

## Experiment 4

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# Newton's Second Law

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### 4.1 Objectives

- Test the validity of Newton's Second Law.
- Measure the frictional force on a body on a “low-friction” air track.

### 4.2 Introduction

Sir Isaac Newton's three laws of motion laid the foundation for classical mechanics. While each one is important, in today's lab we will test the validity of Newton's Second Law,  $F = ma$ .<sup>1</sup> This equation says that when a force acts on an object, it will accelerate and the amount of acceleration is dependent on the object's mass.

### 4.3 Key Concepts

You can find a summary on-line at Hyperphysics.<sup>2</sup> Look for keywords: Newton's laws

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<sup>1</sup>Newton's First Law states that an object in a state of uniform motion tends to remain in that state of motion unless an external force is applied. Newton's Third Law states that for every action there is an equal and opposite reaction.

<sup>2</sup><http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html>

## 4.4 Theory

Newton's Second Law states that the acceleration of a body is proportional to the net force acting on the body ( $a \propto F_{NET}$ ) and inversely proportional to the mass of the body ( $a \propto \frac{1}{m}$ ). Combining these two, we can replace the proportionality with equality. That is,

$$a = \frac{F_{NET}}{m}$$

or

$$F_{NET} = ma$$

$F_{NET}$  is the **sum of all of the forces** acting on the body. In many textbooks this is denoted by  $\sum F$ . So, **Newton's Second Law** is:

$$\sum_{i=1}^n F = ma \tag{4.1}$$

The standard SI unit for force is kg m/s<sup>2</sup>, which is given the name Newton (N). In this lab it will be more convenient to make measurements in grams (g) and centimeters (cm) so you will be using the cgs system of units instead. **Note: In the cgs system of units, the unit for force is the dyne:**

$$1 \text{ dyne} = 1 \text{ g cm/s}^2$$

In this experiment a low friction air track will be used to test the validity of Newton's Second Law. A hanging mass will be attached to a glider placed on the air track by means of a light (negligible mass) string. By varying the amount of mass that is hanging we will vary the net force acting on this two body system. While doing this we will make sure to keep the total mass of the two body system constant by moving mass from the glider to the hanger. With the air track turned on, the hanging mass will be released and the glider will pass through two photogate timers. The photogate timers will be used to measure two velocities. Recall that  $v = \frac{\Delta x}{\Delta t}$ . In our case  $\Delta x$  will be the length of a fin placed on top of the glider. If you know the separation between the two photogate timers, you can use the following equation to determine the acceleration of the glider:

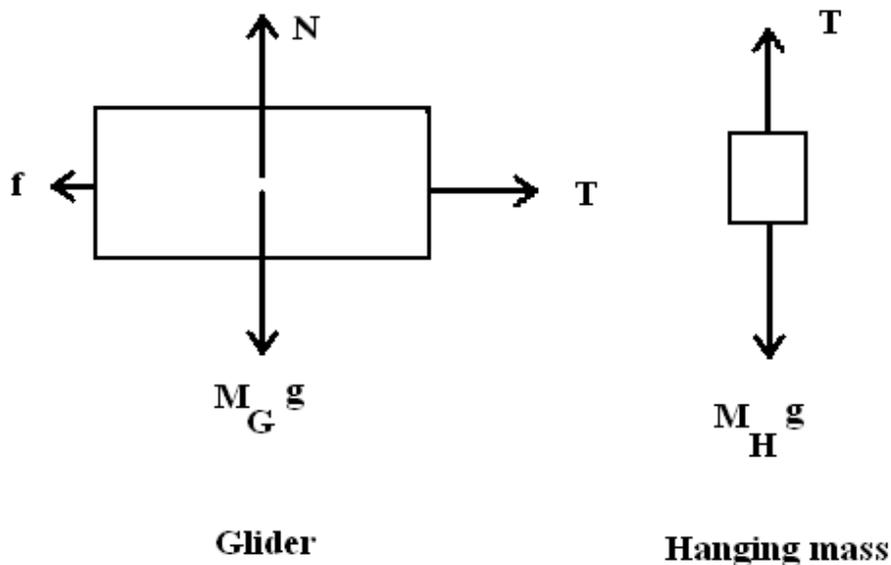


Figure 4.1: Freebody diagrams

$$v_2^2 = v_1^2 + 2aS$$

where  $v_2$  is the velocity measured with the **second photogate**,  $v_1$  is the velocity measured with the **first photogate**,  $a$  is the **acceleration** and  $S$  is the distance **between the two photogate timers**. Solving for the acceleration yields:

$$a = \frac{v_2^2 - v_1^2}{2S} \quad (4.2)$$

Free body diagrams of the forces acting on the glider and hanging mass are shown in Fig. 4.1. In the figure,  $f$  is the net **frictional** force acting on the glider (we will assume this includes both the frictional force between the airtrack and the glider and between the string and the pulley);  $N$  is the upward force (called the “**normal force**”) the air track exerts on the

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glider;  $T$  is the **tension** in the string;  $F_G = M_G * g$  is the **weight**<sup>3</sup> of the **glider**; and  $F_H = M_H * g$  is the **weight** of the **hanging mass** where  $g$  is the acceleration due to gravity. Since the air track is horizontal and the glider does not accelerate in the vertical direction, the normal force and the weight of the glider are balanced,  $N = M_G * g$ . Applying Newton's Second Law to the glider in the horizontal direction and using to the right as the positive direction yields:

$$T - f = M_G * a \quad (4.3)$$

where  $a$  is the acceleration of the glider in the horizontal direction. Applying Newton's Second Law to the hanging mass and defining **downward** as the positive direction yields:

$$M_H * g - T = M_H * a \quad (4.4)$$

where  $a$  is the acceleration of the hanging mass in the vertical direction. Note that since the glider and hanging mass are attached to each other by a string they are both moving with the same acceleration  $a$  and feel the same tension  $T$ .

We have no way of directly measuring the tension  $T$  in the string, but if we combine Equations 4.3 and 4.4 the tension can be eliminated. Solve Eq. 4.3 for  $T$  and plug it into Eq. 4.4 to get:

$$M_H * g - M_G * a - f = M_H * a \quad (4.5)$$

Then rearrange the equation and notice that the weight of the hanging mass,  $M_H * g$ , can be written as  $F_H$  giving:

$$F_H - f = (M_H + M_G) * a \quad (4.6)$$

There are only two unbalanced forces acting on our two-mass system (i.e. the weight of the hanging mass,  $F_H$ , and friction,  $f$ ). Notice what Eq. 4.6 states: the left-hand side is the net force and the right-hand side is the product of the system's mass (the mass of the glider plus the hanging mass)

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<sup>3</sup>In these equations I am explicitly including an asterisk (\*) to indicate multiplication to avoid confusion between the letters used as subscripts to indicate the object ( $G$  for glider and  $H$  for hanger) and the letters used as variables ( $g$  for acceleration due to gravity and  $a$  for acceleration).

and its acceleration. This is Newton's Second Law,  $\Sigma F = ma$ , applied to our two body system.

Let's rearrange equation 4.6 to obtain:

$$F_H = (M_H + M_G) * a + f \quad (4.7)$$

Equation 4.7 has the same form as the equation of a straight line  $y = mx + b$ , where the weight of the hanging mass ( $F_H$ ) plays the role of  $y$  and the acceleration ( $a$ ) plays the role of  $x$ . Before coming to lab you should figure out what physical quantities the slope and  $y$ -intercept correspond to. If you don't remember how to match up the equations refer back to the "Introduction to Computer Tools and Uncertainties" lab and look at Step 1 in the section called "Plotting a best-fit line".

## 4.5 In today's lab

Today we will use an almost frictionless air track to measure how force affects acceleration. We will measure the velocity of the glider while keeping the total mass of the system the same (total mass = mass of hanger + mass of glider). We'll redistribute the mass by moving it from the glider to the hanger. Using the velocities at both photogates, we will then be able to find the acceleration of the cart.

## 4.6 Equipment

**Do not move the glider on the track while the air is turned off!**

- Air track
- Glider
- Hanger
- 2 Photogates
- 5g, 10g, 20g masses

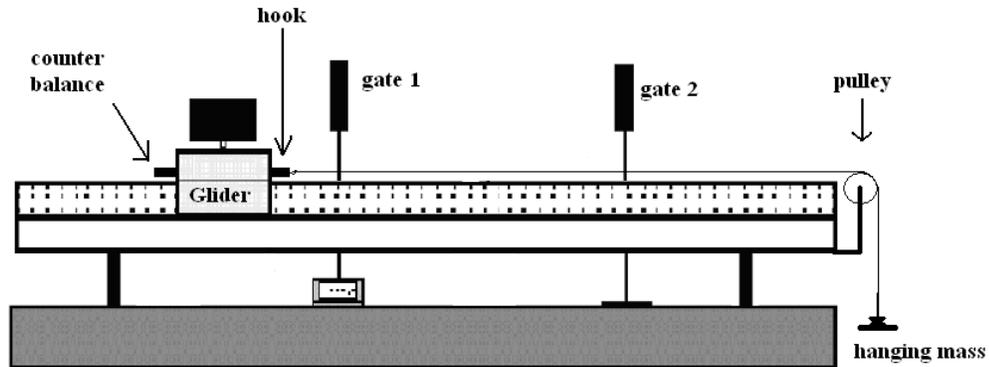


Figure 4.2: Diagram of the apparatus

## 4.7 Procedure

1. Set up the air track as shown in Figure 4.2. With the hanging mass disconnected from the glider and the air supply on, level the air track by carefully adjusting the air track leveling feet. The glider should sit on the track without accelerating in either direction. There may be some small movement due to unequal air flow beneath the glider, but it should not accelerate steadily in either direction.
2. Measure the length ( $L$ ) of the fin on top of the glider and record it along with its uncertainty in your spreadsheet. See Figure 4.3 for a definition of various lengths that will be used throughout this experiment.
3. Make sure the hook and counter balance are both inserted in the **lower** hole on the glider.
4. Measure the mass of the glider ( $M_{G_0}$ ) and empty hanger ( $M_{H_0}$ ) and record these masses in your spreadsheet. In today's lab you will neglect any uncertainties in measuring the masses as they are negligible compared to the uncertainty in the acceleration.
5. Using the 5, 10 and/or 20 gram masses, place 40 grams of mass on the glider. Make sure to distribute the masses **symmetrically** so that the

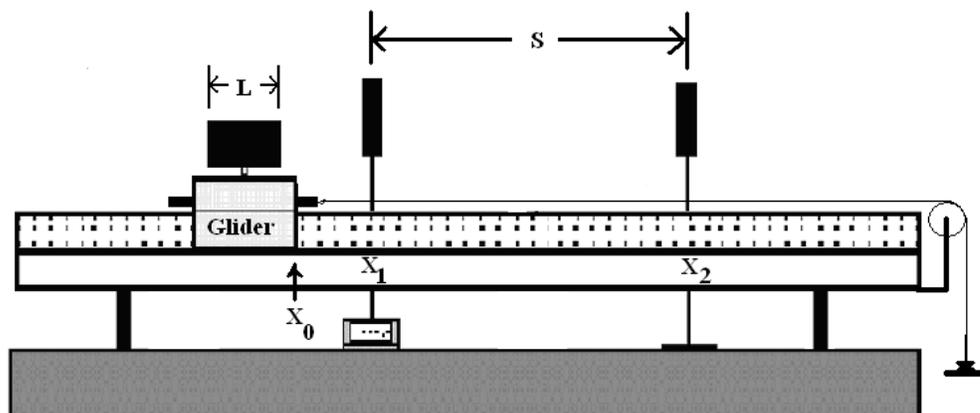


Figure 4.3: Definition of various lengths used throughout this experiment.

glider is balanced on the track and not tipping to one side. Determine the total mass of the glider ( $M_{G_0}$  + the mass you just added) and record this in your spreadsheet in the column labeled  $M_G$ . (See the attached spreadsheet.)

6. Place 10 grams of mass on the hanger. Record this in your spreadsheet in the column labeled “Mass added to hanger” and then have Excel calculate the total mass of the hanger ( $M_H = M_{H_0} +$  the mass you just added to the hanger) in the column labeled  $M_H$ .
7. Note that the total mass of your system ( $M_G + M_H$ ) should remain constant throughout the experiment and always be equal to the value entered next to “Total system mass ( $M_{H_0} + M_{G_0} + 50$ ):”. You are just redistributing 50 grams of mass between the glider and the hanger during the experiment.
8. Next you need to choose a starting position for the glider ( $X_0$ ) and the locations of the two photogate timers ( $X_1$  and  $X_2$ ). See Figure 4.3. When choosing these positions make sure to adhere to the following guidelines:
  - The hanging mass must **not** touch the ground before the cart moves past the second photogate at position  $X_2$ .

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- The position of the first photogate  $X_1$  must be at least 20 cm away from the second photogate.
  - The starting position  $X_0$  of the glider must be at least 25 cm away from the first photogate at  $X_1$ .
9. Using the ruler permanently affixed to the air track, record the locations of  $X_0$ ,  $X_1$  and  $X_2$  in your spreadsheet and assign a reasonable uncertainty to these positions ( $\delta X$ ). **It is very important that your glider always starts from the same location  $X_0$  and that the two photogates are not moved. If they are accidentally bumped or moved, return them to their original location.**
  10. Calculate the magnitude of the displacement  $S$  between the photogates using  $S = |X_2 - X_1|$  and record this in your spreadsheet. (Remember  $S$  needs to be  $> 20$  cm.) Calculate the uncertainty in  $S$  using  $\delta S = 2\delta X$ .
  11. Set your photogate timer to **GATE** mode and make sure the memory switch is set to **ON**. The **GATE** mode will only record time when the glider is passing through one of the two photogates. In this mode, the timer will only display the time the glider took to pass through the first photogate ( $t_1$ ). The time the glider took to pass through the second photogate will be added to the memory. Flipping the toggle switch will display the total time ( $t_{mem}$ ) the glider took to pass through both photogates. To obtain the time the glider took to pass through just the second photogate  $t_2$ , subtract  $t_1$  from the time stored in the photogate's memory so  $t_2 = t_{mem} - t_1$ . The uncertainty in a measurement of time using the photogates is  $\delta t = 0.5$  milliseconds (ms). Using the rules for addition and subtraction of errors, the uncertainty in  $t_2$  (which is gotten by subtracting 2 measured times) is  $\delta t_2 = 2\delta t = 1.0$  ms.
  12. With the air supply on, hold the leading edge of the glider stationary at  $X_0$ ; press the reset button on the photogate timer, then release the glider. Catch the glider after it has passed all the way through the second photogate. Make sure that the hanger doesn't crash into the floor and that the glider does not bounce off the end of the air track and pass back through the second photogate. The time displayed on the photogate's screen will be the time the glider took to pass through the first photogate ( $t_1$ ). Record  $t_1$  in your spreadsheet. Flip

the memory toggle switch and record the total time ( $t_{mem}$ ) to pass through both photogates in your spreadsheet. Calculate  $t_2$  and its uncertainty.

13. Return the glider to  $X_0$ . Make sure all of the masses are still on the hanger and the string is still over the pulley. Then move 10 grams from the glider to the hanger **making sure to keep the masses distributed symmetrically on the glider**.
14. Calculate the new values for the total glider mass ( $M_G$ ) and hanger mass ( $M_H$ ). Check that the total mass of the system ( $M_G + M_H$ ) has not changed. Repeat steps 12–13.
15. Keep taking measurements, moving 10 grams at a time, until no masses remain on the glider. You should have data for 10g, 20g, 30g, 40g and 50g added to your hanger.
16. Have Excel calculate  $v_1$ ,  $v_2$  and their respective uncertainties using the following equations:

$$v_i = \frac{L}{t_i}$$

and

$$\delta v_i = v_i \left( \frac{\delta L}{L} + \frac{\delta t_i}{t_i} \right)$$

17. Use Equation 4.2 to calculate acceleration ( $a$ ) in Excel. This equation was derived assuming that the point where the instantaneous velocity  $v_2$  was measured was a distance  $S$  away from the point where the instantaneous velocity  $v_1$  was measured. This is not quite true, as you measured an average velocity over the length of the fin, and introduces a small systematic error into our calculations. By keeping  $S > 20$  cm and  $|X_1 - X_0| > 25$  cm, this systematic error is  $< 0.5\%$ . The contribution due to this systematic uncertainty is included in the calculation of  $\delta a$ .
18. The formula for the uncertainty in the acceleration ( $\delta a$ ) has already been programmed into Excel for you but looks like this:

$$\delta a = a \left[ \frac{\delta S}{S} + \frac{2(v_2 \delta v_2 + v_1 \delta v_1)}{(v_2^2 - v_1^2)} \right]$$

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19. Have Excel calculate the weight  $F_H = M_H * g$  (where  $g = 981 \text{ cm/s}^2$ ) for each of the hanging masses.
20. Transfer your data into KaleidaGraph and make a graph of  $F_H$  vs.  $a$ . Make sure you include **horizontal error bars** associated with the acceleration ( $a$ ) and fit your graph with a best-fit line. Note that your error bars will **not** have a constant value. You will need to import the  $\delta a$  data column into KaleidaGraph. When adding the error bars rather than choosing Fixed Value from the Error Bars Settings window, you must select Data Columns and choose the data column containing your values for  $\delta a$ .

### 4.8 Checklist

1. Excel sheets (both data view and formula view).
2. Plot including **horizontal** error bars and best-fit line.
3. Answers to questions.



