passes: most of the weight of atoms is in the nucleus, and about half the weight of the nucleus is in protons. The number of protons is also the number of electrons, so whether the absorption is happening mostly due to interaction with the charged protons in the nucleus, or the number of electrons in the atom, the probability of such interactions should be proportional to the number of protons the particle encounters.
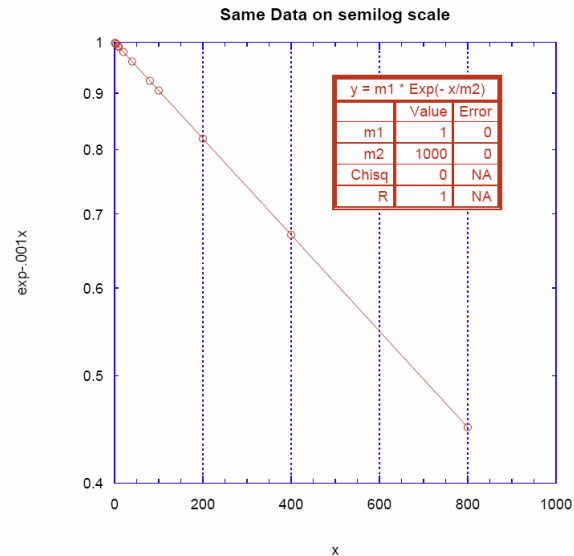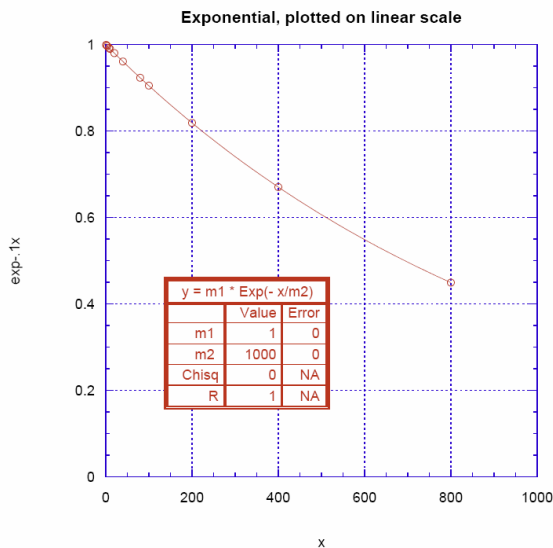
**Semi-log Graphs.** We want to determine absorption of nuclear radiation, especially β's andγ's, obeys this law, and to determine ($\mu/\rho$), if it does. To do this, we make a graph of R vs. ($\rho$t) using a logarithmic y-axis. Taking natural logarithms of both sides of Eq. (1):

$$\ln(R) = ln(R_0) - (\mu/\rho) \cdot (\rho t) \qquad\qquad (2)$$

which is the equation for a straight line, y = b + mx, with b, the y-intercept, equal to log $N_O$ ($N_O$ can be read from the non-linear logarithmic scale at that point) and -($\mu/\rho$)/2.3 as the slope. Thus you should expect to observe a straight line if you plot with a logarithmic y-axis.

In the example plots above, we have plotted the *same* data (an exponential curve) on both a linear and a semi-log scale; we use the term "semi-log" when only the y axis is logarithmic, and "log-log" when both the x and y axes are logarithmic. Notice the unequal spacing of the tick marks on the semi-log scale on the right. The semi-log plot allows you to simply enter data without taking the logarithm, but displays the y axis with y-coordinates data points spaced logarithmically instead of linearly.

For the chosen range of x, the curvature is only strongly visible for the last data points on the linear scale, though it is clearly linear on the semi-log scale. This is a warning that you may need to use a sufficient range of x axis values to observe the curvature.

Some further hints for using *Kgraph* successfully for this experiment:
Semi-log plotting in *Kgraph*:
    Axis Options | Y | Scale | Log      then     Ticks | Minor Interval  |Display Tick |In
General Fits:
1)    You must give non-zero starting values for all parameters
2)    For nonlinear fits, you may actually have to give a decent guess as a starting value
3)    Think about whether you need a + b Exp(-cx), or just the exponential.

**Subtracting two absorption curves.** When your data is curved on a semi-log plot, that probably is a hint that you are dealing with two sources of radiation with different attenuation curves, one steeper than the other.
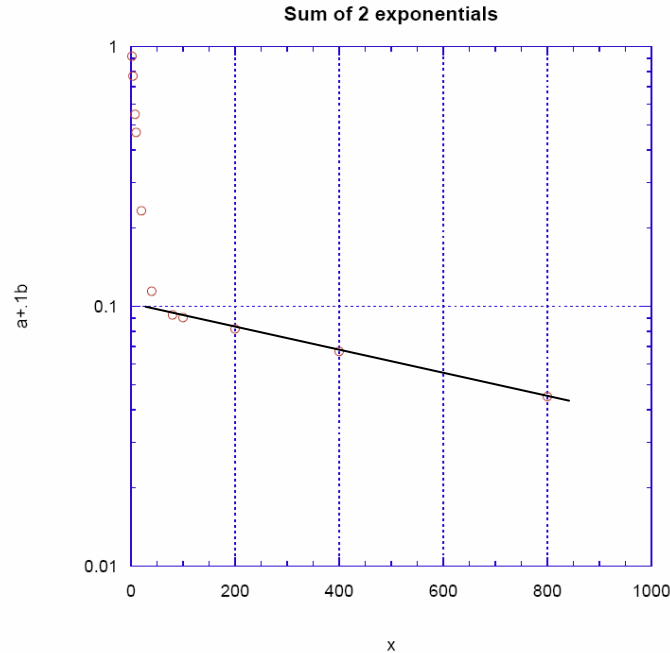
A good way to deal with this situation is to fit data for the flatter attenuation curve first (ie at larger x), since the other radiation source will have died out and not contaminate the second, "slower" attenuation curve very much.  In the example curve above, the last 4-5 points lie on a good straight line, even though the data is actually a sum of two exponentials,

$$f = Exp(-.1x) + .1\ Exp(-.001x).$$

These last 4-5 points would correspond to absorption of the harder-to-absorb component of the combined radiation.  Because in the example the absorption coefficients differ by a factor of 100, under the last 4-5 points, the first (easier-to-absorb) component has died out to an almost completely negligible level.  In fact, for the $4^{th}$ and $5^{th}$ points the two exponentials contribute (.00034 + .09231) and (.000045 and .090484) respectively.

If you fit only the data points at larger x, where the first component is negligible, then you can take the resulting fit curve and, point for point in x, *subtract* the fit from your data at smaller x, where the first radiation source is commonest: this removes the effects of the second (harder to absorb) kind of radiation from the data describing the more easily-absorbed (soft) radiation.  Fitting to a sum of 2 exponentials, while elegant, seems to be hard to get right.  And if you started including too many points at small x, you would not only get a poor fit, but you would also be subtracting not just the hard component, but also some of the soft component as well—so that subtracting this combination would seriously distort the subsequent fit to the easy-to-absorb component.

You will have to be sure to select only points not contaminated by the first kind of radiation in order to get the curve for the harder-to-absorb radiation right; selecting only Pb points would not help, since $\mu/\rho$ but differs between Pb and polyethelene (even for the same type of radiation). You need to find polyethelene points which are unaffected by the easily-absorbed component.  You should also check to be sure that all the points you are going to fit lie on a straight line on the semilog plot.

Next: how can we fit to only selected points?  One way is just to erase data points at low x which are contaminated.  A more elegant way is to mask them off:

<mark>**Masking data in *Kgraph***</mark> provides a way to leave data in the data window but not have it used in any calculations, curve fits, or plots. Any cells that are masked have a red frame in the data window.  Data can be masked in the data window by clicking them and clicking the left red grid-looking Mask frame button; the other buttons reverse the mask or unmask.

**Photon mass attenuation coefficient.** The following figure shows the representative values for energy dependence of the mass attenuation coefficients ($\mu/\rho$).  The total $\mu/\rho$ includes the contribution from the Compton effect, photo-absorption, and pair creation processes.
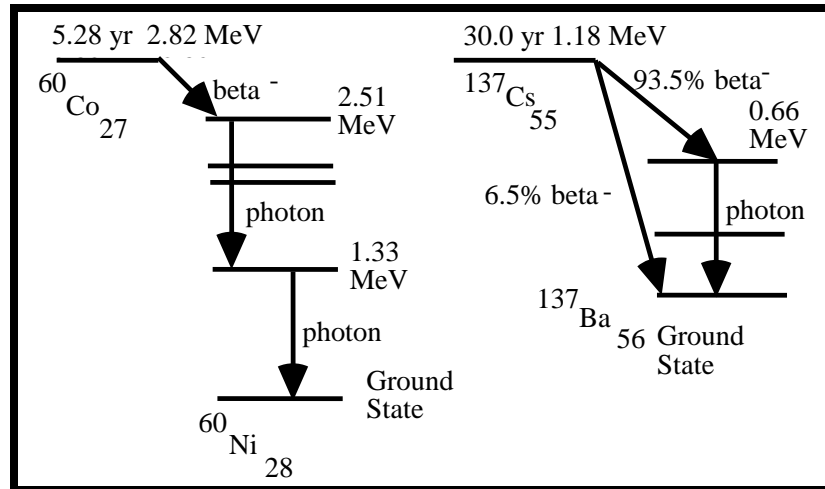
**Photon mass attenuation coefficient in aluminum and lead.**
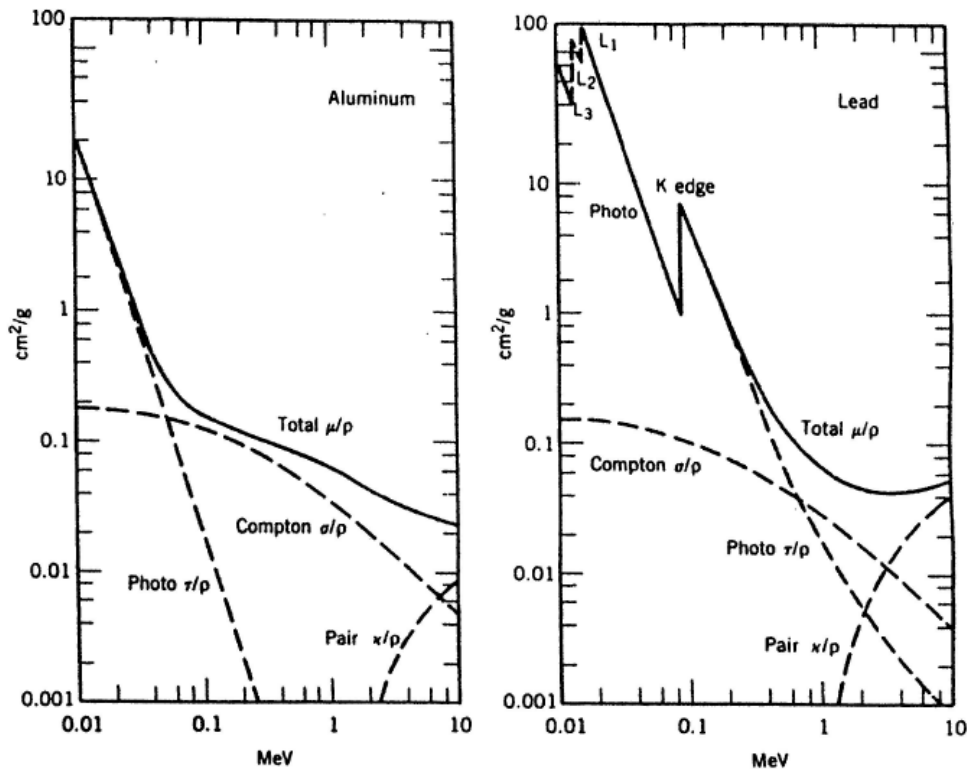
# APPENDIX C

## (Keep for reference in later experiments)

### Decay schemes for $^{60}$Co and $^{137}$Cs



The isotope notation $^{137}$Cs$_{55}$ indicates an isotope of Cesium (element 55, with 55 protons) which has 137-55=82 neutrons. Vertical arrows represent simple transitions from one nuclear energy level to another, which result in emission of a photon whose energy is equal to the difference in energy levels. The sloped arrows represent a beta decay which results in an electron and a neutrino being emitted. The beta electron does not have a fixed energy but rather a spectrum which has a maximum value; a typical value might be about half the maximum value.

**The Cs sources we use have thin windows (for easier transmission of betas), while the Co sources do not have a beta window.** Also the Co beta energy is much less than for Cs. Some more details of the decays are given below:

$^{60}$**Co:** This isotope has a half life of 5.28 years and beta decays to the excited states of the stable nucleus of $^{60}$Ni. In 99.9% of the cases the $^{60}$Ni (2.51 MeV 4th excited state) is formed, with a maximum beta energy of 0.31 MeV. The excited state subsequently decays in less than $10^{-12}$ sec to the ground state via the emission of 2 photons, one with energy 1.17 MeV and one with 1.33 MeV.

$^{137}$**Cs:** This isotope beta decays with a half-life of 30.0 years, with 93.5% of the decays creating $^{137}$Ba in its 2nd excited state at 0.66 MeV and 6.5% creating $^{137}$Ba in the ground state. The maximum beta energy is 0.52 MeV for the decay to the excited state, and 1.18 MeV for the decay directly to the ground state. The excited state has a half-life of 2.55 min. and decays 90% of the time through the emission of a 0.66 MeV photon and 10% of the time with the emission of an atomic conversion electron of a similar energy, equal to 0.66 MeV minus the binding energy. So 84% (= 0.935 * 0.90) of all beta decays of this isotope produce a photon in the final state.