PHY410 Homework Set 3

1. [10 pts] The following pertains to an ideal gas in different dimensions.

   (a) It is common to derive partition function of an ideal gas by considering noninteracting particles enclosed in a cubic box with side length L, see the textbook. The high-temperature results do not depend in fact, though, on any details in the shape of the macroscopic enclosure. Consider now a more general enclosure than in the textbook, in the form of a parallelepiped with side lengths of L_x, L_y and L_z. The energy values in the enclosure are $\epsilon_{\boldsymbol{n}} = \frac{\pi^2 \hbar^2}{2M} \left[\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2} \right]$. Determine the partition function Z_1 for this case, proceeding similarly to the textbook, and compare it to that obtained in the book.

   (b) [Kittel-Kroemer 3-11] Consider next an ideal gas of N particles, each of mass M, confined to an interval in one-dimension of length L. Find the entropy of that gas at temperature τ.

   (c) From the free energy F for the latter gas, find the force $f = -(\partial F/\partial L)_\tau$ that that ideal gas exerts onto the walls of the one-dimensional box confining the gas.

2. [10 pts] Consider two containers, adjacent to each other, each containing an ideal gas with N particles kept at the same temperature τ, but at different pressures p_1 and p_2.

   (a) By making use of the equation of state for an ideal gas, determine the pressure p that arises after an opening is made in the wall that separates those containers, allowing for a free flow of gas in-between.

   (b) Combine the Sackur-Tetrode formula with the ideal-gas equation of state and express the entropy of an ideal gas in terms of temperature, number of particles and pressure.

   (c) Find the entropy for the gas in the containers above, before and after the connection, and compute the change of entropy in the process.

3. [10 pts] First solve problem 3-4 in Kittel-Kroemer. Next, as a concrete example, consider the binary system of N magnetic dipoles m in contact with a reservoir at temperature τ. What is the mean square fluctuation of the total magnetization $\langle M^2 \rangle - \langle M \rangle^2$, when that system is placed in the magnetic field of magnitude B?

4. [5 pts] First solve problem 3-3 in Kittel-Kroemer. Use the result to find the average energy U of the harmonic oscillator.

5. [5 pts] Kittel-Kroemer, problem 3-7. Astoundingly, this simple model can be relevant to something as complex as DNA.

6. [10 pts] Kittel-Kroemer, problem 3-10. A one-dimensional polymeric chain is analogous to a binary spin system.