

Practical 1

RC Circuits

Objectives

- 1) Observe and qualitatively describe the charging and discharging (decay) of the voltage on a capacitor.
- 2) Graphically determine the time constant for the decay, $\tau = RC$.

Apparatus

DC Power Supply
Desktop timer
Resistor and Capacitor
Digital Multimeter
Circuit Breadboard and Cable

Introduction

The Capacitor

A capacitor is a device that stores electrical charge. The simplest kind is a "parallel plate" capacitor that consists of two metal plates separated by an insulating material such as dry air, plastic or ceramic. Such a device is shown schematically below.

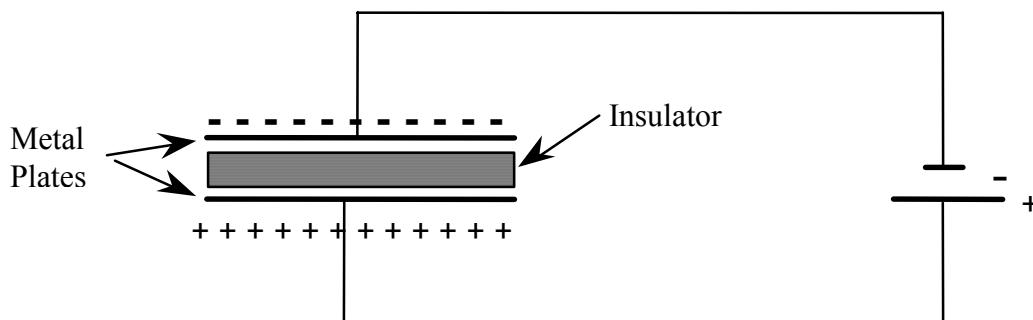


Figure 1. A simple capacitor circuit.

It is straightforward to see how it could store electrical energy. If we connect the two plates to each other with a battery in the circuit, as shown in the figure above, the battery will drive charge around the circuit as an electric current. But when the charges reach the plates they can't go any further because of the insulating gap; they collect on the plates, one plate becoming positively charged and the other negatively charged. The voltage across the plates due to the electric charges is opposite in sign to the voltage of the

battery. As the charge on the plates builds up, this back-voltage increases, opposing the action of the battery. As a consequence, the current flowing in the circuit decays, falling to zero when the back-voltage is exactly equal and opposite to the battery voltage.

If we quickly remove the wires without touching the plates, the charge remains on the plates. Because the two plates have different charge, there is a net electric field between the two plates. Hence, there is a voltage difference between the plates. If, sometime later, we connect the plates again, this time with a light bulb in place of the battery, the plates will discharge: the electrons on the negatively charged plate will move around the circuit to the positive plate until all the charges are equalized. During this short discharge period a current flows and the bulb will light. The capacitor stored electrical energy from its original charge up by the battery until it could discharge through the light bulb. The speed with which the discharge (and conversely the charging process) can take place is limited by the resistance of the circuit connecting the plates and by the capacitance of the capacitor (a measure of its ability to hold charge). In this lab you will test the theory that describes this behavior by measuring some discharge rates with an oscilloscope and comparing them to predictions of the theory.

RC circuit

An RC circuit is simply a circuit with a resistor and a capacitor in series connected to a voltage source (battery).

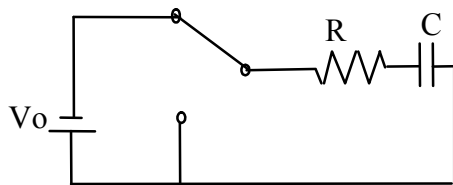


Figure 2a Charging

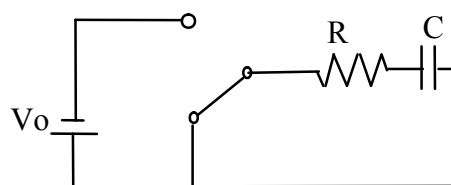


Figure 2b Discharging

As with circuits made up only of resistors, electrical current can flow in this RC circuit, with one modification. A battery connected in series with a resistor will produce a constant current. The same battery in series with a capacitor will produce a time varying current, which decays gradually to zero. If the battery is removed and the circuit reconnected without the battery, a current will flow (for a short time) in the opposite direction as the capacitor "discharges". A measure of how long these transient currents last in a given circuit is given by the time constant τ .

The time it takes for these transient currents to decay depends on the resistance and capacitance. The resistor resists the flow of current; it thus slows down the decay. The capacitance measures capacity to hold charge: like a bucket of water, a larger capacity container takes longer to empty than a smaller capacity container. Thus, the time constant of the circuit gets larger for larger R and C. In detail:

$$\tau \text{ (seconds)} = R(\text{Ohms}) \times C(\text{Farads})$$

The current does not fall to zero at time τ ; τ is the time it takes for the voltage of the discharging capacitor to drop to 37% its original value. It takes 5 to 6 τ 's for the current to decay to 0 amps. Just as it takes time for the charged capacitor to discharge, it takes time to charge the capacitor. Due to the unavoidable presence of resistance in the circuit, the charge on the capacitor and its stored energy only approaches a final (steady-state) value after a period of several times the time constant of the circuit elements employed.

What's going on in this RC Circuit

- 1) Initially, the switch is open, and no current is flowing.
- 2) The switch is closed as in Figure 2. (a). The capacitor will charge up, its voltage will increase. During this time, a current will flow, producing a voltage across the resistor according to Ohm's Law, $V_R = IR$. As the capacitor is being charged up, the current will be decreasing, with a certain time constant τ due to the stored charge producing a voltage across the capacitor.

In terms of $\tau = RC$, the voltage across the resistor and the voltage across the capacitor when the capacitor is charging look like:

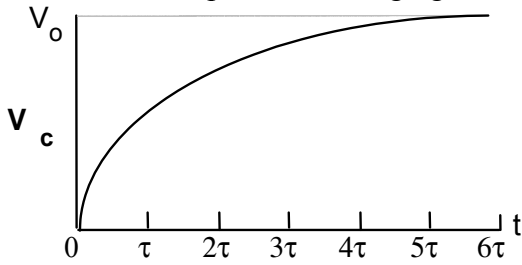


Figure 3a: Voltage across the capacitor V_c as a function of time. Time constant $\tau = RC$

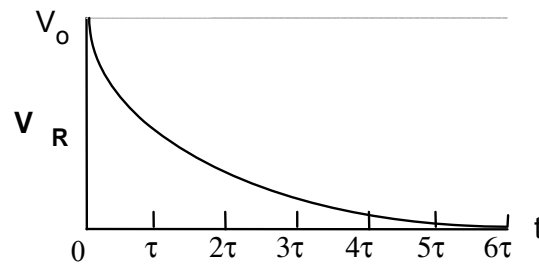


Figure 3b: Voltage across the resistor V_R as a function of time. Time constant $\tau = RC$

V_c and V_R while the capacitor is charging can be expressed as:

$$V_C(t) = V_o \left(1 - e^{-\frac{t}{RC}} \right) \quad \text{and} \quad V_R(t) = V_o e^{-\frac{t}{RC}}$$

where, e is an irrational number and is the base of the natural logarithm. The value of e is approximately 2.718.

When $t = \tau = RC$,

$$\begin{aligned} V_C &= V_o(1 - e^{-1}) & \text{and} & & V_R &= V_o(e^{-1}) \\ V_C &= 0.63V_o & & & V_R &= .37V_o \end{aligned}$$

So, after $t = RC$ seconds, the capacitor has been charged to 63% of its final value and the voltage across the resistor has dropped to 37% of its peak (initial) value. After a very long time, the voltage across the capacitor will be V_o and the voltage across the resistor will be zero.

3) If we flip the switch as shown in Figure 2(b), we will discharge the capacitor.

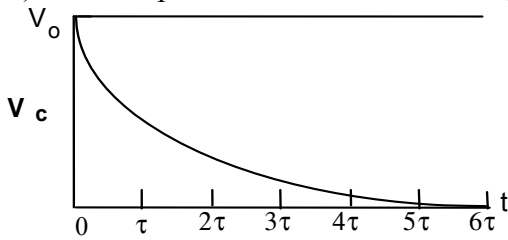


Figure 4a Voltage across the capacitor V_c for the discharging capacitor. $\tau = RC$

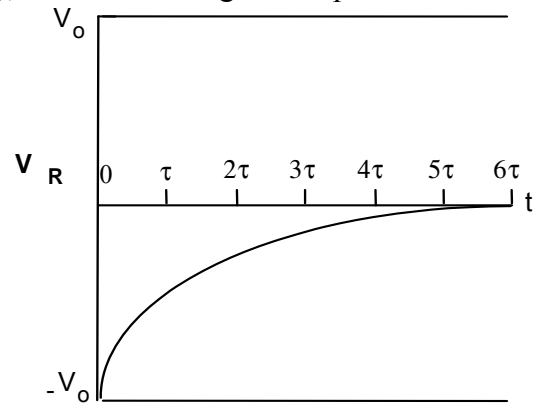


Figure 4b Voltage across the resistor V_R for the discharging capacitor. $\tau = RC$

The voltage of the resistor is exponentially increasing from $-V_0$ to zero. It is critical to remember that the total voltage between the capacitor and the resistor must add up to the applied voltage. If the circuit is disconnected from power supply, then the sum of the voltage must be zero.

The voltage across a capacitor and a resistor in a discharging RC circuit is given by:

$$V_C = V_0 e^{-\frac{t}{RC}} \quad \text{and} \quad V_R = -V_0 e^{-\frac{t}{RC}}$$

Dividing the voltage across the capacitor, V_C by the initial voltage, V_0 :

$$\frac{V_C}{V_0} = e^{-\frac{t}{RC}}$$

Calculating the natural logarithm of this voltage ratio yields:

$$\ln\left(\frac{V_C}{V_0}\right) = \ln\left(e^{-\frac{t}{RC}}\right) = -\frac{t}{RC}$$

Notice, the result has the same form as the equation of a straight line $y=mx+b$. In our case, the slope is $\frac{-1}{RC}$ and the intercept is zero.

Procedure

In this experiment you will make measurements using a digital multimeter and an electronic timer. You will charge the capacitor and measure the voltage across the resistor as the capacitor discharges. Set the multimeter to read DC voltage.

1. Build the circuit shown in Figure 7a. Use the DC power supply, a resistor, and a $1,000 \pm 200 \mu\text{F}$ capacitor ($1\mu\text{F} = 10^{-6} \text{F}$). The color code of the resistor you will use will be given to you on your practical lab answer sheet (you'll get the answer sheet when you do the experiment); enter the color code in your Excel spreadsheet. Use the color code to determine the nominal value of the resistance and its uncertainty. Use your Excel spreadsheet to do these calculations.
2. Use the power supply to charge the capacitor to about 12 V to 13 V. Then disconnect the power supply from the circuit and notice that the voltage across the capacitor slowly decreases – what could possibly cause this effect (see question 5)?
3. Build the circuit shown in figure 7b. After building the circuit shown in 7b, the capacitor should still be charged and you should still notice that the voltage is slowly decreasing. For the branch of the circuit containing the switch, use a single wire with a banana plug on each end. For now, only connect one end of this wire. Connecting the second end serves to close the switch. When this is done, the capacitor will start to discharge through the resistor.

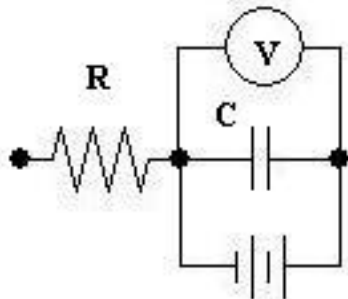


figure 7a

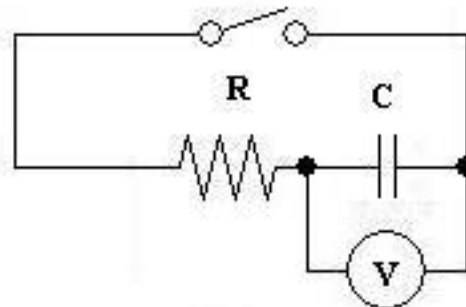


figure 7b

4. Close the switch and simultaneously start the desktop timer. Record the voltage across the capacitor when you closed the switch (at $t=0$) in your spreadsheet. Continue recording the voltage across the capacitor once every 20 seconds until your timer has reached 100 seconds.
5. Have *Excel* calculate (V_c / V_0) and $\ln(V_c / V_0)$; recall V_0 is the voltage measured across the capacitor when $t = 0$ seconds. Then, import your data into *Kaleidagraph*.

6. Make a plot of $\ln(V_c/V_0)$ versus time. Have *Kaleidagraph* fit your graph with a best-fit line. Use the curve fit parameters to determine the time constant of the circuit and its uncertainty.

Uncertainties:

The uncertainty in the time constant τ obtained from the graph of $\ln(V_c/V_0)$ versus time is given by:

$$\delta\tau = \tau \left| \frac{\delta slope}{slope} \right|$$

From the multiplication rule for uncertainties,

$$\delta(RC) = RC \left(\frac{\delta R}{R} + \frac{\delta C}{C} \right)$$

Questions:

1. What is the nominal resistance of your resistor that you obtained from the color code and its uncertainty?
2. From the nominal values of R and C, what is the value of RC and its uncertainty?
3. From the equation for the best fit line of your graph, what is the time constant and its uncertainty?
4. Discuss the consistency of the time constant obtained from your graph and the value of RC from question 2.

Comparing Data

It is often necessary to compare two different pieces of data or results of two different calculations and determine if they are compatible (or consistent). **In just about every experiment in this course you will be asked if two quantities are compatible or consistent.** The following describes how to determine if two pieces of data are consistent (or compatible). Use this procedure to answer the question at the end and use it as a reference whenever you are asked if two pieces of data are compatible or consistent. Let's denote the pieces of data by d_1 and d_2 . If $d_1 = d_2$ or $d_1 - d_2 = 0$, clearly they are compatible. We often use Δ (pronounced "Delta") to denote the difference between two quantities:

$$\Delta = d_1 - d_2 \quad (8)$$

This comparison must take into account the uncertainties in the observation of both measurements. The data values are $d_1 \pm \delta d_1$ and $d_2 \pm \delta d_2$. To perform the comparison, we need to find $\delta\Delta$. The addition/subtraction rule for uncertainties is:

$$\delta\Delta = \delta d_1 + \delta d_2 \quad (9)$$

Our comparison becomes, "is zero within the uncertainty of the difference Δ ?" This is the same thing as asking if:

$$|\Delta| \leq \delta\Delta \quad (10)$$

Equation (9) and (10) express in algebra the statement " d_1 and d_2 are compatible if their error bars touch or overlap." The combined length of the error bars is given by (9). $|\Delta|$ is the separation of d_1 and d_2 . The error bars will overlap if d_1 and d_2 are separated by less than the combined length of their error bars, which is what (10) says. Sometimes rather than a second measured value you are comparing your data to an expected value. If this is the case, replace $d_2 \pm \delta d_2$ with $e \pm \delta e$, where $e \pm \delta e$ is the expected value including its uncertainty.

RESISTOR COLOR CODES

Most resistors are coded with color bands around one end of the resistor body. Using the resistor color code system is similar to using scientific notation. Scientific notation uses a number between 0 and 9.9 multiplied by some power of ten. The resistor color code system uses a number between 01 and 99 multiplied by some power of ten. These color bands tell the value of the resistance. Starting from the end, the first band represents the first digit of the resistance value and the second band the second digit. The third band represents the power of ten multiplying the first two digits. The fourth band represents the tolerance. If the fourth band is absent, it means the tolerance is 20%. The following is a color code chart, from which one can tell the resistance of a resistor:

Table 2 Resistor Color Codes

Color	1 st Digit	2 nd Digit	Power of 10	Multiplier	Tolerance
Black	0	0	0	1	-
Brown	1	1	1	10	-
Red	2	2	2	100	-
Orange	3	3	3	1,000	-
Yellow	4	4	4	10,000	-
Green	5	5	5	100,000	-
Blue	6	6	6	1,000,000	-
Violet	7	7	7	10,000,000	-
Gray	8	8	8	100,000,000	-
White	9	9	9	1,000,000,000	-
Gold	-	-			5%
Silver	-	-			10%
None	-	-			20%

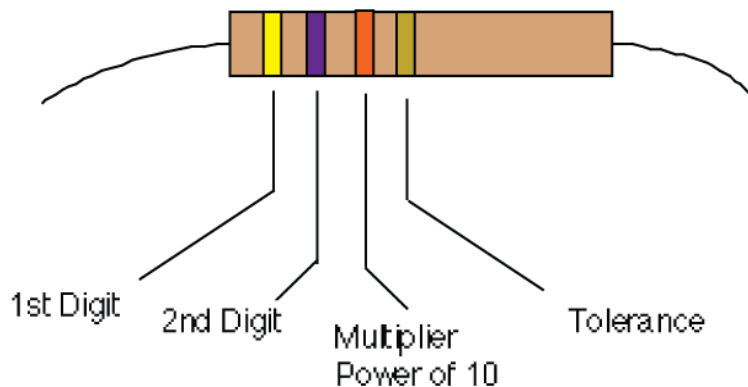


Figure 11 Resistor

EXAMPLE:

Suppose the color code on a resistor is yellow, violet, orange and gold like the resistor depicted above in Figure 11. What is its resistance and what is the uncertainty of this resistance?

The value of the resistance can be found from the first three colors. From the table above, the first digit is 4 (corresponding to the yellow band), the second digit is a 7 (corresponding to the violet band) and the power of 10 multiplier is 3 (corresponding to the orange band).

So, the resistance is:

$$\begin{array}{ccc} \text{second digit} & & \text{multiplier} \\ \text{first digit} & \searrow & \swarrow \\ & 47 \times 10^3 \Omega & \end{array}$$

The fourth color is used to calculate the uncertainty in the resistance. The tolerance of this resistor is 5% (corresponding to the gold band). So, the uncertainty of the resistor is:

$$\delta R = R \times \textit{tolerance} = 47,000 \Omega \times \frac{5}{100} = 2,350 \Omega$$

The resistance of this particular resistor is $47,000 \pm 2,000 \Omega$ or $47 \pm 2 \text{ k}\Omega$. Because the tolerance is only given to one significant figure, the uncertainty can only be known to one significant figure.

Spreadsheet for Practical Lab:

RC Practical Lab

R color code:

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Time	V _c	V _c /V ₀	ln(V _c /V ₀)
(sec)	(volts)		
0			
20			
40			
60			
80			
100			

Resistance (R) from color code =		±		(units)
C =	1000	±	200	μF
RC =		±		(units)
Slope of graph =		±		(units)
Time constant (τ) from graph =		±		(units)