Homework Assignment #1 due Friday March 20

/1/ The standard (Dirac) representation of the gamma matrices is

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad \text{in 2x2 block diagonal form.}$$

The chiral (Weyl) representation is

$$\gamma_c^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \gamma_c^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

Let U be the unitary matrix such that $\gamma_c^{\mu} = U \gamma^{\mu} U^{+}$.

/a/ What is U? Prove $UU^{+} = 1$.

/b/ Show $U \gamma^0 U^{+} = \gamma_c^0$.

/c/ Show $U \gamma^i U^{+} = \gamma_c^i$.

/2/ Dirac spinors $v(p, \lambda)$ for antiparticles.

In class we derived the Dirac spinors, $u_{\alpha}(p,1)$ and $u_{\alpha}(p,2)$, for particles.

/a/ Use the standard (Dirac) representation for the gamma matrices. What are the Dirac spinors, $v_{\alpha}(p,1)$ and $v_{\alpha}(p,2)$, for antiparticles?

/b/ Prove $(i \gamma . \partial - m) \psi = 0$ for antiparticles.

/c/ Normalize the spinors by $\bar{v} v = - 2m$. Then determine $\Sigma_{\lambda} v(p, \lambda) \bar{v} (p, \lambda)$.

/d/ Derive the spinors, $v^{(c)}(p,1)$ and $v^{(c)}(p,2)$, for the chiral (Weyl) representation, from the unitary matrix U.