Citation:
K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014) (URL: http://pdg.lbl.gov)

The muon

$J = 1/2$

Mass $m = 0.1134289267 \pm 0.0000000029$ u

Mass $m = 105.6583715 \pm 0.0000035$ MeV

Mean life $\tau = (2.1969811 \pm 0.0000022) \times 10^{-6}$ s

$\tau(\mu+) / \tau(\mu-) = 1.00002 \pm 0.00008$

$c\tau = 658.6384$ m

Magnetic moment anomaly

$(g-2)/2 = (11659209 \pm 6) \times 10^{-10}$

$(g_{\mu+} - g_{\mu-}) / g_{average} = (-0.11 \pm 0.12) \times 10^{-8}$

Electric dipole moment

$d = (-0.1 \pm 0.9) \times 10^{-19}$ e cm

The muon

Decay parameters [b]

$\rho = 0.74979 \pm 0.00026$

$\eta = 0.057 \pm 0.034$

$\delta = 0.75047 \pm 0.00034$

$\xi P_\mu = 1.0009 \quad + 0.0016 / - 0.0007$ [c]

$\xi P_\mu \ \delta/\rho = 1.0018 + 0.0016 / - 0.0007$ [c]

$\xi' = 1.00 \pm 0.04$

$\xi'' = 0.7 \pm 0.4$

$\alpha/A = (0 \pm 4) \times 10^{-3}$

$\alpha'/A = (-10 \pm 20) \times 10^{-3}$

$\beta/A = (4 \pm 6) \times 10^{-3}$

$\beta'/A = (2 \pm 7) \times 10^{-3}$

$\acute{\eta} = 0.02 \pm 0.08$

μ^- DECAY MODES

(μ^+ modes are charge conjugates of the modes below.)

	Fraction ($\Gamma i / \Gamma$)	Confidence level	p(MeV/c)
$e^- \ \bar{\nu}_e \ \nu_\mu$	$\approx 100\%$		53
$e^- \ \bar{\nu}_e \ \nu_\mu \ \gamma$	[d] $(1.4\pm0.4)\%$		53
$e^- \ \bar{\nu}_e \ \nu_\mu \ e^- e^+$	[e] $(3.4\pm0.4) \times 10-5$		53

Lepton flavor violating modes

e- γ	$< 5.7 \times 10^{-13}$	90%	
e- e-e+	$< 1.0 \times 10^{-12}$	90%	
e- 2γ	$< 7.2 \times 10^{-11}$	90%	

Properties of the muon

mass = 106 MeV
charge = -e

Particle decays obey certain conservation laws.

From what we understand today, the following quantities must be conserved,

without exception,

- energy and momentum
- angular momentum
- electric charge
- color quantum number

In theory, the muon can decay in many ways.

These decay processes are allowed, within the above list of conservation laws:

- ❏ $\mu^- \rightarrow e^- +$ even # neutrinos

  $M_\mu > m_e$; m_ν is very small ; so energy and momentum can be conserved ;
  charge OK;
  ang. momentum OK.

- ❏ $\mu^- \rightarrow e^- +$ photons

  $M_\mu > m_e$; $m_\gamma = 0$; so energy and momentum can be conserved ;
  charge OK;
  ang. momentum OK.

Decays that cannot occur.

$\mu^- \to e^-$ cannot occur;
it cannot satisfy both momentum conservation and energy conservation.

Consider $\vec{P}_\mu = 0$; $E_\mu = M_\mu$;

then $\vec{p}_e = 0$; $E_e = m_e$ not equal

$\mu^- \to \tau^-$ + neutrinos cannot occur;
it cannot satisfy energy conservation because $M_\tau > M_\mu$.

Consider $\vec{P}_\mu = 0$; $E_\mu = M_\mu$;

but $E_\tau + E_{neutrinos} > M_\tau > M_\mu$ not equal

Neutrinos

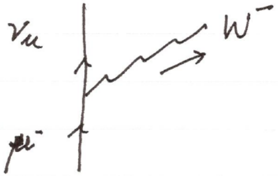
We know three distinct kinds of neutrinos

ν_e
ν_μ
ν_τ

In weak interactions, e always interacts with ν_e ;

$$\mathcal{L}_{int} = g\,\bar{\psi}_e\,\gamma^\lambda(1-\gamma_5)\,\psi_{\nu_e}\,W_\lambda + h.c.$$

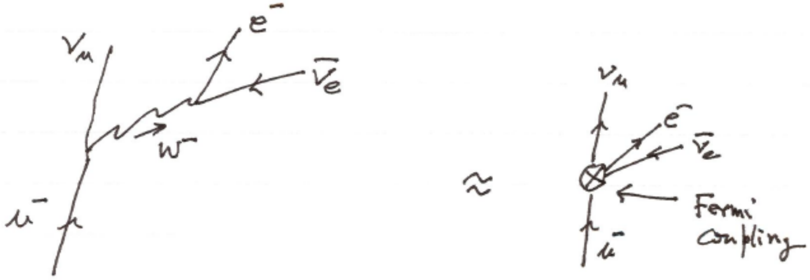
In weak interactions, μ always interacts with ν_μ ;



$$\mathcal{L}_{int} = g\,\bar{\psi}_\mu\,\gamma^\lambda(1-\gamma_5)\,\psi_{\nu_\mu}\,W_\lambda + h.c.$$

The dominant decay mode

$\mu^- \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$



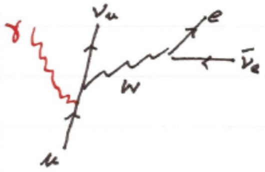
$$\frac{1}{\tau} = \Gamma = \frac{G_F^2 \, m_\mu^5}{192\pi^3}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = 1.166 \times 10^{-5} \ \mathrm{GeV}^{-2}$$

$(\hbar = 1, c = 1)$

Two rare decays are listed

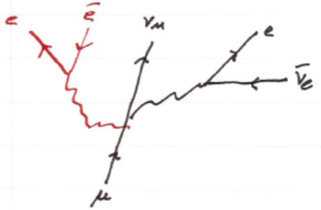
$e^- \ \bar{\nu}_e \ \nu_\mu \ \gamma$



+ 2 other F. diagrams

BR = 1.4 / 100

$O(\alpha)$



+ 2 other F. diagrams

BR = 3.5 / 10^5

$O(\alpha^2)$

According to our knowledge of the electroweak interactions, there is another conservation law -- Lepton Flavor Conservation.

- Lepton Flavor Conservation

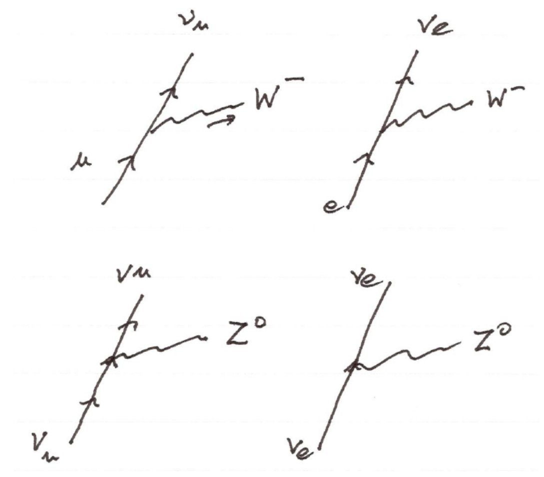
$$N_e = N(e-) + N(\nu_e) - N(e+) - N(\bar{\nu}_e)$$
$$N_\mu = N(\mu-) + N(\nu_\mu) - N(\mu+) - N(\bar{\nu}_\mu)$$
$$N_\tau = N(\tau-) + N(\nu_\tau) - N(\tau+) - N(\bar{\nu}_\tau)$$

all three are constant.

For example,

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

Electroweak gauge interactions in the Standard Model are symmetric with respect to the Lepton Flavor quantum numbers.



For a long time we believed that the LF numbers are conserved.
But they are not.

Neutrino Oscillations

- ν_e, ν_μ, ν_τ are not mass eigenstates

- ν_1, ν_2, ν_3 are mass eigenstates, with masses = m_1 m_2 m_3

$$(\Delta m_{21})^2 = 7.65 \pm 0.22 \times 10^{-5}\ eV^2$$

$$(\Delta m_{31})^2 = \pm 2.40 \pm 0.12 \times 10^{-3}\ eV^2$$

If we know that LF number is conserved, then the decay $\mu \rightarrow e\,\gamma$ ***cannot*** occur.

But we know that LF number is not conserved. So the challenge is to observe the decay $\mu \rightarrow e\,\gamma$.

To be continued ...