Chapter 7 -- Radiative Corrections: some formal developments

7.1. Field-strength renormalization
7.2. The LSZ reduction formula
7.3. The optical theorem
7.4. The Ward-Takahashi identity
7.5. Renormalization of the electric charge

A quotation from Peskin & Schroeder, Chapter 7:

“We cheated four times in the last three chapters, stating (and sometimes motivating) a result but postponing its proof. Those results were:

1. decay rates
2. master formula
3. ward identity
4. subtraction for UV divergences

“

Regarding renormalization…

Paul Dirac wrote:
Sensible mathematics involves neglecting a quantity when it turns out to be small - not neglecting it just because it is infinitely great and you do not want it! (Paul Dirac)

Richard Feynman wrote:
But no matter how clever the word, it is what I call a dippy process! Having to resort to such hocus pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self consistent. ... I suspect that renormalization is not mathematically legitimate. (Richard Feynman, 1985)

Why are we willing to tolerate infinite renormalizations?

Why do we tolerate infinite renormalizations, in QED -- the most precise theory in physics?

$$e^2 = e_0^2 + e_0^4 \ln \frac{\Lambda^2}{m^2} + \text{higher order}$$

Why do we tolerate this?

$m = 0.5$ MeV ; need $\Lambda \gg m$

$\sqrt{s} = 14$ TeV at the LHC

$$M_{Planck} = \sqrt{\frac{\hbar c}{G}} = 1.2 \times 10^{19} \text{ GeV}$$

Planck mass:

$$\frac{G M_p^2}{r} = \frac{\hbar c}{r}$$

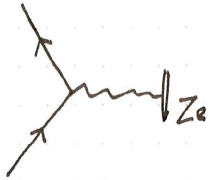
$$\ln \frac{M_p^2}{m_e^2} = 103$$

$$e^2 \sim \frac{1}{137} \sim 0{,}01$$

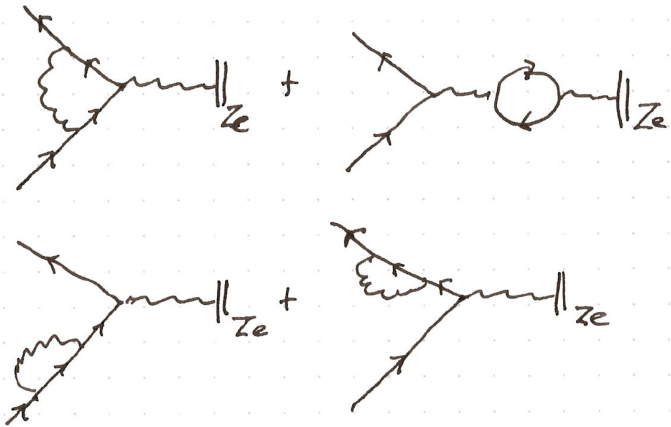
$$e^2 \ln \frac{M_p^2}{m_e^2} \sim 1 \quad (\text{not } \ll 1!)$$

Radiative corrections to electron scattering

Lowest order approximation
(LO ; or, the Born approximation)



First order radiative corrections
(or, the NLO calculation)



There are 3 loop diagrams

- . vertex function, Γ^{μ} =

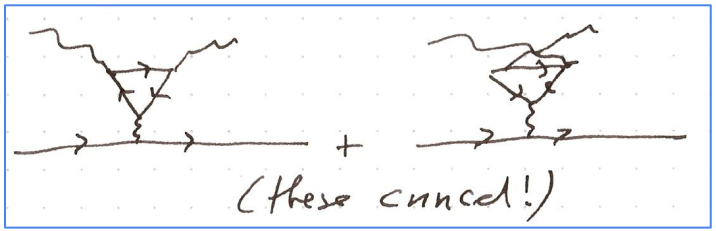
- . electron self-energy, Σ =

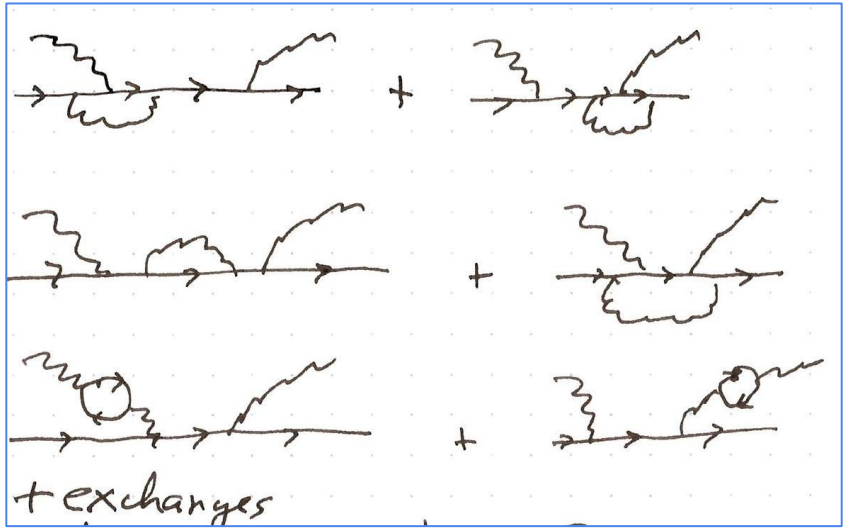
- . photon self energy; or vacuum polarization, $\Pi^{\mu\nu}$ =

All three are UV divergent.

An important point: The same UV-divergent loop diagrams appear in all scattering processes.

For example, consider the radiative corrections for Compton scattering.





An important point: Since the same UV-divergent loop diagrams appear in all scattering processes, *renormalization is universal.* i.e., it's not just a different renormalization calculation for each scattering process.)

Vacuum polarization

Vacuum polarization affects the photon propagator.

For this reason, vacuum polarization is sometimes called “photon self energy”.

But the photon mass remains 0 in the interacting theory. For this reason I don’t like the term “photon self-energy”.

(The photon mass is 0 in all orders of perturbation theory because of gauge invariance. That is a fundamental symmetry of nature.)

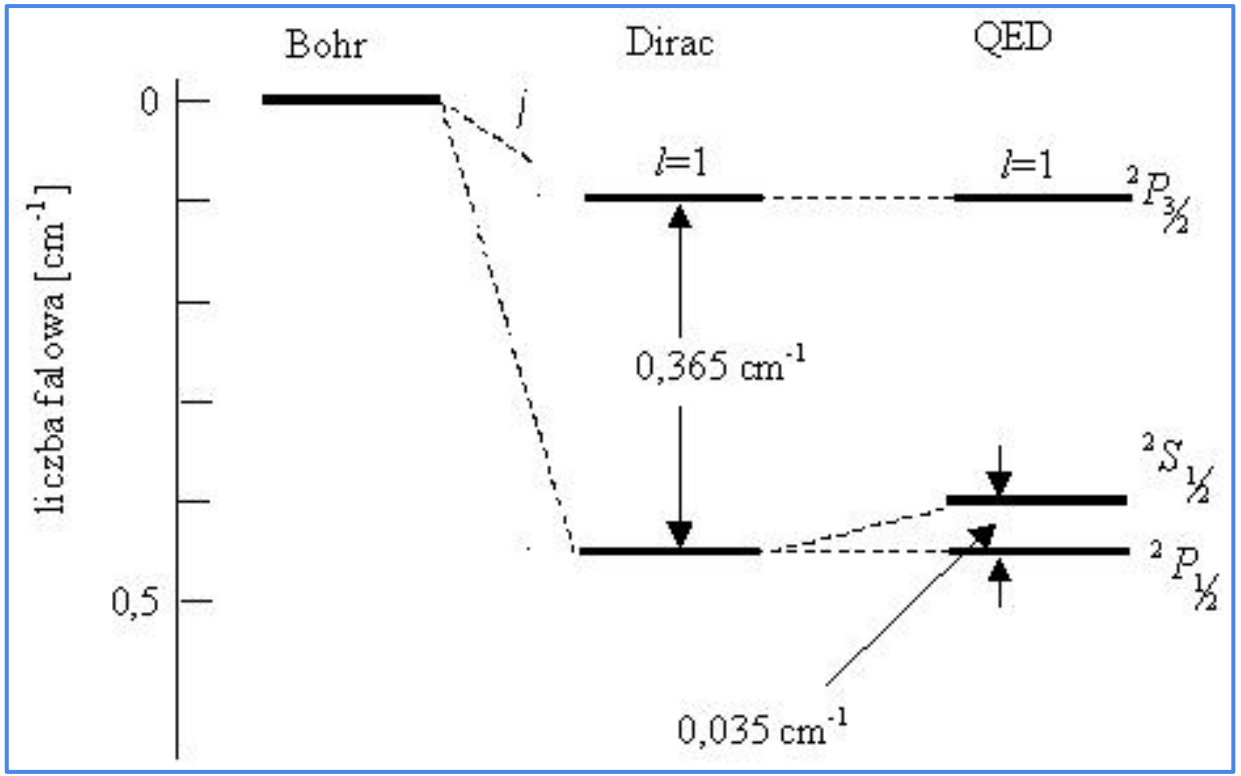
The free field propagator is

That is the photon propagator in a theory with no charged fields. (not very interesting!)

The lowest order correction is

The Lamb shift

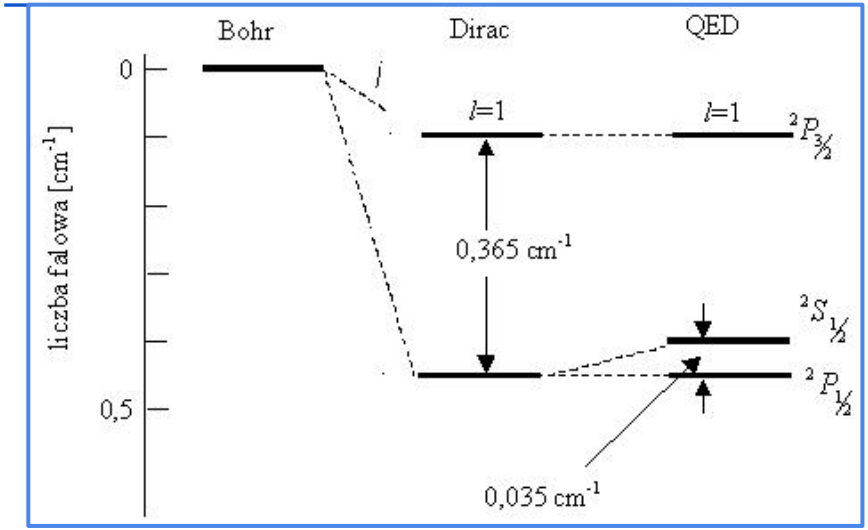
Consider the n = 2 states of hydrogen



text copied from a web site:

"The technique used was quite interesting. They made a **beam of Hydrogen atoms in the state**, which has a very long lifetime because of selection rules. **Microwave radiation** with a (fixed) frequency of 2395 MHz was used to cause transitions to the state and a **magnetic field was adjusted to shift the energy of the states** until the rate was largest. The **decay of the state to the ground state was observed** to determine the transition rate. From this, they were able to deduce the shift between the and states."

http://dydaktyka.fizyka.umk.pl/Wystawy_archiwum/z_omegi/lamba-en.html



Lamb's shift: a subtle structure of the n=2 level in hydrogen according to Bohr's, Dirac's and QED with assumption Lamb's shift.
The Lamb shift removes the degeneration due to quantum number *j*.

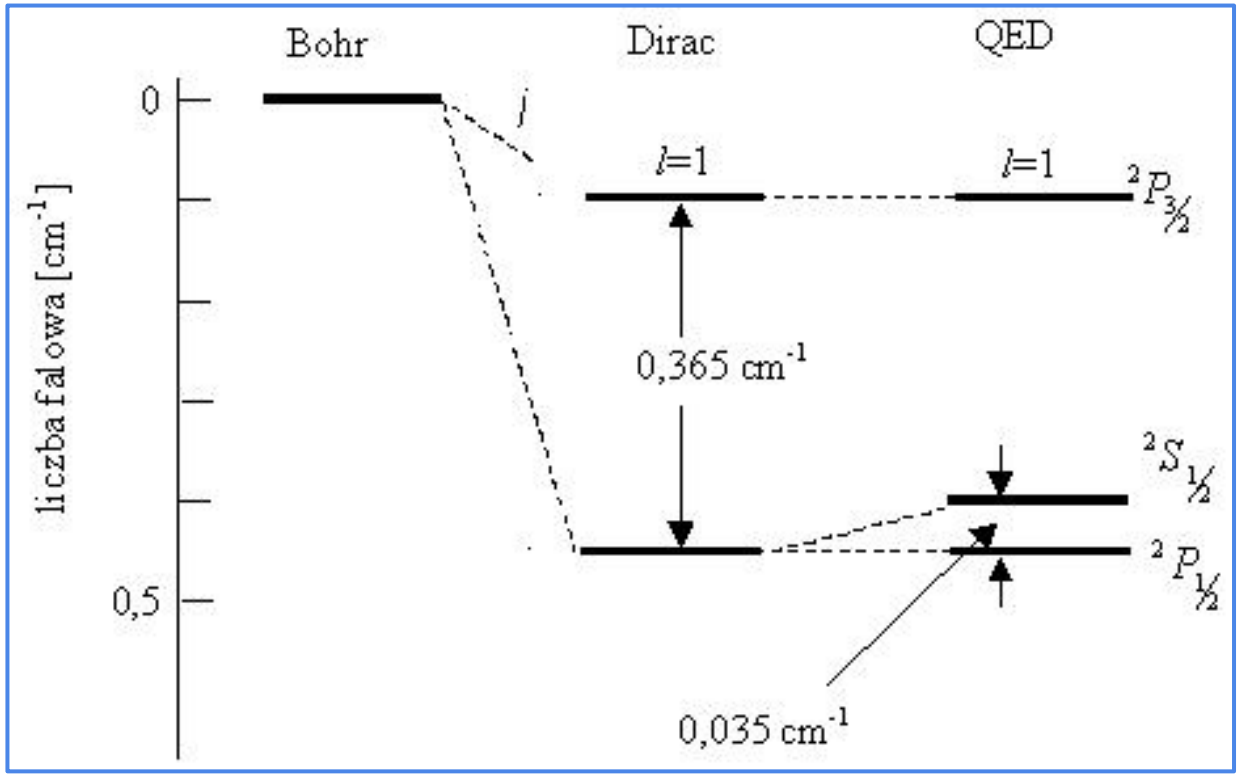
Lamb's shift

W hen it seemed that about hydrogen atom we knew almost everything in 1947 W.E. Lamb and R.C. Retherford decided to check results of Dirac. They used microwaves technique, available from the constructions of radar
The Lamb's shift*, a minimal difference in lowest energetic level of the excited hydrogen atom can't be explained in any way without introduction of the absolutely new concept in Physics: **Quantum Electrodynamics** .

The essential of the QED is that vacuum is never empty, but filled with virtual particles – appearing suddenly and then quickly disappearing. Processes like those do not violate the energy conservation principle: the loan of energy from *nowhere* is very short, like it does the bank employee, who brings back in the morning the money borrowed in the evening.

Part of the Lamb shift (not all!) is due to the vacuum polarization correction to the photon propagator.

Homework Problem 16.


Bohr
Dirac
QED
0
0,5
liczba falowa [cm-1]
j
l=1
l=1
2P3/2
0,365 cm-1
2S1/2
2P1/2
0,035 cm-1