The standard model consists of three gauge theories.

Theory	Fields	Interactions (L_{int})
QED	ψ = (quarks and leptons)	$e \bar{\psi} \gamma^{\mu} \psi A^{\mu}$
	A^{μ} = (photon)	
QCD	ψ = (quarks)	$g \bar{\psi} \gamma^{\mu} \lambda_a/2 \psi A_a^{\mu}$
	A_a^{μ} = (gluons)	$g c_{abc} A_a^{\mu} A_b^{\nu} F^{(0)}_{c\mu\nu}$ and quartic
Electroweak	ψ = (quarks and leptons)	γ^{μ} *and* $\gamma^5\gamma^{\mu}$ *couplings*
	A_a^{μ} and B^{μ} = (SU_2 x U_1 gauge fields)	interactions between gauge fields
	Φ = (Higgs field)	Higgs field interactions

The simplest case is QED.

$$L = L_{\psi} + L_A + L_{int}$$

$$L_{\psi} = \overline{\psi} (i \gamma . \partial - m) \psi$$

$$L_A = -1/4 \, F_{\mu\nu} F^{\mu\nu}$$

$$L_{int} = e \, \overline{\psi} \, \gamma . A \, \psi$$

QCD and QED have similarities and differences.

Section 4.2: Perturbation Expansion for Correlation Functions

Equation (4.31) (familiar from PHY 855)

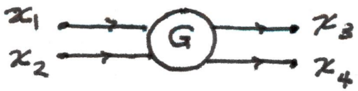
$$\langle \Omega | T\{ \phi(x) \phi(y) \} | \Omega \rangle = \lim_{T \to \infty} \frac{\langle 0 | T\{ \phi_1(x) \phi_1(y) e^{-i\int_{-T}^{T} dt\, H_1(t)} \} | 0 \rangle}{\langle 0 | T\{ e^{-i\int_{-T}^{T} dt\, H_1(t)} \} | 0 \rangle}$$

Correlation Functions

also called Green's functions, or N-point functions

Consider, for example, in the Heisenberg picture,

$$G = \langle \Omega | T\, \psi(x_3)\, \psi(x_4)\, \overline{\psi}(x_1)\, \overline{\psi}(x_2) | \Omega \rangle$$



Or, in the interaction picture,

$$G = \frac{\langle 0 | T\, \psi(x_3)\, \psi(x_4)\, \overline{\psi}(x_1)\, \overline{\psi}(x_1)\, U(T, -T) | 0 \rangle}{\langle 0 | T\, U(T, -T) | 0 \rangle}$$

where

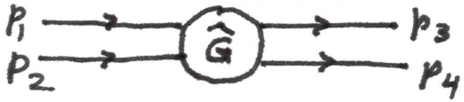
$$U(T, -T) = exp\{ -i \int_{-T}^{T} H_1\, dt' \}$$

Perturbation theory…

Expand U in powers of

$$H_1 = - \int L_{int}(\mathbf{x}', t')\, d^3x' .$$

Also, transform to momentum space,



$$\hat{G} = \int G\, e^{i(p3.x3 + p4.x4 - p1.x1 - p2.x2)}\, d^4x_1\, d^4x_2\, d^4x_3\, d^4x_4$$

Section 4.3: Wick's Theorem

Equation (4.38).

$$T\{ \varphi_i(x_1)\, \varphi_i(x_2)\, \varphi_i(x_3) \dots \varphi_i(x_m) \}$$

$$= N\{ \varphi_i(x_1)\, \varphi_i(x_2)\, \varphi_i(x_3) \dots \varphi_i(x_m) + \text{all possible contractions} \}$$

Corollary. A contraction is the propagator of the contracted fields.

Corollary.

$$\langle 0|\, T\{ \varphi_i(x_1)\, \varphi_i(x_2)\, \varphi_i(x_3) \dots \varphi_i(x_m) \}\, |0\rangle$$

= the sum of all *complete* contractions

Section 4.4: Feynman Diagrams

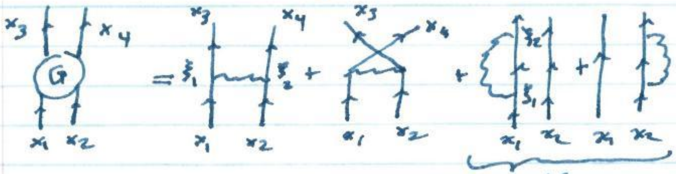
Feynman Rules for correlation functions in coordinate space

Combining perturbation theory for U(T,-T) [Eq. 4.31] and Wick's theorem [Eq. 4.38], we calculate fully contracted correlation functions (G) in the interaction picture. Later we will convert that to a transition probability amplitude ($\mathcal{M}$) and then a reaction cross section ($d\sigma/d\Omega$).

For example, here is $G^{(2)}(x_3 x_4 x_1 x_2)$

$$\langle 0 | T\, \psi(x_3)\, \psi(x_4)\, \bar\psi(x_1)\, \bar\psi(x_2)$$
$$\bar\psi e\gamma^\mu \psi A_\mu(\xi_1)\ \ \bar\psi e\gamma^\nu \psi A_\nu(\xi_2) | 0 \rangle\, d^4\xi_1\, d^4\xi_2 / 2!$$

Now apply Wick's theorem.
Draw all the full contractions



but these are disconnected diagrams, which cancel terms in denominator.

1st diagram = $S_F(x_3-\xi_1)\,(e\gamma^\mu)\,S_F(\xi_1-x_1)$
$S_F(x_4-\xi_2)\,(e\gamma^\nu)\,S_F(\xi_2-x_2)$
$D_{\mu\nu}(\xi_1-\xi_2)\ d^4\xi_1\ d^4\xi_2$

(1) Draw all topologically distinct connected diagrams.

(2) Vertex $= e\gamma^\mu$

(3) Fermion propagators

$$S_F(x-y) = \underbrace{\psi(x)\,\bar\psi(y)}$$

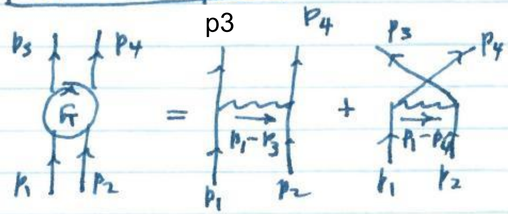
(4) Photon propagators

$$D_{\mu\nu}(\xi_1-\xi_2) = \underbrace{A_\mu(\xi_1)\,A_\nu(\xi_2)}$$

(5) Integral over vertex positions
$\int d^4\xi_1$ etc

Feynman Rules for correlation functions in momentum space

Momentum space



(1) Draw topologically distinct diagrams.

(2) Vertex $= e\gamma^\mu$; 4-momentum is conserved

(3) $\hat{S}_F(p) = \frac{i(\not{p}+m)}{p^2 - m^2 + i\epsilon}$

(4) $\hat{D}_{\mu\nu}(k) = \frac{i g_{\mu\nu}}{k^2 + i\epsilon}$ "Feynman gauge"

(5) overall 4-momentum conservation gives a factor $(2\pi)^4 \delta^4(P_f - P_i)$.

(6) $\int \frac{d^4 q}{(2\pi)^4}$ over loop momenta.

$$\hat{G} = \int G\, e^{i(p3.x3 + p4.x4 - p1.x1 - p2.x2)}\, d^4x_1\, d^4x_2\, d^4x_3\, d^4x_4$$

1st diagram = $S_F(x_3-\xi_1)(e\gamma^\mu)\,S_F(\xi_1-x_1)$
$S_F(x_4-\xi_2)(e\gamma^\nu)\,S_F(\xi_2-x_2)$
$D_{\mu\nu}(\xi_1-\xi_2)\;d^4\xi_1\;d^4\xi_2$

4-momentum conservation

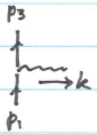
- Consider the vertex at ξ_1:

$$\int d^4x_3\,d^4x_1\,e^{ip_3\cdot x_3}\,e^{-ip_1\cdot x_1}\int d^4\xi_1$$
$$S_F(x_3-\xi_1)\,e\gamma^\mu\,S_F(\xi_1-x_1)\,D_{\mu\nu}(\xi_1-\xi_2)$$

$$=\int d^4x_3\,d^4x_1\,d^4\xi_1\;e^{ip_3\cdot(x_3-\xi_1)}\,e^{-ip_1(x_1-\xi_1)}\,e^{i(p_3-p_1)\cdot\xi_1}$$
$$S_F(x_3-\xi_1)\,e\gamma^\mu\,S_F(\xi_1-x_1)\;D_{\mu\nu}(\xi_1-\xi_2)$$

$$=\hat{S}_F(p_3)\,e\gamma^\mu\,\hat{S}_F(p_1)\int d^4\xi_1\,e^{i(p_3-p_1)\cdot\xi_1}$$
$$\int\frac{d^4k}{(2\pi)^4}\,e^{ik\cdot(\xi_1-\xi_2)}\,\hat{D}_{\mu\nu}(k)$$

$$\int d^4\xi_1\Rightarrow(2\pi)^4\,\delta^4(p_3-p_1+k)$$



$$=\hat{S}_F(p_3)\,e\gamma^\mu\,\hat{S}_F(p_1)\,\hat{D}_{\mu\nu}(p_1-p_3)\,e^{-i(p_1-p_3)\cdot\xi_2}$$

- Now the vertex at ξ_2:

$$\int d^4x_4\,d^4x_2\;e^{ip_4\cdot x_4}\,e^{-ip_2\cdot x_2}\;d^4\xi_2$$
$$S_F(x_4-\xi_2)\;e\gamma^\nu\;S_F(\xi_2-x_2)\;e^{-i(p_1-p_3)\cdot\xi_2}$$

$$=\hat{S}_F(p_4)\,e\gamma^\nu\,\hat{S}_F(p_2)\int d^4\xi_2\;e^{ip_4\cdot\xi_2}\,e^{-ip_2\cdot\xi_2}$$
$$e^{-i(p_1-p_3)\cdot\xi_2}$$

$$=\hat{S}_F(p_4)\,e\gamma^\nu\,\hat{S}_F(p_2)\;(2\pi)^4\,\delta^4(p_3+p_4-p_1-p_2)$$

∴ 1st diagram =

$\hat{S}_F(p_3)(e\gamma^\mu)\,\hat{S}_F(p_1)$

$\hat{S}_F(p_4)(e\gamma^\nu)\,\hat{S}_F(p_2)$

$D_{\mu\nu}(p_1-p_3)\qquad(2\pi)^4(p_3+p_4-p_1-p_1)$

But now we need Feynman Rules for
<u>transition probability amplitudes</u> .

First, how are *transition amplitudes* related to *correlation functions?*

How are N-point functions related to transition probability amplitudes?

Section 4.5 : Cross Sections and S-Matrix Amplitudes

Section 4.7 : Feynman Rules for Fermions

Section 4.8 : Feynman Rules for QED